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M. Sawan and R.W. Conn

Fusion Technology Institute
University of Wisconsin
1500 Engineering Drive
Madison, WI 53706

http://fti.neep.wisc.edu

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by

Mohamed Sawan
and
Robert Conn

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Nuclear Engineering Department
The University of Wisconsin
Madison, Wisconsin 53706

A consistent model for the energy distribution of secondary neutrons at energies above the binding energy per nucleon must consider (n,2n) and (n,n') reactions simultaneously. Previous calculational models have treated the two reactions independently.[1-4] A consistent model is presented here which reveals an inconsistency in models used in ENDF/B.

(n,2n) reactions involve the successive emission of neutrons from a highly excited compound nucleus. The first neutron is emitted at an energy much lower than the incident energy leaving the residual nucleus in a highly excited state from which a second neutron can be emitted. For such a sequential decay, the first neutron emitted is identical to an inelastically scattered neutron and the evaporation model can be used to calculate its energy distribution. We make the assumption in this work that in the cases when a neutron energy of E_o is inelastically scattered below $E_o - E_{th}^{2n}$, (E_{th}^{2n} is the (n,2n) threshold energy) the residual nucleus will emit another neutron rather than decay by gamma emission. This is an excellent assumption except within a few hundred eV of the threshold energy.[5] Therefore, the energy distribution of the first neutron, normalized to unity, is given by a Maxwellian distribution extending from $E = 0$ to $E = E_o - E_{th}^{2n}$. This distribution is

$$f_{2n}^{I}(E_o \rightarrow E) = (Ee^{-E/T_1})H(E_o - E_{th}^{2n} - E)/[T_1^2 - T_1\{E_o - E_{th}^{2n} + T_1\} \cdot \exp(-(E_o - E_{th}^{2n})/T_1)]. \qquad (1)$$

Here, T_1 is the nuclear temperature corresponding to incident energy E_o, and H is the Heaviside function.

The emission of the first neutron leaves the target nucleus with a level density given by

$$W(E) = f_{2n}^{I} (E_o \to E_o - E).\tag{2}$$

The distribution of the excitation energy, which is available as kinetic energy of the second emitted neutron and as excitation energy of the residual nucleus, is then given by

$$U(E) = f_{2n}^{I}(E_o \to E_o - E_{th}^{2n} - E), \quad 0 \le E \le E_o - E_{th}^{2n}.\tag{3}$$

The remaining problem is analogous to one in which a neutron source with energy distribution, $U(E)$, and mean energy, $E_o - E_{th}^{2n} - 2T_1$, is inelastically scattered by a nucleus of mass number, A-1. Using the appropriate inelastic scattering kernel, the energy distribution of the second neutron is given by

$$f_{2n}^{II}(E_o \to E) = \int_{o}^{E_o - E_{th}^{2n}} dE' \, U(E') \, f_{in}(E' \to E).\tag{4}$$

This kernel must also be normalized to unity. The evaporation model can be used to calculate the energy distribution of the second neutron if $(E_o - E_{th}^{2n} - 2T_1)$ is much greater than the inelastic threshold energy.[5] In such a case, the energy distribution of the second neutron is given by

$$f_{2n}^{II}(E_o \to E) = (Ee^{-E/T_2})H(E_o - E_{th}^{2n} - E)/[T_2^2 - T_2\{E_o - E_{th}^{2n} + T_2\}\exp(-(E_o - E_{th}^{2n})/T_2)]\tag{5}$$
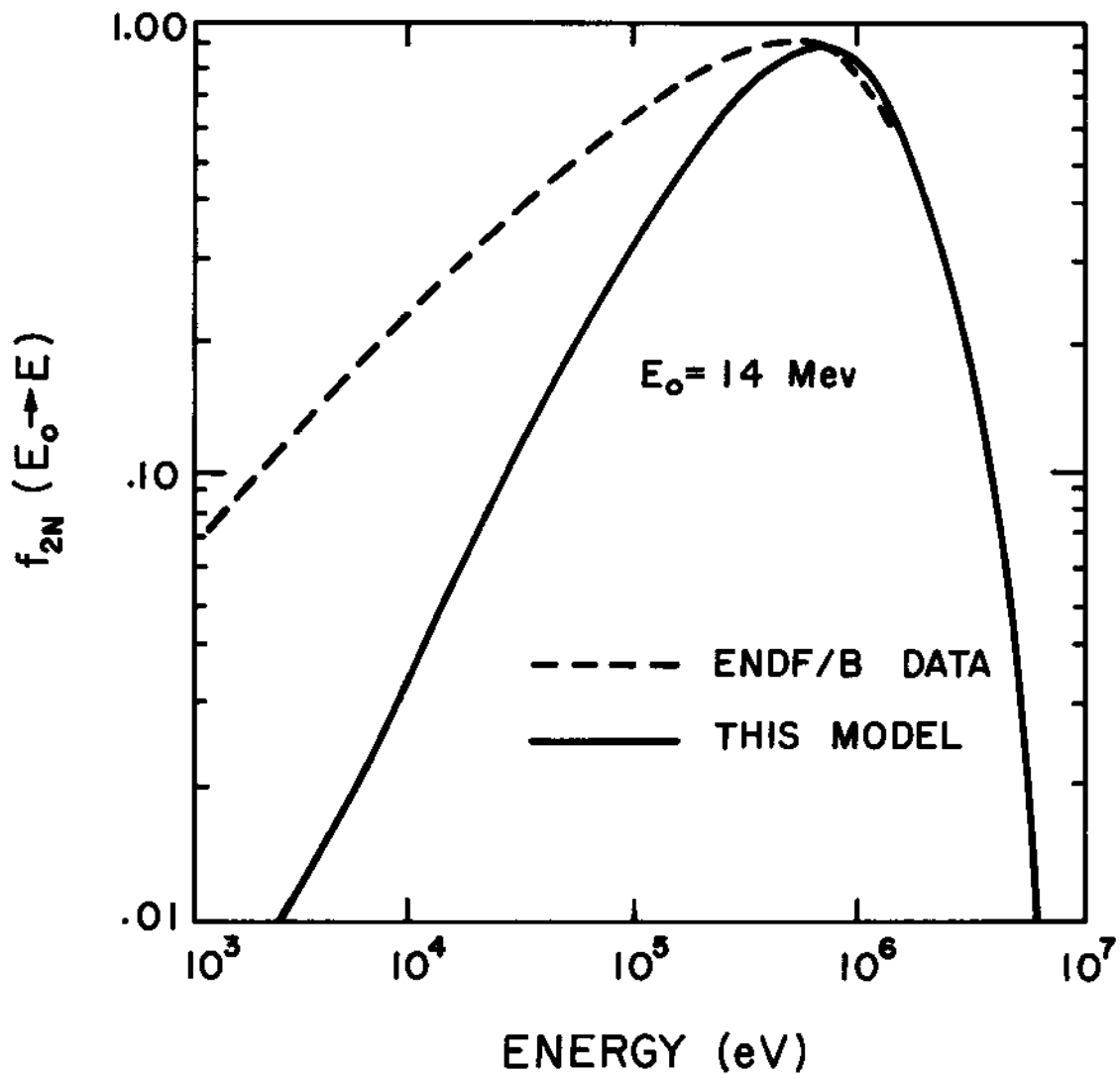
Figure 1 - The energy distribution of neutrons emitted in (n,2n) reactions induced by 14 MeV neutrons in lead as given by ENDF/B-II and the model developed here.

where T_2 is the nuclear temperature corresponding to an incident energy, $E_o - E_{th}^{2n} - 2T_1$.

Segev[2] has given a multigroup formulation for the (n,2n) scattering kernel which agrees to some extent with our results. In the Segev model, T_2 is considered to be the nuclear temperature when the incident neutron energy is $E_o - E_{th}^{2n}$. This is true only if $E_o - E_{th}^{2n} \gg 2T_1$. The calculational procedure suggested by Henryson[3] contains an inconsistency in the specified energy limits. The calculation procedure given by Odette[4] agrees in large part with the procedure developed here. However, no analytic expression was developed for $f_{2n}^{II}(E_o \rightarrow E)$. The description in the ENDF/B data files for treatment of the (n,2n) reaction suggest a model which assumes the simultaneous emission of both neutrons from one compound nucleus. Generally, two nuclear temperatures are provided to account for the distinct spectra of the two neutrons. The difference in the secondary neutron spectra from the (n,2n) reaction between the model discussed here and that used in ENDF/B is shown in Figure 1 for lead. The incident neutron energy is taken to be 14 MeV. It is clear that ENDF/B (n,2n) model would predict too soft a spectrum in lead.

At energies $E_o > E_{th}^{2n}$, where both (n,n') and (n,2n) reactions are energetically possible, the inelastic scattering kernel, $f_{in}(E_o \rightarrow E)$, should involve only that part of the Maxwellian for which $E_o - E_{th}^{2n} \leq E \leq E_o$. This is essential for consistent calculations because it has been assumed that when a neutron of energy E_o is inelastically scattered

below $E_o - E_{th}^{2n}$, the residual nucleus will emit a second neutron. Thus, a consistent inelastic scattering kernel is

$$f_{in}(E_o \rightarrow E) = Ee^{-E/T_1} H(E-E_o+E_{th}^{2n})/[T_1(T_1+E_o-E_{th}^{2n})$$

$$\exp(-E_o-E_{th}^{2n})/T_1) - T_1(T_1+E_o)\exp(-E/T_1)]. \qquad (6)$$

The average energy for this distribution is

$$<E> = 2T_1 + \frac{\{(E_o-E_{th}^{2n})^2 e-(E_o-E_{th}^{2n})/T_1 - E_o^2 e^{-E_o/T_1}\}}{\{(T_1+E_o-E_{th}^{2n})e^{-(E_o-E_{th}^{2n})/T_1} - (T_1+E_o)e^{-E_o/T_1}\}} \qquad (7)$$

The current ENDF/B evaluations treat the secondary neutron spectrum for inelastic scattering as being a complete Maxwellian even for $E_o > E_{th}^{2n}$. The average energy for such a distribution is

$$<E>_{ENDF/B} = 2T_1 - \frac{E_o^2 e^{-E_o/T_1}}{T_1 - (E_o+T_1)e^{-E_o/T_1}}. \qquad (8)$$

At $E_o = 14$ MeV, eqn. (8) gives an average energy of 2.03 MeV for lead whereas the procedure presented here would yield, from eqn. (7), 8.37 MeV. This implies that the use of ENDF/B (n,n') data would also predict too soft a spectrum for the secondary distribution compared to the results of the consistent model developed here. However, this problem with the (n,n') reaction will not lead to significant errors in transport calculations since $\sigma(n,n')$ is generally much less than $\sigma(n,2n)$ when the latter reaction is energetically possible.

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