Space Travel Overview:

You Can Get There from Here!

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Lecture 5

Resources from Space

NEEP 533/ Geology 533 / Astronomy 533 / EMA 601

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Outline

- Leaving Earth
  - Laws of gravitation and motion
  - Rocket physics
- Near-Earth space
  - Celestial coordinates
  - Orbital dynamics
  - Lagrange points
- Traveling to the planets and asteroids
  - Minimum-energy (Hohmann) transfers
  - Gravity assists
  - Low-thrust trajectories

Newton's Law of Gravitation

- Every particle of matter attracts every other particle of matter with a force directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

$$\mathbf{F} = \frac{-GMm}{r^2}\,\hat{\mathbf{r}}$$

$G = 6.67\times10^{-11}\ m^3\ s^{-2}\ kg^{-1}$

is the *gravitational constant*.

Newton's Laws of Motion

- The fundamental laws of mechanical motion were first formulated by Sir Isaac Newton (1643-1727), and were published in his *Philosophia Naturalis Principia Mathematica*.

- Calculus, invented independently by Newton and Gottfried Leibniz (1646-1716), plus Newton's laws of gravitation and motion are the tools needed to understand rocket motion.

Newton's Laws of Motion

- Every body continues in its state of rest or of uniform motion in a straight line except insofar as it is compelled to change that state by an external impressed force.
- The rate of change of momentum of the body is proportional to the impressed force and takes place in the direction in which the force acts.

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}$$

- To every action there is an equal and opposite reaction.

Tsiolkowsky's Rocket Equation

- First derived by Konstantin Tsiolkowsky in 1895 for straight-line rocket motion with constant exhaust velocity, it also applies to elliptical trajectories with only initial and final impulses.
- Conservation of momentum leads to the so-called rocket equation, which shows the dependence of payload fraction on exhaust velocity.
- It assumes short impulses with coast phases between them, such as used for chemical and nuclear-thermal rockets.

Konstantin Tsiolkowsky

Tsiolkowsky's Rocket Equation

- Assuming constant exhaust velocity, conservation of momentum for a rocket and its exhaust leads to

$$\frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt} = 0$$

$$\Rightarrow \frac{dm}{m} = \frac{-dv}{v_{ex}}$$

Rocket equation

$$\Rightarrow \frac{m_f}{m_i} = \exp\left(\frac{-\Delta v}{v_{ex}}\right)$$

$m_f \equiv$ **final mass**

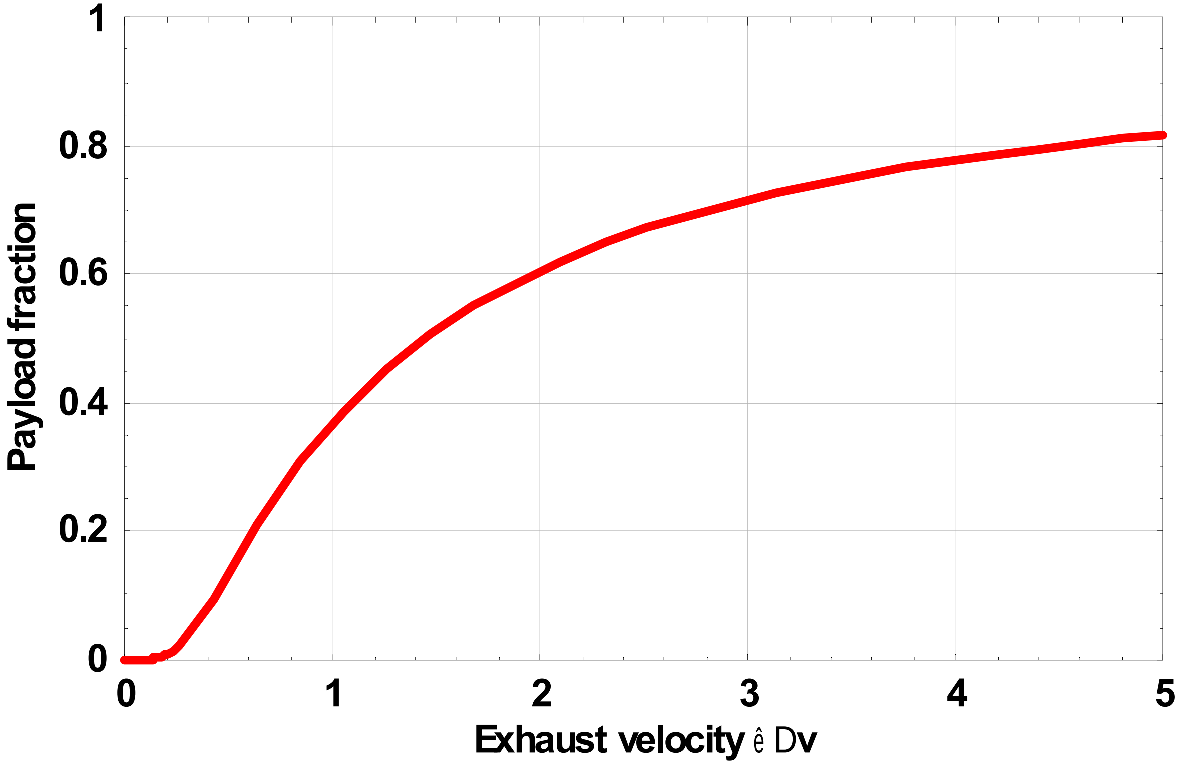
$m_i \equiv$ **initial mass**

$\Delta v \equiv$ **velocity increment**

$v_{ex} \equiv$ **exhaust velocity**

High Exhaust Velocity Gives Large Payloads

- This plot of the rocket equation shows why high exhaust velocity historically drives rocket design: payload fractions depend strongly upon the ratio of the exhaust velocity to the Δv.



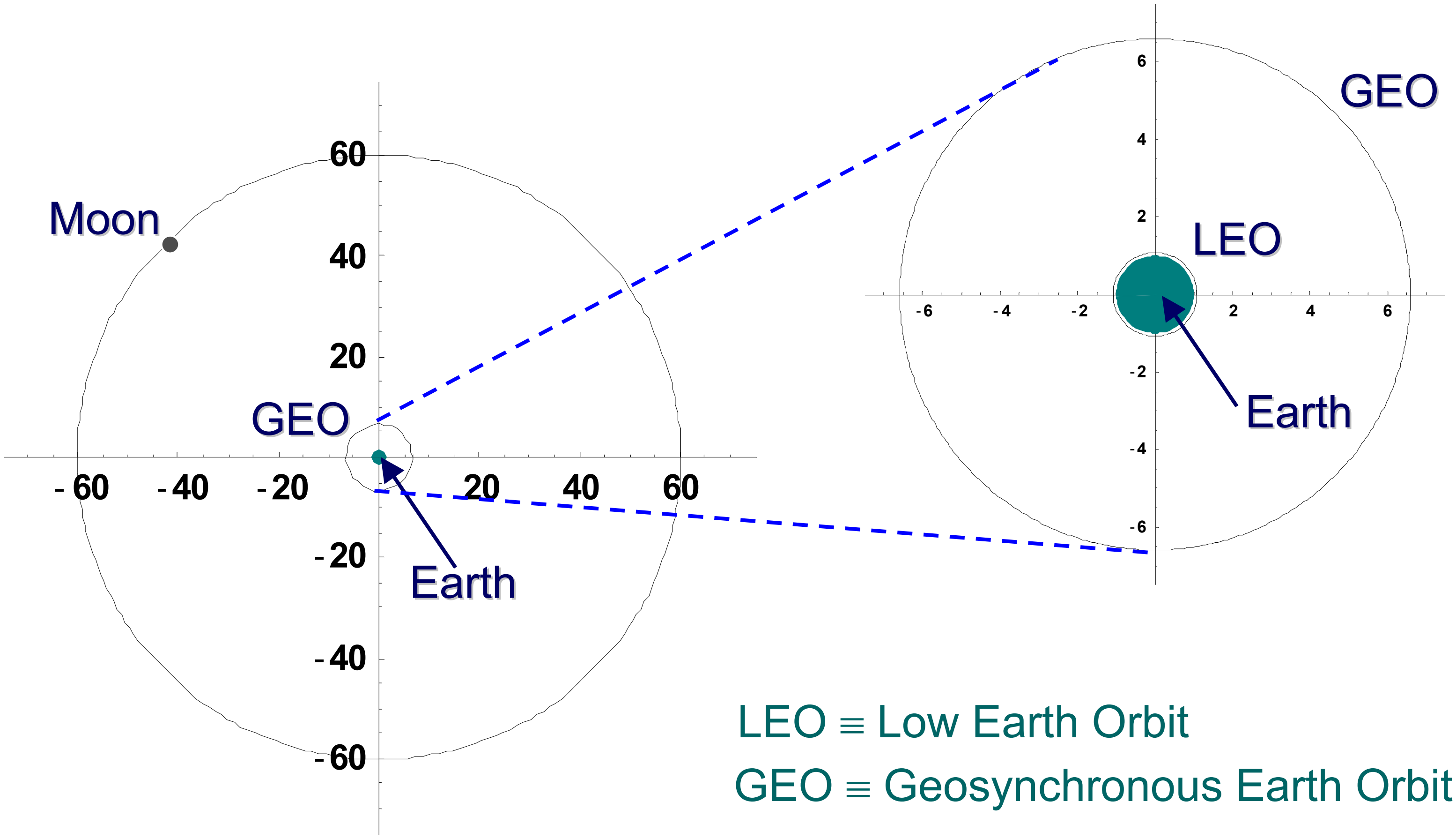
- $V_{chem} \leq 5$ km/s
- Earth-escape Δv~11.2 km/s
- Earth-orbit to Mars-orbit Δv~5.6 km/s
- Earth-orbit to Jupiter-orbit Δv~14.4 km/s

Many Factors Affect a Rocket's Launch

- Number of stages
  - Staging gains efficiency by eliminating mass of lower stages when accelerating a payload.
  - Analyze by applying the rocket equation in series to the stages.
- Gravitational loss
  - Due to finite launch time; ($\sim g \Delta t$), typically~25%.
- Atmospheric drag
  - Typically ~5%.
- Divergence of exhaust from nozzle

Near-Earth Space

- Distances are in Earth radii: ~6400 km



LEO ≡ Low Earth Orbit

GEO ≡ Geosynchronous Earth Orbit

Escape from Earth

- Earth's mass = 6 x 10^{24} kg
- Earth's average radius = 6380 km = 6.38 x 10^6 m
- $G=6.67\times10^{-11}$ m^3 s^{-2} kg^{-1}

$$v_{esc} = \left(2\frac{GM}{r}\right)^{1/2} = \left(\frac{2\times6.67\times10^{-11}\times6\times10^{24}}{6.38\times10^{6}}\right)^{1/2}$$

$$= 11.2 \text{ km/s}$$

- The actual velocity increment required to escape from Earth is somewhat higher, because of the finite time of flight and atmospheric friction.

Some Useful Orbital Dynamics Formulas

Circular velocity

$$v_{cir} = \left(\frac{GM}{r} \right)^{1/2}$$

Escape velocity

$$v_{esc} = \left(2\frac{GM}{r} \right)^{1/2}$$

Velocity of a body on an elliptical orbit

$$v_{ellipse} = \left[GM \left(\frac{2}{r} - \frac{1}{a} \right) \right]^{1/2}$$

Energy (potential plus kinetic) of a body on any conic-section orbit ($E<0 \Rightarrow$ trapped orbit)

$$E_{conic} = \frac{GM}{2a}$$

$G = 6.67\times10^{-11}\ \mathrm{m^3\ s^{-2}\ kg^{-1}}$

$a \equiv$ semi-major axis

Defining a Celestial Body's Position Requires Six Parameters

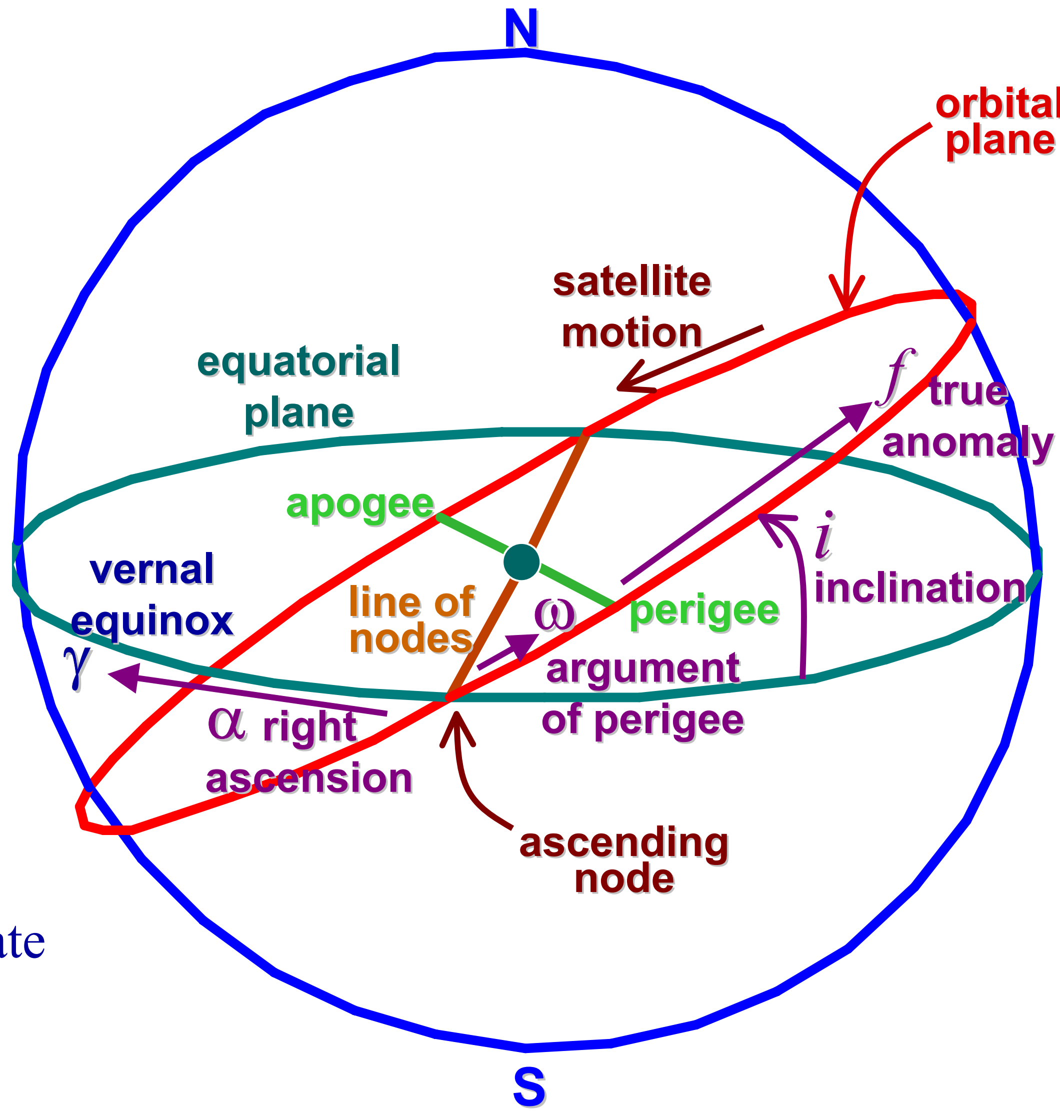
- Two parameters are not shown:

  a, semimajor axis

  e, eccentricity

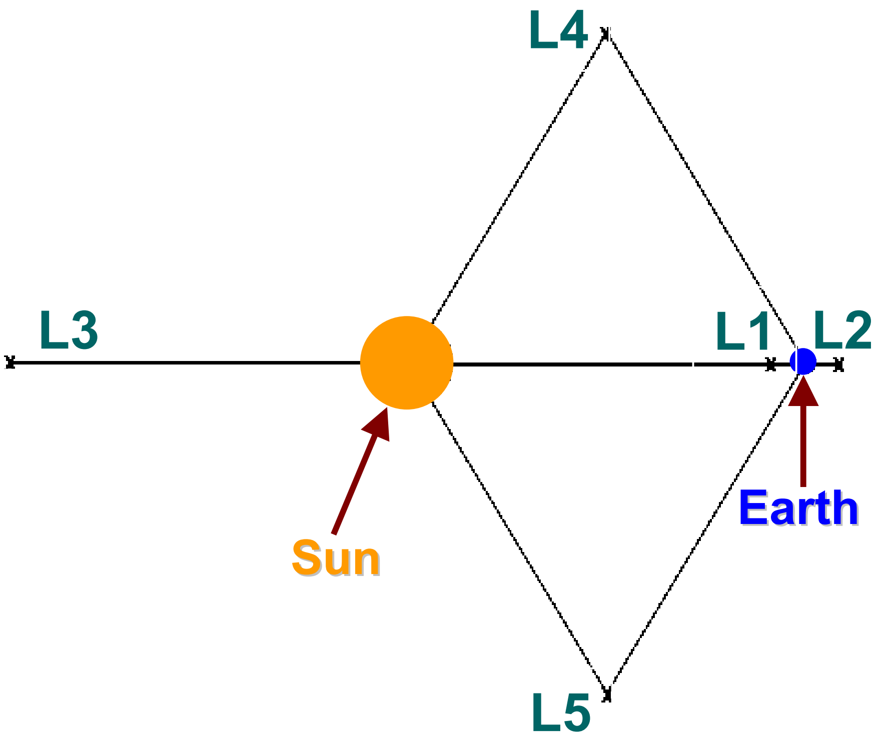
- Four of the six parameters appear in purple at right.

- These are equatorial coordinates--appropriate for an Earth satellite.



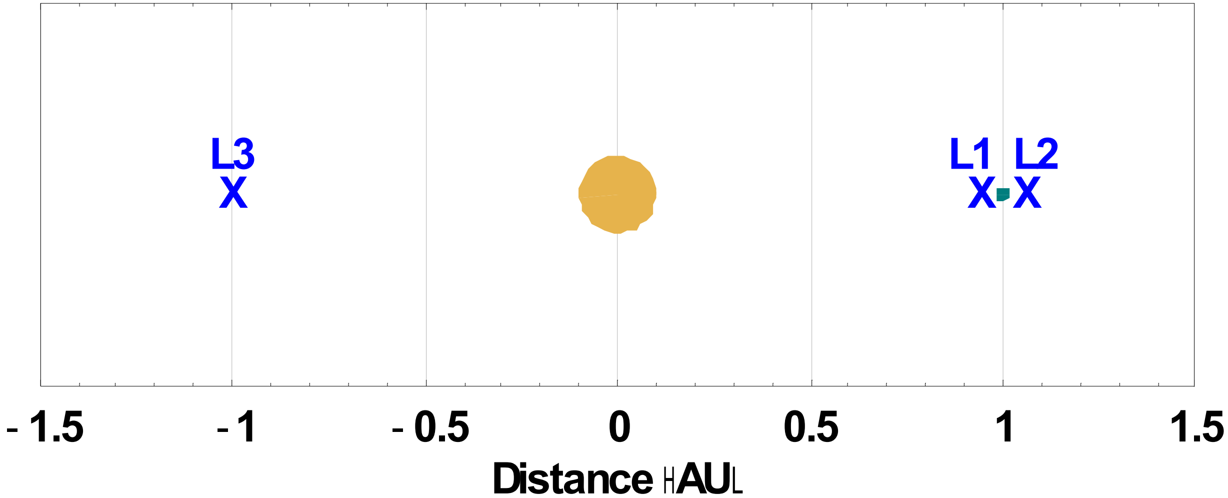
Lagrange Points are Equilibrium Positions in a Three-Body System

- The points *L1, L2, and L3* lie on a straight line through the two main bodies and are points of *unstable equilibrium*. That is, a small perturbation will cause the third body to drift away.
- The *L4 and L5* points are at the third vertex of an *equilateral triangle* formed with the other two bodies; they are points of *stable equilibrium*.
- In general, the three bodies can have arbitrary masses.

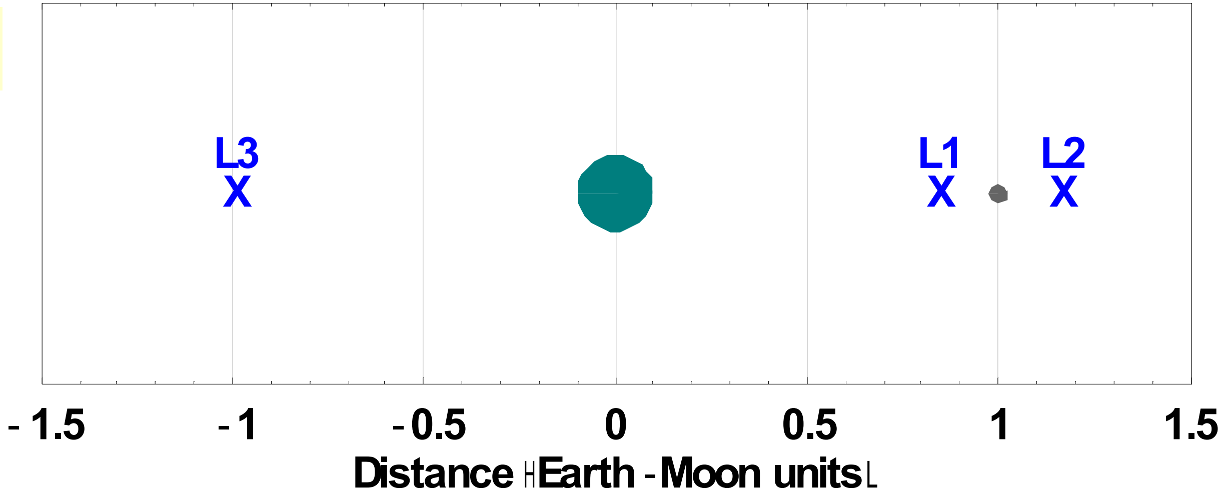


L1, L2, and L3 Lagrange Points for the Sun-Earth and Earth-Moon Systems

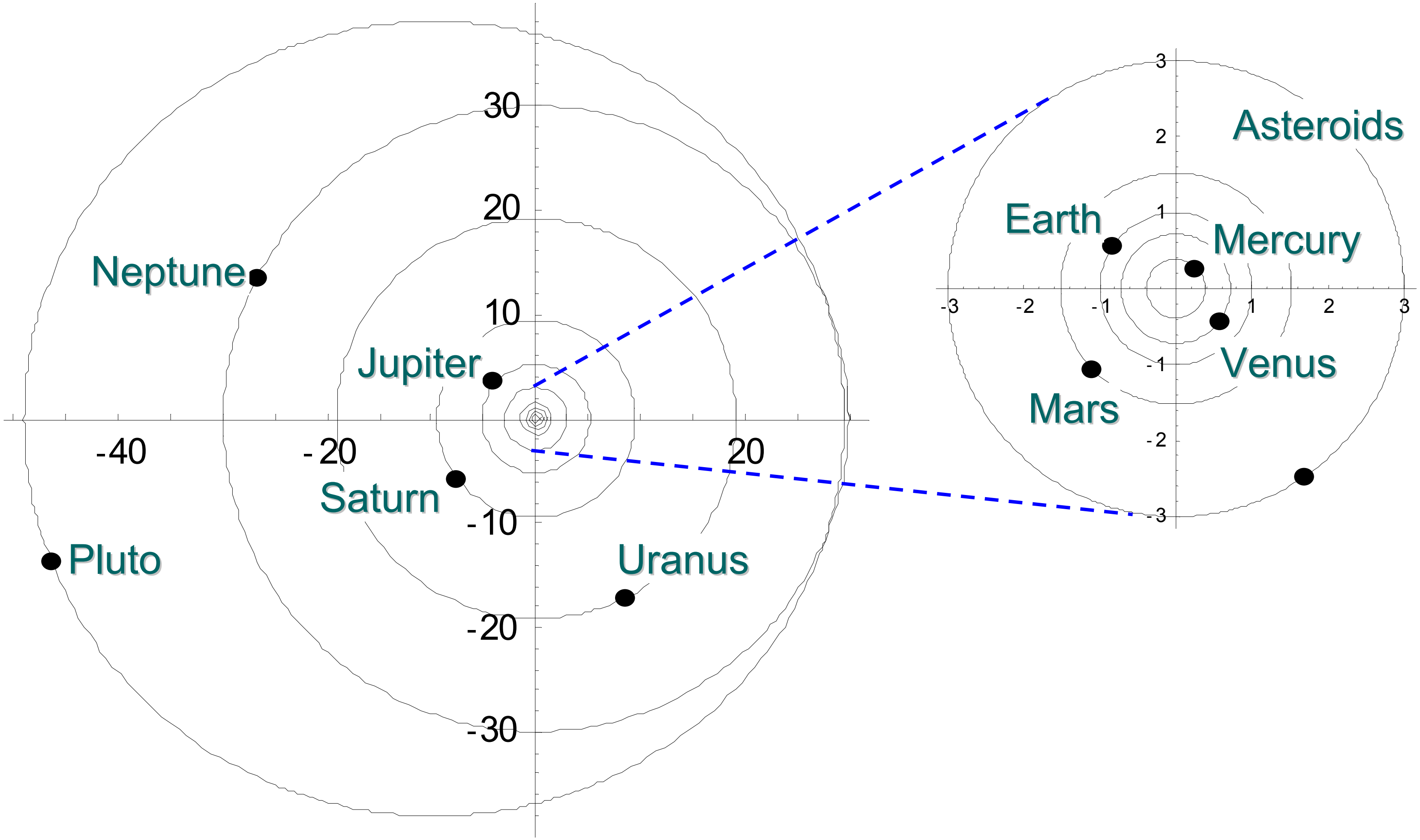
Sun-Earth



Earth-Moon

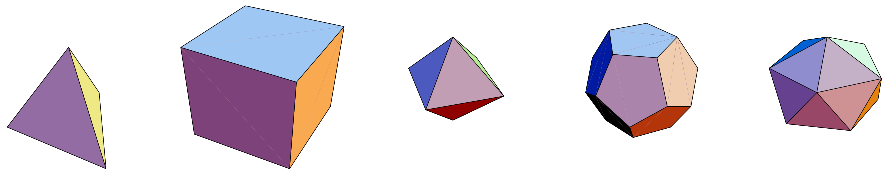


The Solar System



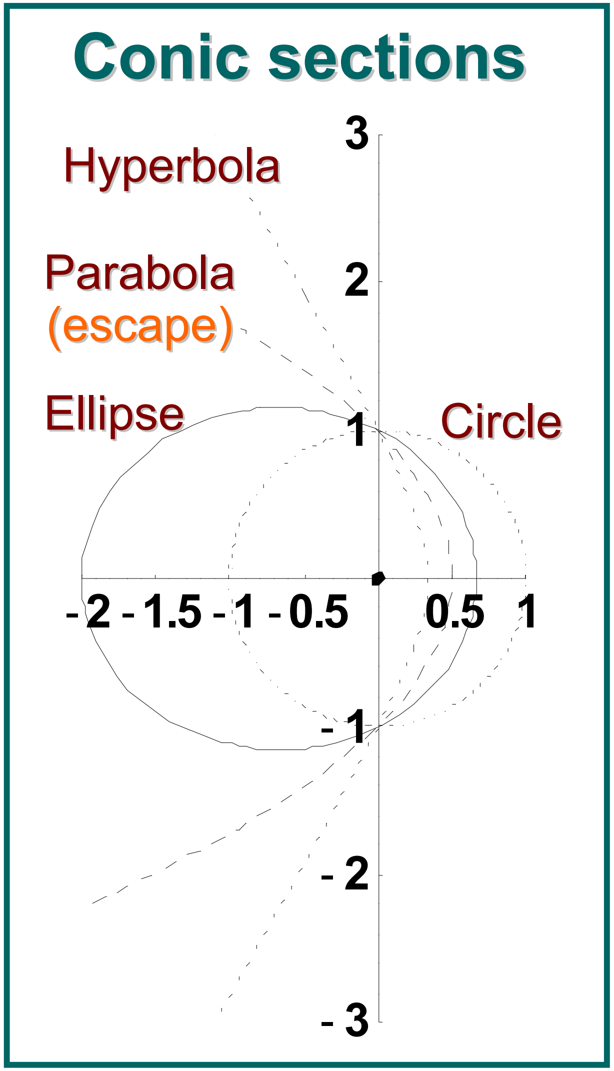
- Distances in astronomical units: 1 AU=1.5x10^8 km

Kepler's Laws of Planetary Motion



- The planets move in *ellipses* with the sun at one focus.
- Areas swept out by the radius vector from the sun to a planet in equal times are equal.
- The square of the period of revolution is proportional to the cube of the semimajor axis.

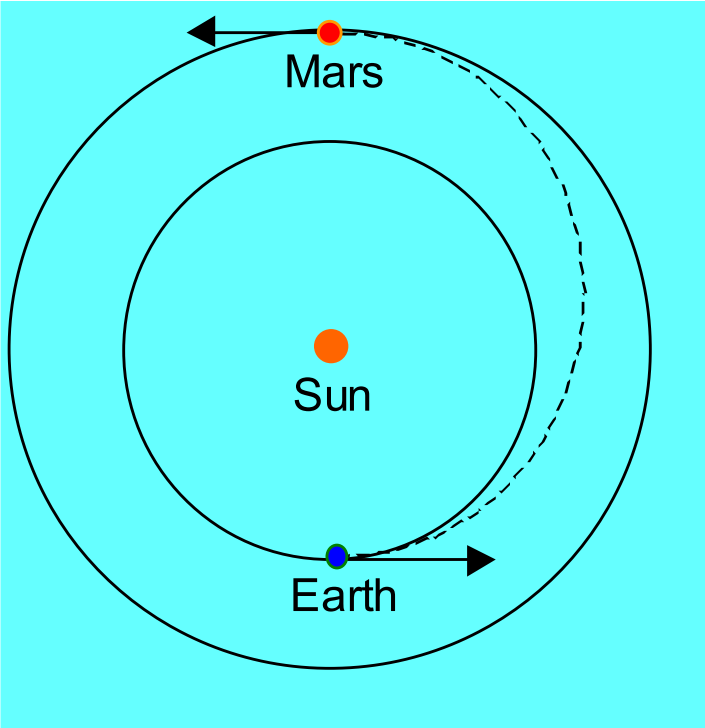
$$T^2 \propto a^3$$



Hohmann's Minimum-Energy Interplanetary Transfer

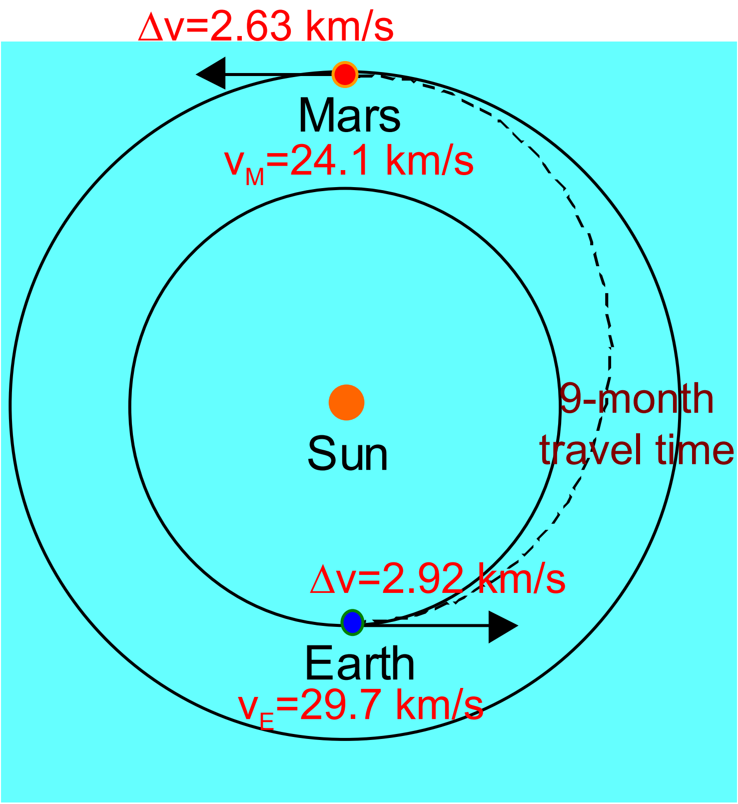
- The minimum-energy, two-impulse transfer between circular orbits is an elliptical trajectory called the Hohmann trajectory, shown at right for the Earth-Mars case.
- A Hohmann transfer consists of three phases:
    1. Large impulse to leave circular orbit
    2. Long coast phase on an elliptical orbit
    3. Large impulse to match target planet's velocity

Earth-Mars Hohmann transfer



Hohmann's Minimum-Energy Interplanetary Transfer Can Be Calculated Easily

- The Hohmann trajectory appears at right for the Earth-Mars case, where the minimum total delta-v expended is 5.6 km/s.
- Shown are the velocities on Earth and Mars orbits plus the Δv values required to put the rocket onto the elliptical Hohmann trajectory at perihelion (closest to Sun) and aphelion (farthest from Sun).
    - Assumes circular, co-planar orbits.



- The sum of the velocity increments at Earth and Mars is the total delta-v value needed in the rocket equation.

Hohmann Transfer Time

- Kepler's third law, $T^2 \propto a^3$, can be used to calculate the time required to traverse a Hohmann trajectory by raising to the 3/2 power the ratio of the semimajor axis of the elliptical Hohmann orbit to the circular radius of the Earth's orbit and dividing by two (for one-way travel):

$$\frac{T_1}{T_2} = \left(\frac{a_1}{a_2}\right)^{3/2}$$

- For example, call the travel time for an Earth-Mars trip T_1 and the semimajor axis of the Hohmann ellipse a_1:

$a_1 = (1\ \mathrm{AU} + 1.5\ \mathrm{AU})/2 = 1.25\ \mathrm{AU}$

$T_1 = 0.5\ (a\ /\ 1\ \mathrm{AU})^{3/2}$ years

$= 0.7$ years $= 8.4$ months

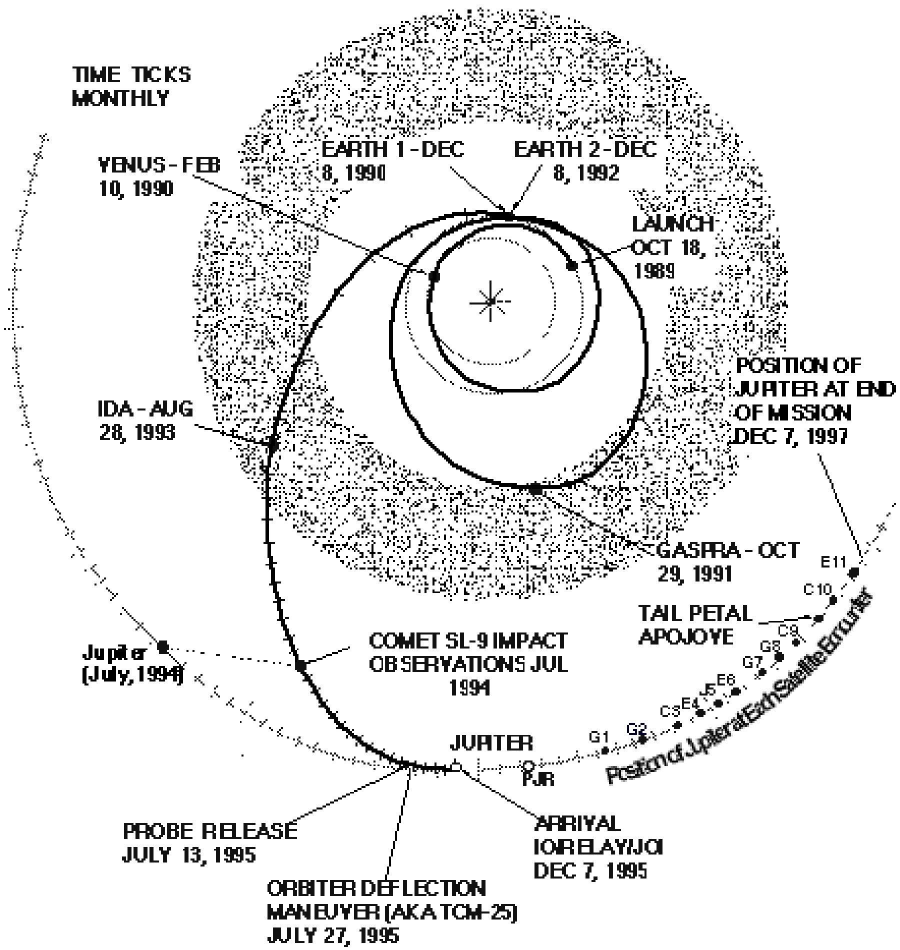
Use Earth as reference:

$a_2 = 1\ \mathrm{AU}$

$T_2 = 1$ year

Many Spacecraft Trajectories Can Be Approximated by Hohmann Orbits

Galileo Spacecraft's Trajectory



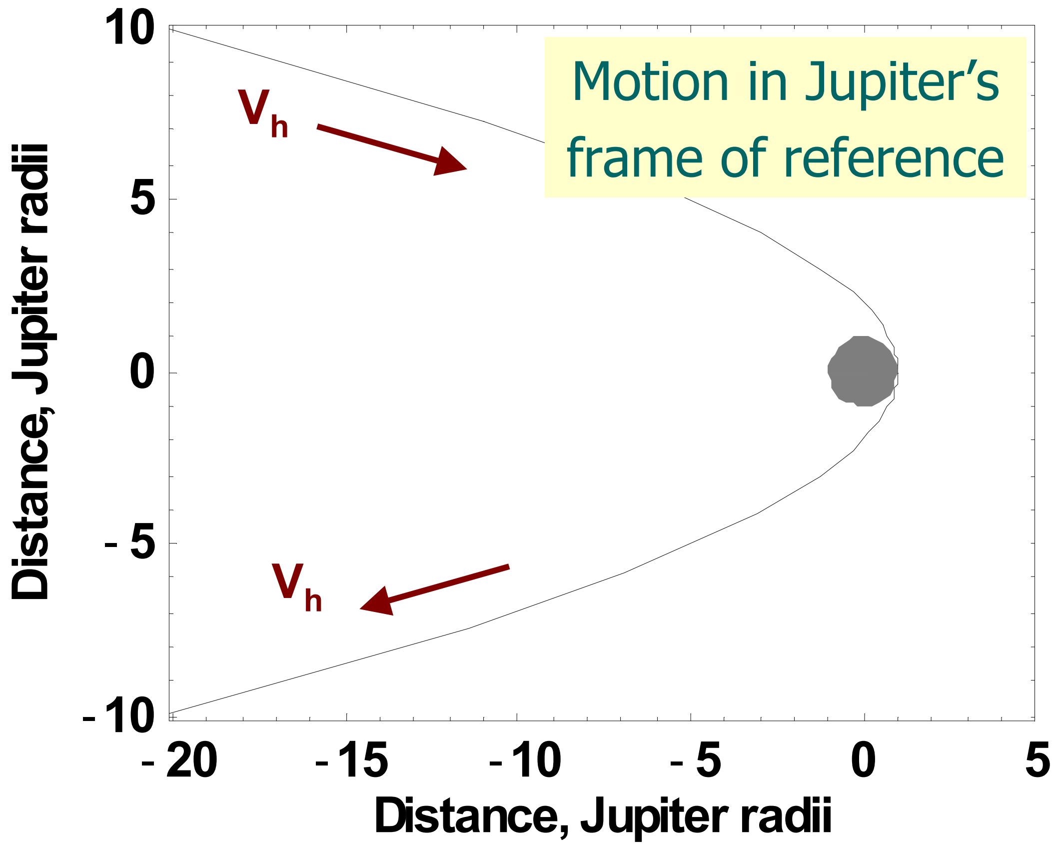
$a = (1\text{ AU} + 5.2\text{ AU})/2$
$= 3.1\text{ AU}$

$T = 0.5\,(a / 1\text{ AU})^{3/2}$ years $= \sim 2.7$ years

The final phase (nearly elliptical) of Galileo's trip to Jupiter took ~3 years.

Gravity Assists
Enable or Facilitate Many Missions

- In the planet's frame of reference, a spacecraft enters and leaves the sphere of influence with a so-called hyperbolic excess velocity, v_h, equal to the vector sum of its incoming velocity and the planet's velocity.



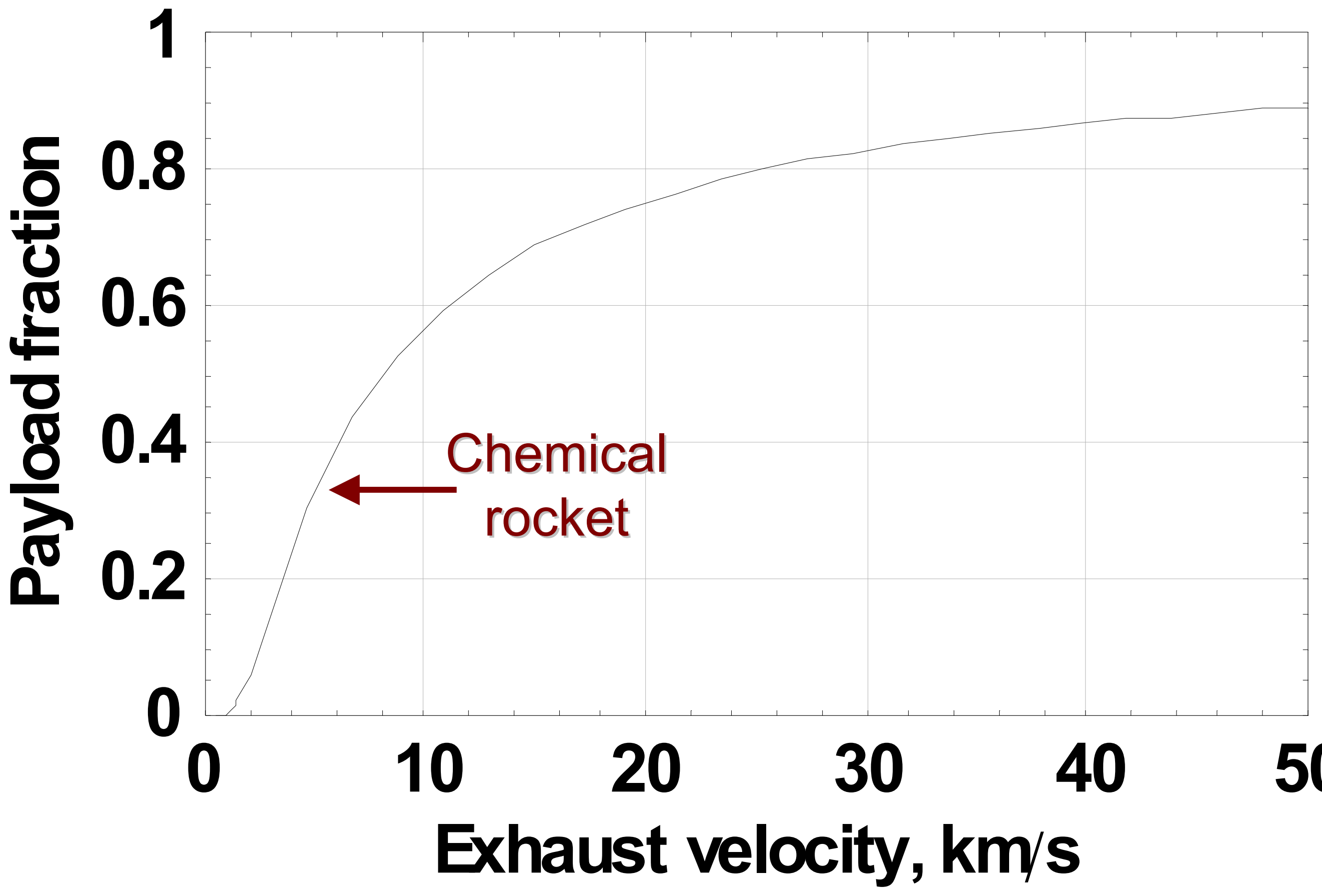
Gravity Assists
Enable or Facilitate Many Missions

- In the planet's frame of reference, the direction of the spacecraft's velocity changes, but not its magnitude. In the spacecraft's frame of reference, the net result of this trade-off of momentum is a small change in the planet's velocity and a very large delta-v for the spacecraft.

- Starting from an Earth-Jupiter Hohmann trajectory, a flyby of Jupiter at one Jovian radius has these parameters:
  - Hyperbolic excess (beyond escape) velocity v_h~5.6 km/s
  - Flyby gives an angular change in direction of ~160^o if the rocket skims Jupiter's atmosphere.
  - Flyby gives a Δv of ~13 km/s
  - Final velocity = 13+5.6 km/s=18.6 km/s (greater than Solar-System v_{esc})

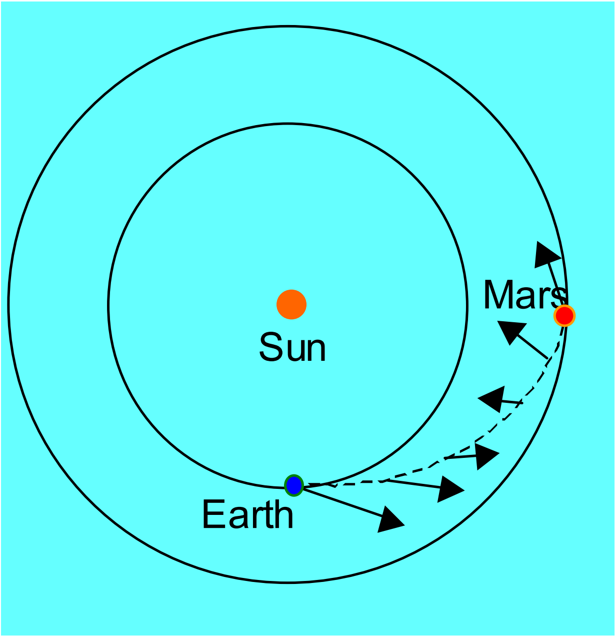
Missions to Mars and Beyond Require Larger Exhaust Velocities than Chemical Rockets Can Produce

Earth-Mars one-way trip: $\Delta v \sim 5.6$ km/s (Hohmann)



Separately Powered Systems Differ Significantly from Chemical Rockets

- Propellant not the power source; power system adds mass.
- High exhaust velocity ($\geq 10^5$ m/s).
- Low thrust ($\leq 10^{-2}$ m/s} $\equiv 10^{-3}$ Earth gravity) in most cases.
- Thrusters operate for a large fraction of the mission duration.
- High-exhaust-velocity trajectories *differ fundamentally* from chemical-rocket trajectories (figure at right).



Note: Trajectory is schematic, not calculated.

Separately powered rocket equation

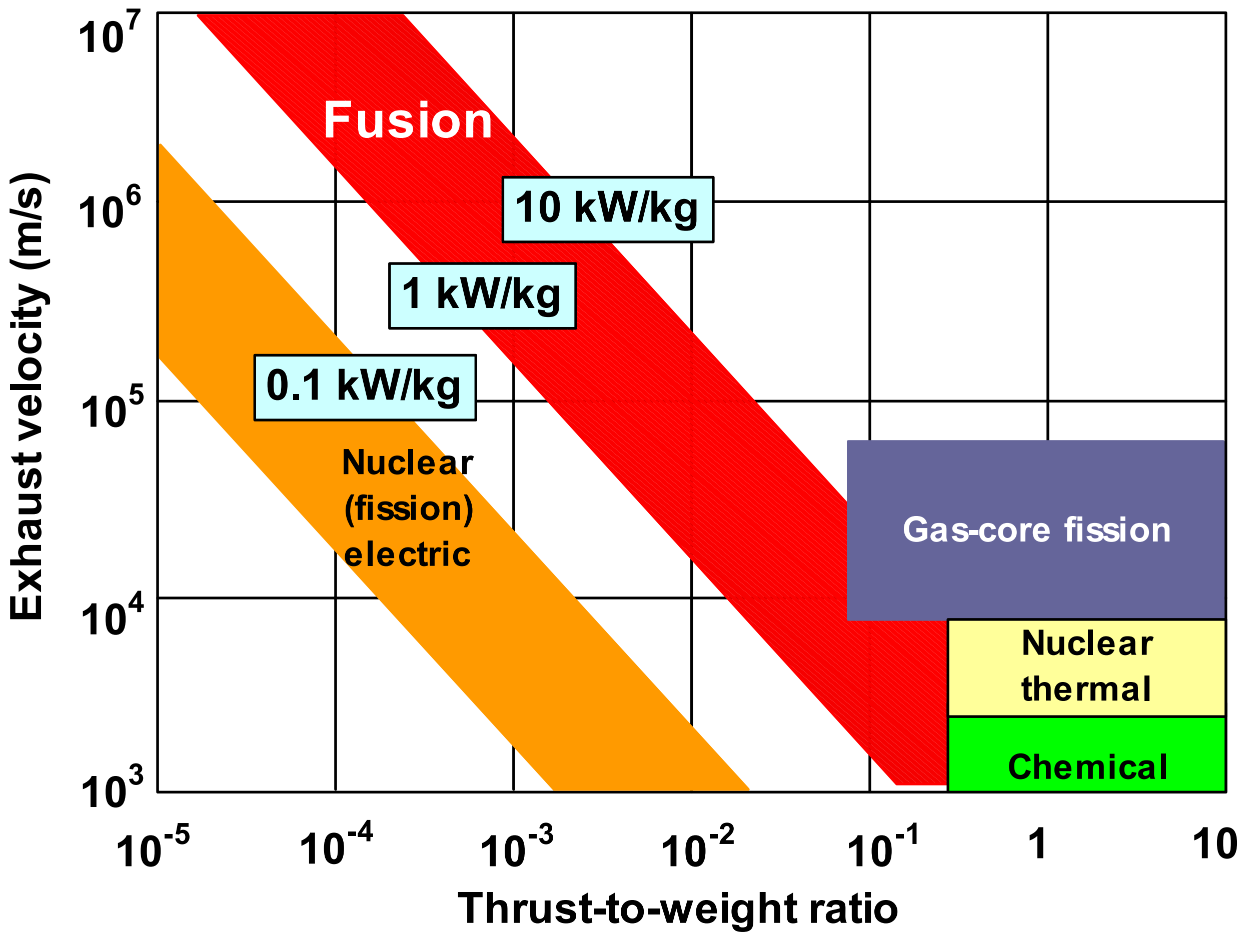
$$\frac{M_l + M_w}{M_l + M_w + M_p} = \exp\left(\frac{-\Delta v}{v_{ex}}\right)$$

M_l=payload mass

M_w=power and propulsion system mass

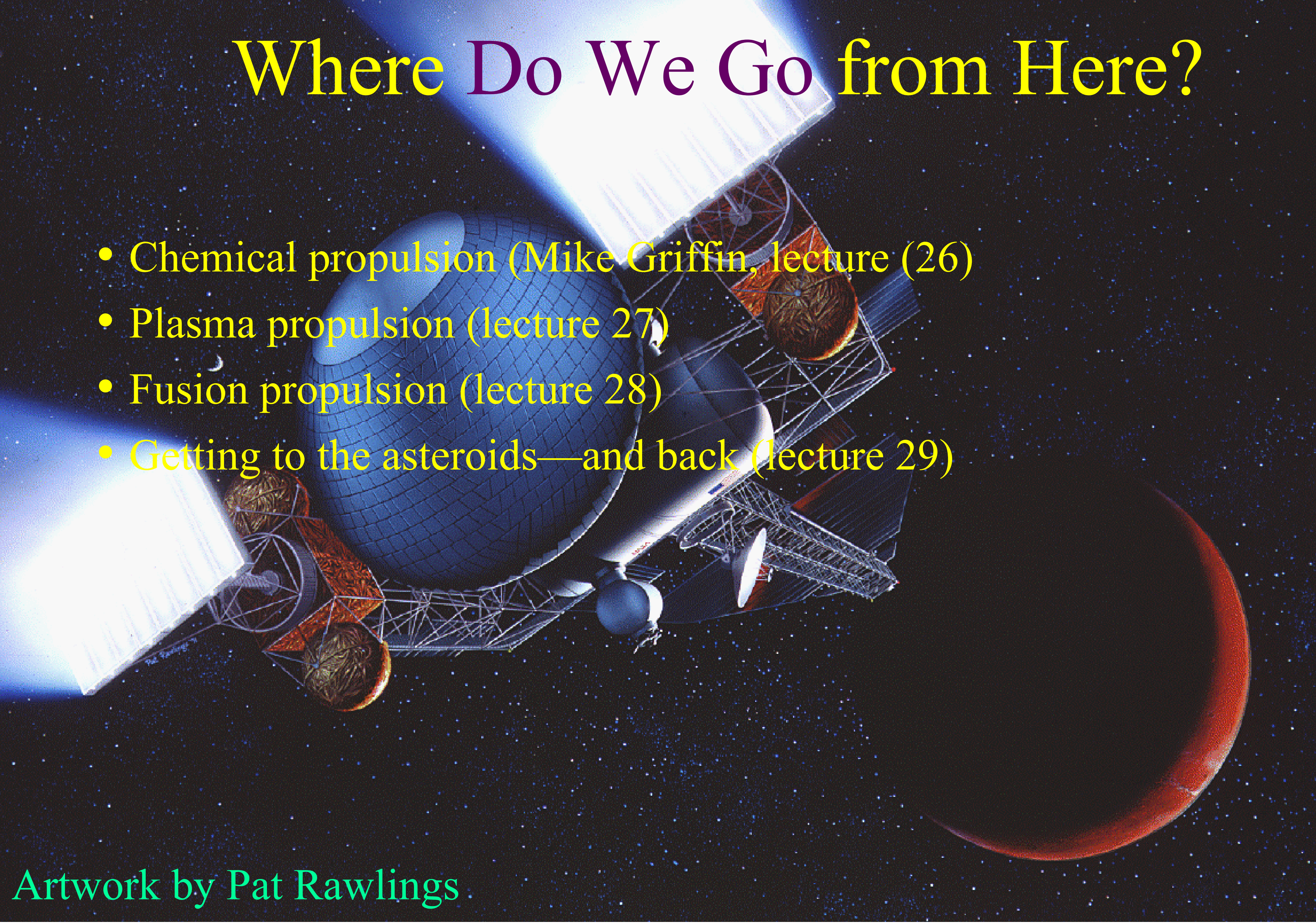
M_p=propellant mass

Figures of Merit for Separately Powered Systems are Exhaust Velocity (m/s) and Specific Power (kW/kg)



Where Do We Go from Here?

- Chemical propulsion (Mike Griffin, lecture (26)
- Plasma propulsion (lecture 27)
- Fusion propulsion (lecture 28)
- Getting to the asteroids—and back (lecture 29)



Artwork by Pat Rawlings