Nuclear Fundamentals

1.) Definitions

$\alpha, \beta, \gamma, \nu$, **n, p**

2.) Mass Energy Relationships

Particles;

$$E_o = m_o c^2$$

E_o is the energy equivalent to rest mass (m_o) of particle,

c = speed of light, ≈ 3 x 10^8 m/s

Photons

$$E = h\nu$$

h = Planck's constant = 6.626 x 10^{-34} joule-s

ν = frequency of radiation, s^{-1}

deBroglie equation,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{hc}{E} = 12.4 \frac{Å}{E(keV)}$$

3.) Radioactive Decay of Unstable Isotopes, $^{A}X_{Z}$

Particles

$$^{A}X_{Z} \rightarrow {}^{A-4}X_{Z-2} + {}^{4}\alpha_{2}$$

(parent) (daughter)

$$^{238}Pu_{94} \rightarrow {}^{234}U_{92} + {}^{4}He_{2}$$

From nuclear reactor physics we find;

$$E_{\alpha} = \frac{E}{\left[1 + \left(\frac{4.0026}{A_{d}}\right)\right]}$$

$$E = Q_{a} + E_{p} - E_{d}$$

E	Q_a	E_p	E_d
Total transitional energy	ΔE between ground state of parent & daughter	Excitation energy of α emitting level in parent	Excitation of daughter nucleus fed by decay

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Electron Events

Too many n's

$$^{1}n_{o} \rightarrow {}^{1}p_{1} + {}^{0}\beta_{-1} + \bar{\nu}_{e} \qquad {}^{A}X_{Z} \rightarrow {}^{A}Y_{Z+1} + {}^{0}\beta_{-1} + \bar{\nu}_{e}$$

Too few n's

$^{1}\boldsymbol{p}_{1} \rightarrow {}^{1}\boldsymbol{n}_{0} + {}^{0}\beta_{+1} + \nu_{e}$ $\quad$ $^{A}\boldsymbol{X}_{Z} \rightarrow {}^{A}\boldsymbol{Y}_{Z-1} + {}^{0}\beta_{+1} + \nu_{e}$

Too few n's

$$^{1}\boldsymbol{p}_{1} + {}^{0}\beta_{+1} \rightarrow {}^{1}\boldsymbol{n}_{0} + \nu_{e} \quad {}^{A}\boldsymbol{X}_{Z} + {}^{0}\beta_{-1} \rightarrow {}^{A}\boldsymbol{Y}_{Z-1} + \nu_{e}$$

Example $^{238}\boldsymbol{Pu}_{94}$ decay chain

4.) <u>Law of Radioactive Decay</u>

$$\frac{\boldsymbol{dN}}{\boldsymbol{dt}} = -\lambda \boldsymbol{N}$$

N= # of radioactive atoms

λ = decay constant $= \dfrac{\mathbf{0.693}}{\boldsymbol{t}_{\frac{1}{2}}}$

$\boldsymbol{t}_{\frac{1}{2}}$ **= half life**

1 Curie = 3.7 x 10^{10} disintegrations per second

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Can rewrite decay equation:

$$\int dN/N = \int \lambda\, dt$$

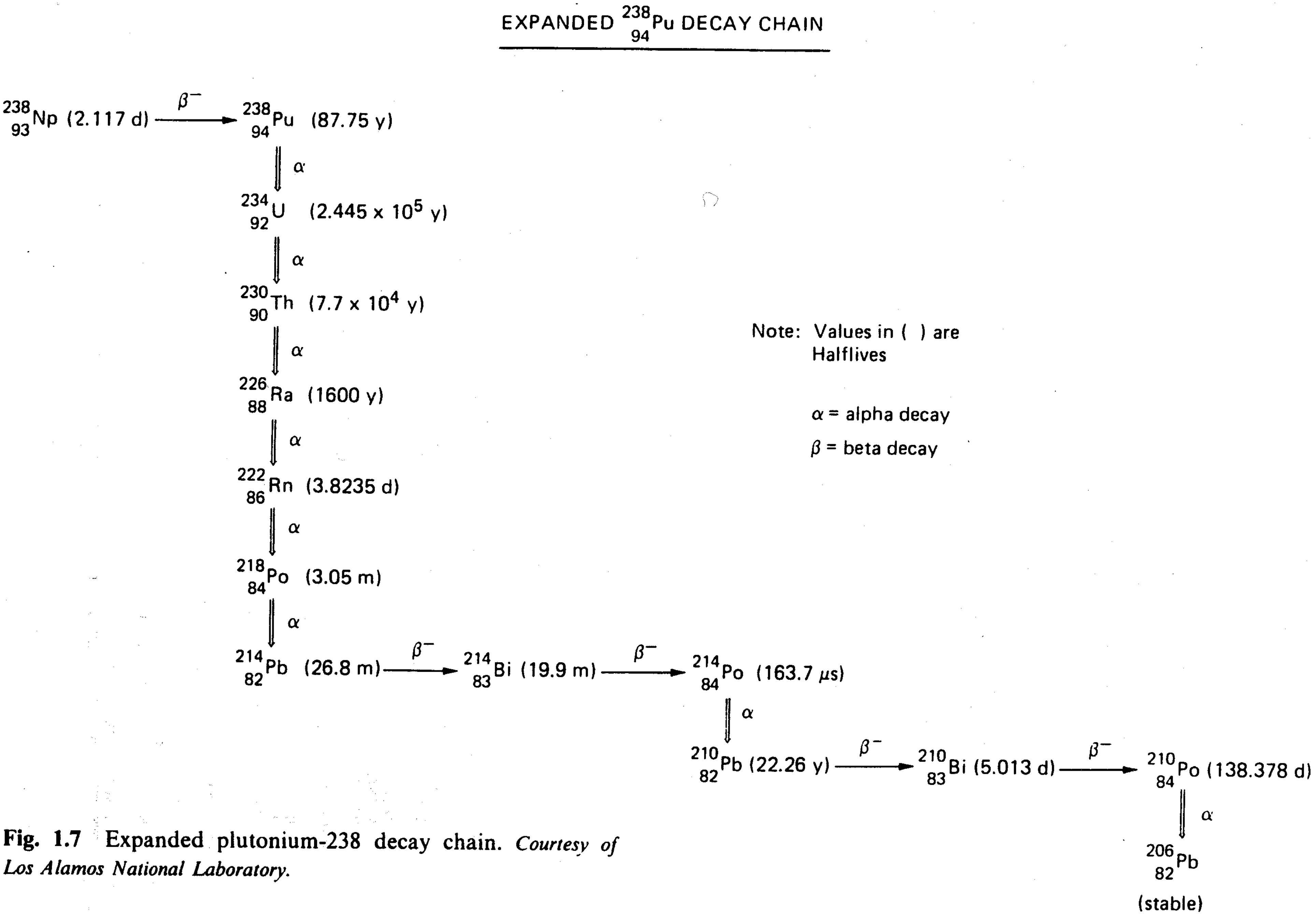

Fig. 1.7 Expanded plutonium-238 decay chain. *Courtesy of Los Alamos National Laboratory.*

which gives

$$N = N_o \exp(-\lambda t)$$

Can also express the instantaneous power released:

$$Q(t) = Q_o \exp(-\lambda t)$$

Problem

Assume a radioisotope system which contains 1 kg of PuO_2 (90% ^{238}Pu) which initially operates at 0.4 W/g of ^{238}Pu. What is the thermal power after 10 years?

A.) $t_{\frac{1}{2}} = 87.75$ y

$$\lambda = \frac{0.693}{87.75} = 0.0079\ y^{-1}$$

B.)

$$Q_o = \{1000\ g\ PuO_2 \bullet \left(\frac{238\ g\ Pu}{270\ g\ PuO_2}\right) \bullet \left(\frac{0.9\ 238Pu}{Pu}\right)\} \bullet \left[\frac{0.4\ W}{g\ 238\ Pu}\right]$$

$$= 317\ \text{Watts}$$

C.) After 10 years;

$$Q = 317 \bullet \exp(-0.0079 \times 10)$$
$$= 293\ \text{Watts}$$

5.) Cross Sections

Interaction probability for a given reaction by a particle traveling through a given medium;

microscopic, σ cm^2 (1 barn = 10^{-24} cm^2)

macroscopic, $\Sigma = N\sigma$ cm^{-1}

so,

$\Sigma\, dx$ = probability of interaction per unit track length

Removal of particles from a given state;

$$-d\Phi(x) = \Phi(x)\,\Sigma\, dx$$

Or,

$$\Phi(x) = \Phi_0 \exp(-\Sigma x)$$

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Mean distance before undergoing a reaction:

$$\lambda = 1/\Sigma$$

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6.) Neutron Spectra

When neutrons, which are emitted during the fission process, slow down and are thermalized, they come close to a Maxwell -Boltzmann distribution.

$$n(E) = \frac{dn}{dE}\left\{\frac{2\pi n}{(\pi kT)^{\frac{3}{2}}}\right\} \bullet \exp\left(-\frac{E}{kT}\right) \bullet \sqrt{E}$$

Figures 2.12 & 2.13

@ T= 20°C most probable thermal velocity is 2200 m/s which give E = 0.0252 eV

For;

$E_n < 0.0252$ eV	cold neutrons
$1 < E_n < 100$ eV	resonance n’s
$0.1 < E_n < 1$ keV	intermediate n’s
$0.5 < E_n < 15$ MeV	fast neutrons

Lots of Cross Sections!

Figure 2.14

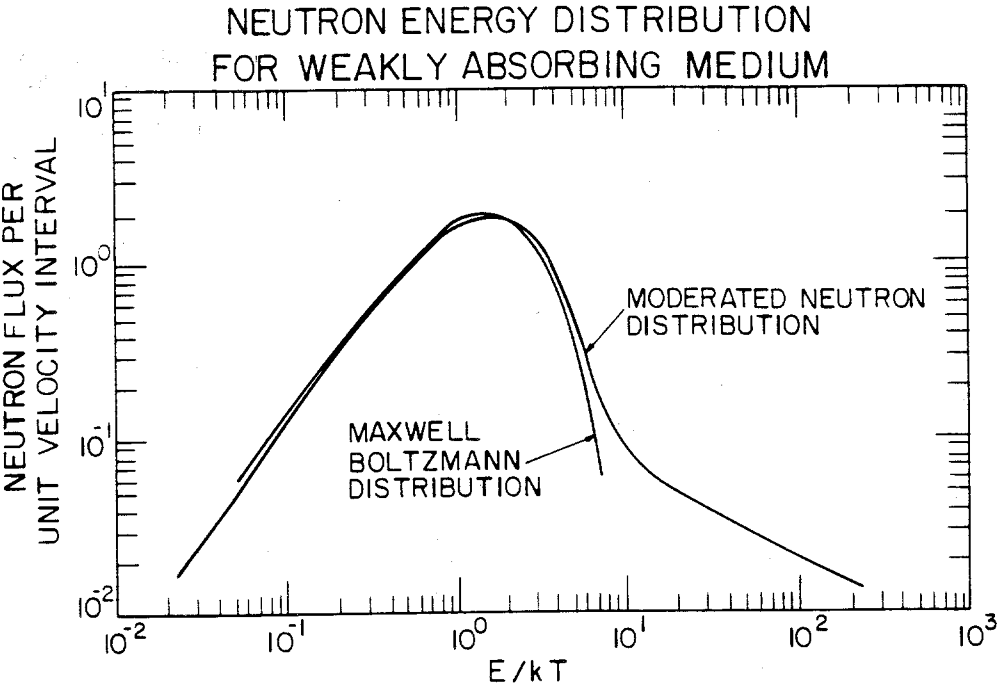

Fig. 2.12 Neutron energy distribution for weakly absorbing medium. *Courtesy of Oak Ridge National Laboratory* [*4*].

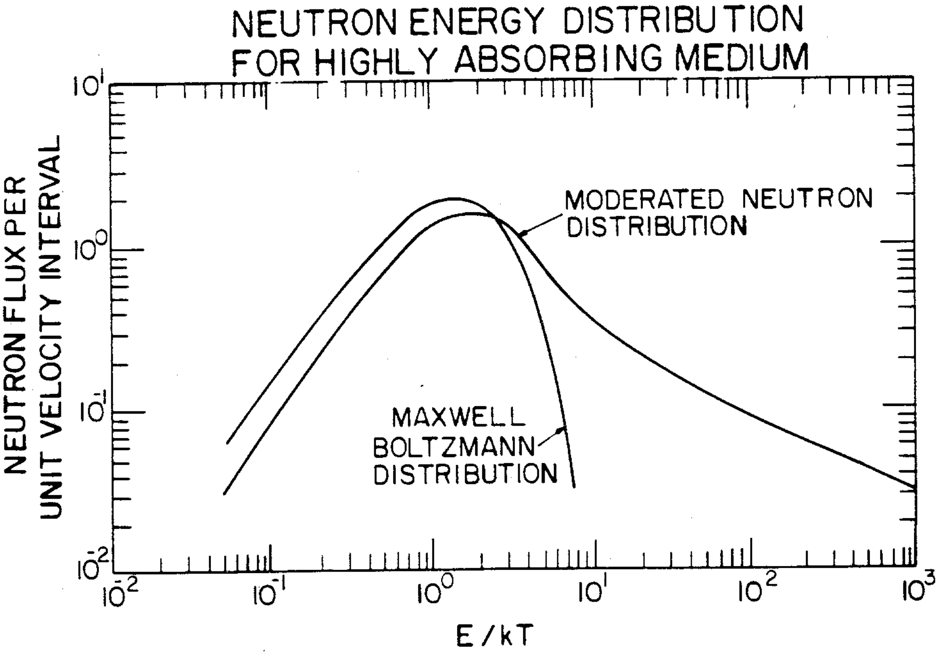

Fig. 2.13 Neutron energy distribution for highly absorbing medium. *Courtesy of Oak Ridge National Laboratory* [*4*].

7.) Neutron Slowing Down

Elastic scattering useful for dropping the n energy from the MeV range to the eV range where fissioning is more probable (σ_{fiss} is the highest)

$$E_{min} = \left[\frac{(A-1)}{(A+1)}\right]^2 E_o = \alpha E_o$$

A = At. mass of nuclide/At. mass of neutron

Logarithmic Energy Decrement

$$\xi = \ln\left(\frac{E_o}{E}\right) = 1 + \frac{(\alpha \bullet \ln \alpha)}{(1-\alpha)}$$

Would like high values of ξ

See Table 2.3

8.) Production of Isotopes

Formation rate = $\Phi\sigma_{act}N - \lambda N$

where:

σ_{act} is cross section for producing isotope Y from parent X

N = no. of parent X atoms per cc.

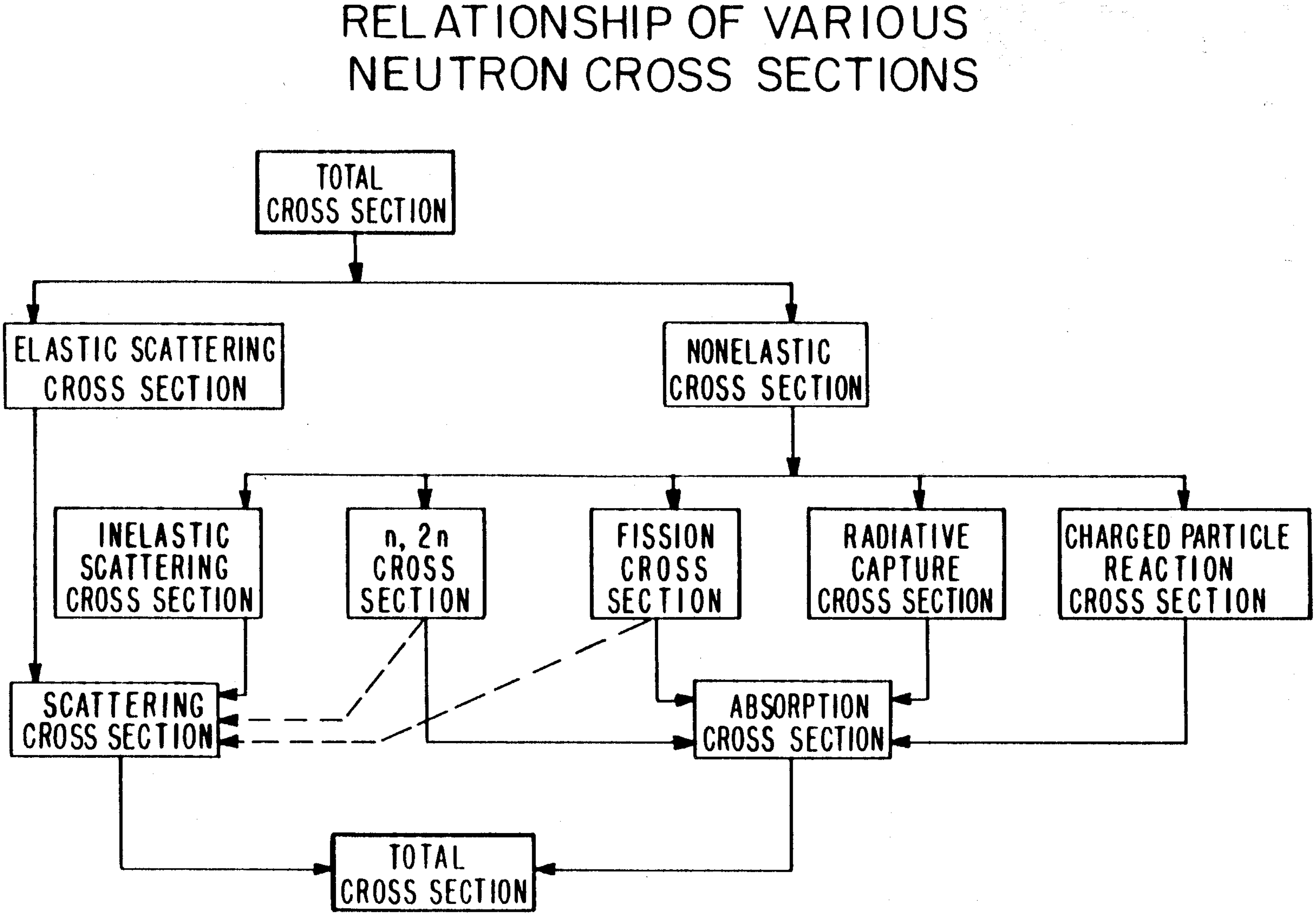

Fig. 2.14 Relationship of various neutron cross sections. *Courtesy of Oak Ridge National Laboratory* [4].

Table 2.3 Typical neutron collision parameters.

Material	Atomic Mass (A)	Normal Density (g/cm^3)	ξ	α
Hydrogen (H)	1	8.9×10^{-5} (gas)	1.000	0
Lithium (Li)	7	0.534	0.268	0.563
Beryllium (Be)	9	1.85	0.209	0.640
Carbon (C)	12	1.60	0.158	0.716
Oxygen (O)	16	0.0014 (gas)	0.120	0.779
Sodium (Na)	23	0.971	0.083	0.840
Iron (Fe)	56	7.86	0.0357	0.931
Tungsten (W)	184	19.2	0.0108	0.979
Uranium (U)	238	19.1	0.0084	0.983
Plutonium (Pu)	239	19.6	0.0083	0.983

However, if isotope Y is radioactive, then N* is the number of Y atoms per cc at any time.

$$\text{decay rate} = \lambda N^*$$

Net production of Y:

$$\frac{dN^*}{dt} = \Phi\sigma_{act}N - \lambda N$$

Can show that,

$$N^* = \left[\left(\frac{\Phi\sigma_{act}N}{\lambda}\right)\right] \bullet \left[1 - \exp(-\lambda t_r)\right]$$

Activity after the neutrons are turned off:

$$\lambda N^* = \Phi\sigma_{act}N \bullet \left[1 - \exp(-\lambda t\ \ )\right] \bullet \exp(-\lambda t_r)$$

Where t_r = time after removal from the n flux

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9.) Some Observations on Reactor Operation

$$\text{Define } k = \frac{\text{\# of fissions (or n's) in one generation}}{\text{\# of fissions (or n's) in the immediately preceding generation}}$$

k<1 subcritical

k =1 critical
k> 1 supercritical

Figure 3.2

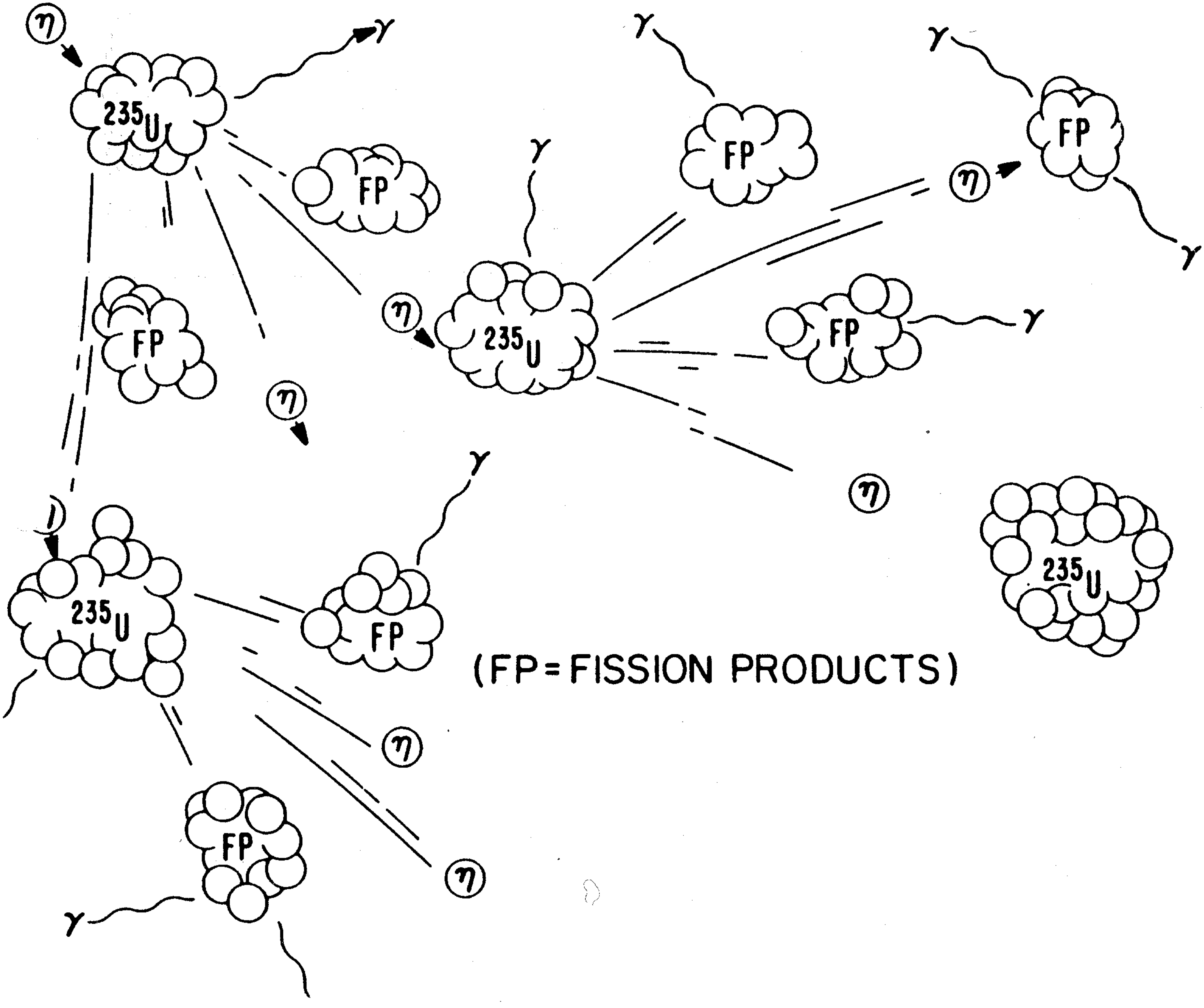
Description of space reactor - Figure 3.3

Fissionable elements-Tables 3.1 & 3.2

Energy released - Table 3.3

Fission Products -Figure 3.4

CHAIN REACTION



(FP = FISSION PRODUCTS)

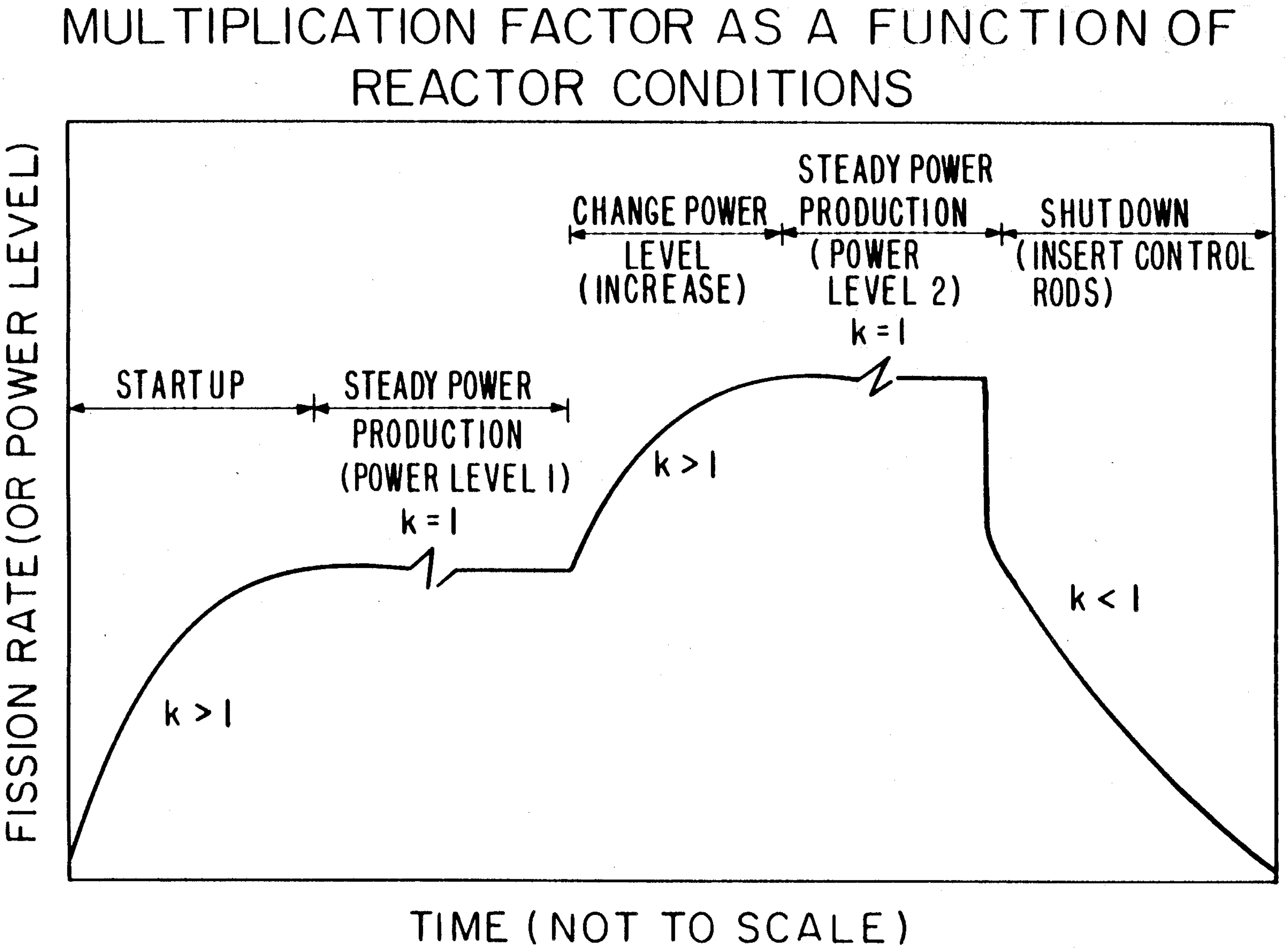


Fig. 3.2 Multiplication factor as a function of reactor conditions.

GENERIC SPACE NUCLEAR REACTOR SYSTEM

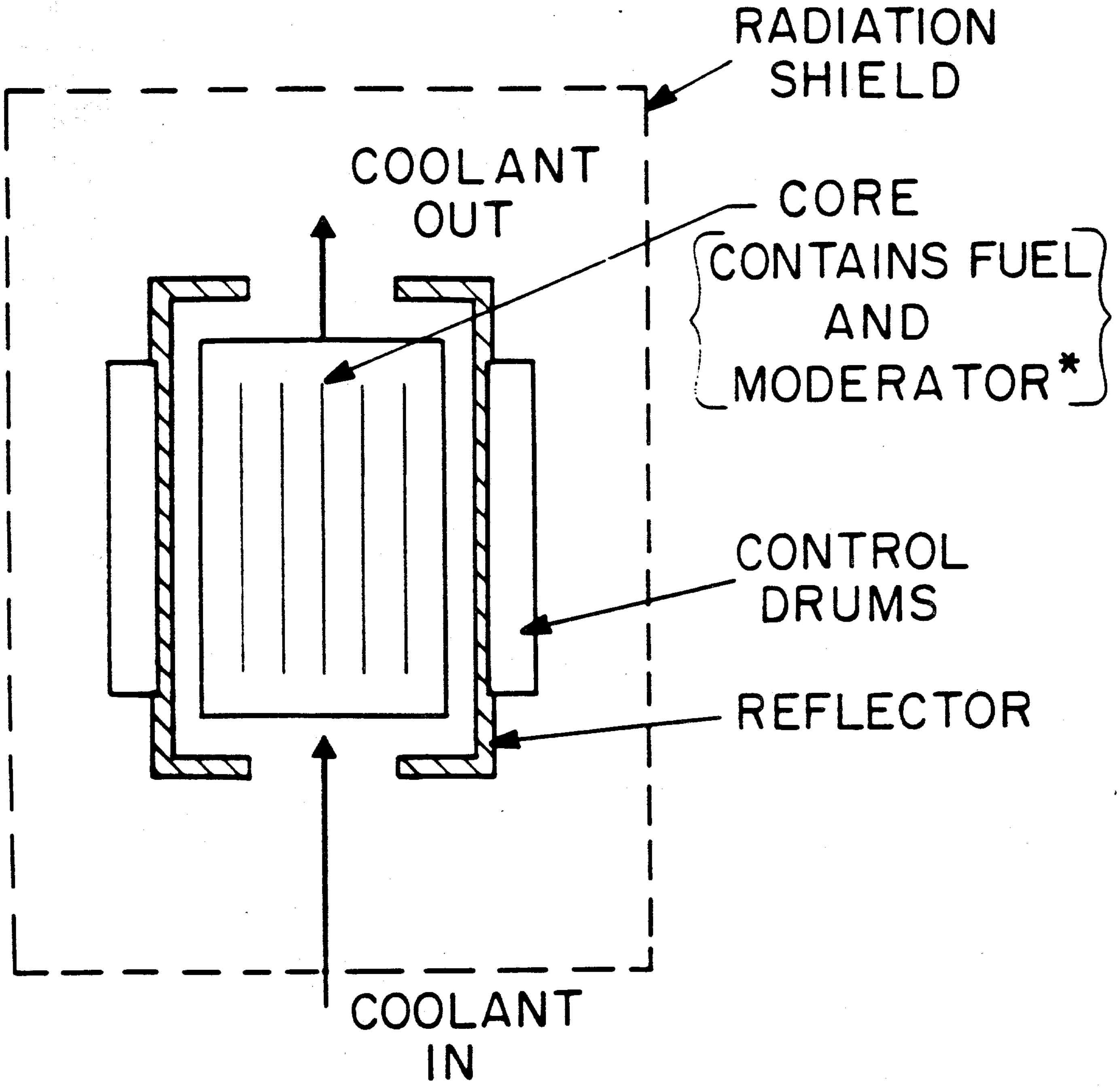


Figure 3.3

* THERMAL REACTORS ONLY

Table 3.1 Neutron fission thresholds as a function of nuclear mass (calculated).

Mass No., A	Fission Threshold (MeV)
16	18.5
60	48
100	47
140	62
200	40
236	~5

Table 3.2 Neutron fission thresholds of heavy nuclides (experimental).

Target Nucleus	Compound Nucleus	Fission Threshold (MeV)
^{232}Th	^{233}Th	1.3
^{233}U	^{234}U	< 0
^{234}U	^{235}U	0.4
^{235}U	^{236}U	< 0
^{236}U	^{237}U	0.8
^{238}U	^{239}U	1.2
^{237}Np	^{238}Np	0.4
^{239}Pu	^{240}Pu	< 0

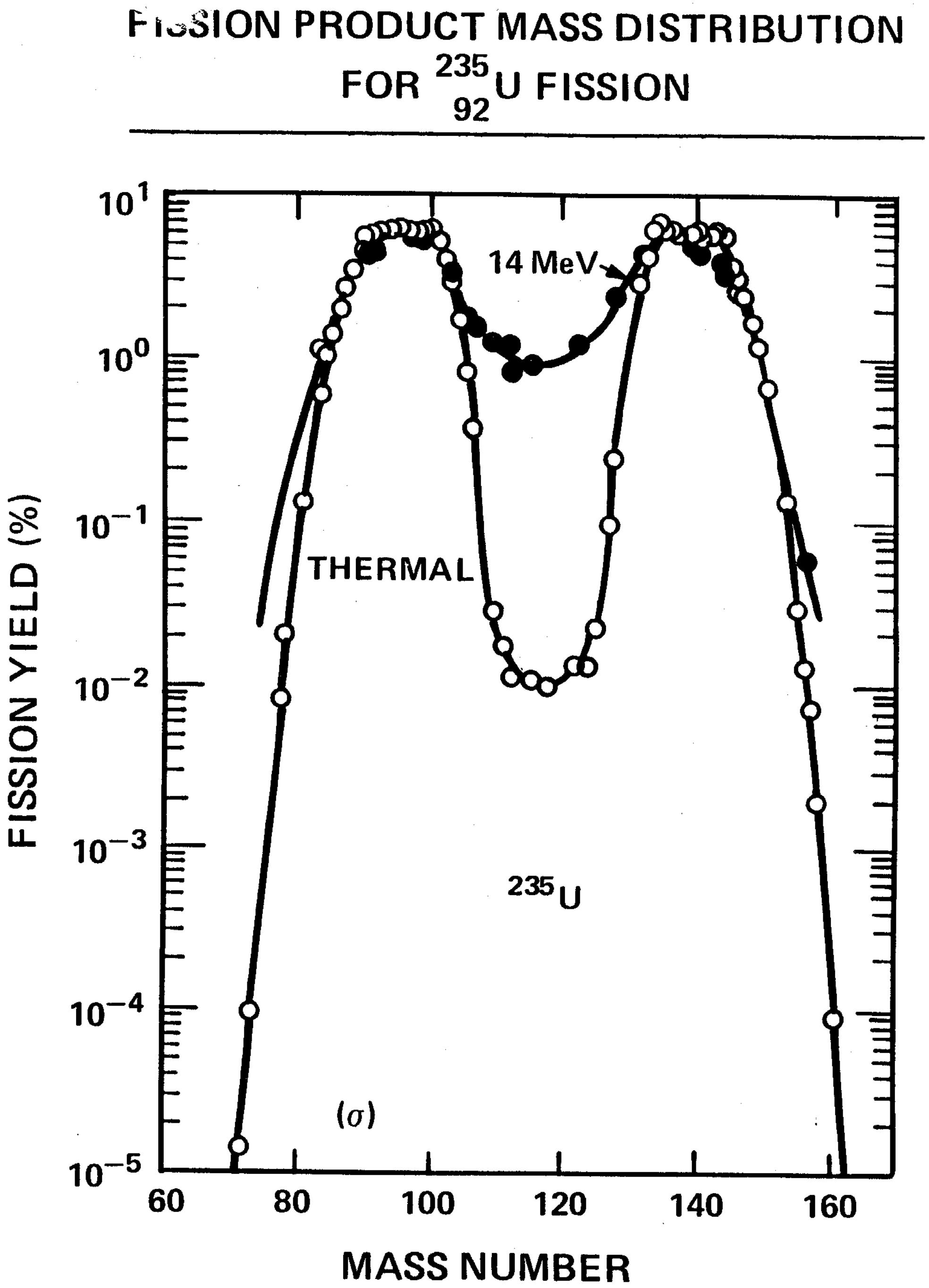


Fig. 3.4 Fission product mass distribution for uranium-235 fission [*8*].

Table 3.3 Typical energy distribution for Uranium-235 fission.

Energy Form	Energy Released (MeV)	Energy Potentially Recoverable (MeV)
Kinetic Energy of Fission Fragments	168	168
Decay of Fission Products		
● Beta Radiation	8	8
● Gamma Radiation	7	7
● Neutrinos	12	—
Prompt (Fission) Gamma Radiation	7	7
Kinetic Energy of Fission Neutrons	5	5
Capture Gamma Radiation	—	3–12
TOTALS	207	198–207