

Problem 10.2

Assume: • **Fast reactor fuel pin characteristics as in Table 10.2**

- **Gap closure**
- **Only He fill gas**
- **linear power, R , is symmetric**

- **kdT from eq. 10.20**

- **$h_{\text{coolant}} = \frac{12\text{Watts}}{\text{cm}^2 \text{ } ^\circ\text{C}}$**

- **$k_c = 0.22 \frac{\text{Watts}}{\text{cm} \text{ } ^\circ\text{C}}$**

Wanted: T_o and T_s @ midplane (before core restructuring)

Schematic

(Diagram)

See Hard Copy of Class Notes

$$T_{\text{Na}} (\text{midplane}) = 470 + \frac{650 - 470}{2} = 560^\circ\text{C}$$

Na Film Temperature Drop (at midplane)

q_{Na} = heat flux from clad surface to coolant

$$q_{Na} = \frac{1}{2 R_c} = h_{Na} (T_{co} - T_{Na})$$

or

$$T_{co} - T_{Na} = \frac{550 \frac{\text{Watts}}{\text{cm}}}{2 \cdot (0.315 \text{cm}) \cdot 12 \frac{\text{Watts}}{\text{cm}^2 \cdot \text{C}}}$$

or,

$$T_{co} = 560 + 23 = 583^\circ\text{C}$$

Temperature Drop Through Clad

q_c = heat flux through cladding (ave)

$$q_c = \frac{1}{2 R_c - \frac{t_c}{2}} = \frac{k_c}{t_c} \cdot (T_{ci} - T_{co})$$

for stainless steel, $k_c = 0.22 \frac{\text{Watts}}{\text{cm} \cdot \text{C}}$

$$\frac{k_c}{t_c} = \frac{0.22}{0.04} = 5.5 \frac{\text{Watts}}{\text{cm}^2 \cdot \text{C}}$$

$$\text{So, } T_{ci} - T_{co} = \frac{550}{2 (0.315 - 0.020) \cdot 5.5} = 54^\circ \text{C}$$

$$\text{or, } T_{ci} = 583 + 54 = 637^\circ \text{C}$$

Temperature Across the Gap

q_F = Surface heat flux from the fuel

$$= \frac{1}{2 R} = \frac{k_g}{t_g} \cdot (T_s - T_{ci})$$

$$\text{from (10.63) } kdT = \frac{1}{4} T_s$$

$$\text{or, } \int_{T_0}^{T_s} kdT - \int_{T_0}^{T_s} kdT = \frac{1}{4} T_s = \frac{550}{4} = 43.8 \frac{\text{W}}{\text{cm}} \quad 1)$$

For (He) gas conductivity use Eq. 10.102

$$k_g = 15.8 \times 10^{-6} (920)^{0.79} = 3.5 \times 10^{-3} \text{ W/cm}^\circ\text{C}$$

Approximate gas T, °K

now,

$$T_s - T_{ci} = t_g \cdot \frac{q}{k_g \cdot 2R}$$

$$= \frac{(0.007 \text{ cm})}{(0.0035 \frac{\text{W}}{\text{cm} \cdot \text{C}})} \cdot \frac{87.6 \frac{\text{W}}{\text{cm}}}{0.268 \text{ cm}}$$

$$T_s = 637 + 654 = 1291 \text{ }^\circ\text{C}$$

w/o gap closure

Gap Closure

$$\text{Fuel Thermal Expansion} = \frac{\Delta R_f}{R_f} = \alpha_f (\bar{T}_f - T_a)$$

$$T_a = \text{Fabrication T} = 25 \text{ }^\circ\text{C}$$

$$\text{Clad Thermal Expansion} = \frac{\Delta R_c}{R_c} = \alpha_c (\bar{T}_c - T_a)$$

$$\text{Hot fuel Radius} = R_f^{\text{hot}} = R_f^{\text{cold}} [1 + \alpha_f (\bar{T}_f - T_a)]$$

$$\text{Hot inner clad radius} = R_{ci}^{\text{hot}} = R_c^{\text{hot}} - t_c^{\text{hot}}$$

$$\text{gap width } t_g^{\text{hot}} = R_{ci}^{\text{hot}} - R_f^{\text{hot}}$$

$$t_g^{\text{cold}} = R_{ci}^{\text{cold}} - R_f^{\text{cold}}$$

so,

$$t_g^{\text{hot}} - t_g^{\text{cold}} = R_{ci}^{\text{cold}} \left(\bar{T}_c - T_a \right) - R_f^{\text{cold}} \left(\bar{T}_f - T_a \right)$$

Inserting numbers

$$\bar{T}_c = \frac{(637 + 583)}{2} = 610^\circ\text{C}$$

$$R_f = 1.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1} \text{ (fig. 10.8) } 1600^\circ\text{C (Pu-15\%)} \\ R_c = 1.8 \times 10^{-5} \text{ }^\circ\text{C}^{-1} \text{ (fig 10.9)}$$

$$R_f^{\text{cold}} = 0.315 - 0.04 - 0.007 = 0.268 \text{ cm}$$

$$R_{ci}^{\text{cold}} = 0.315 - 0.04 = 0.275 \text{ cm}$$

$$t_g^{\text{hot}} - t_g^{\text{cold}} = 0.275 \cdot 1.8 \times 10^{-5} (610 - 25) - \\ 0.268 \times 1.2 \times 10^{-5} (\bar{T}_f - 25)$$

$$t_g^{\text{hot}} - t_g^{\text{cold}} = 2.9 \times 10^{-3} - 3.22 \times 10^{-6} (\bar{T}_f - 25) \quad 2)$$

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Solve for \bar{T}_f

• Use an iterative procedure, start with $T_s = 1291^\circ\text{C}$ w/o gap closure

1291

From figure 10.20 $kdT = 51 \text{ W/cm}$

0

From Equation 1

T_0

$$kdT = 51 + 43.8 = 94.8 \text{ W/cm}$$

0

from figure 10.20

$$T_0 = 2700 \text{ }^\circ\text{C}$$

Since fuel has not restructured assume parabolic profile

$$\frac{(T - T_s)}{(T_0 - T_s)} = 1 - \frac{r^2}{R^2}$$

$$T = T_s + (T_0 - T_s) \left(1 - \frac{r^2}{R^2}\right) = T_0 - [T_0 - T_s] \frac{r^2}{R^2}$$

$$\text{Then } \bar{T}_f = \frac{1}{R^2} \int_0^R T_0 - [T_0 - T_s] \frac{r^2}{R^2} \cdot 2r \, dr$$

$$= T_0 - \frac{T_0 - T_s}{2} = \frac{T_0 + T_s}{2}$$

or,

$$\bar{T}_f = \frac{1291 + 2700}{2} = 1996 \text{ }^\circ\text{C}$$

Hence, from eq. above

$$\begin{aligned}t_g^{\text{hot}} &= t_g^{\text{cold}} + 2.9 \times 10^{-3} - 3.22 \times 10^{-6}(1996-25) \\ &= 0.007 + 0.0029 - 0.00635 \\ &= 0.00355 \text{ cm}\end{aligned}$$

First Iteration

$$\frac{k_g}{t_g} = \frac{0.0035}{0.00355} = 0.985 \text{ W/cm}^2\text{ }^\circ\text{C}$$

therefore,

$$\begin{aligned}T_s - T_{ci} &= \frac{t_g}{k_g} \cdot \frac{1}{2 R} \\ &= \frac{1}{0.985} \cdot \frac{550}{2} \cdot \frac{1}{0.268[1 + 0.000012(996 - 25)]}\end{aligned}$$

R_f^{hot} (above)

$$= \frac{550}{2 \cdot 0.985 \cdot 0.275}$$

$$= 323 \text{ }^\circ\text{C}$$

$$T_s = 637 + 323 = 960 \text{ }^\circ\text{C}$$

Now go back to Figure 10.20

$$T_s = 960$$

$$kdT = 41 \text{ W/cm}$$

0

$$T_0$$

$$kdT = 43.8 + 41 = 84.8 \text{ W/cm}$$

$$T_0 = 2520 \text{ }^\circ\text{C}$$

recalculate,

$$\overline{T_F} = \frac{2520 + 960}{2} = 1740 \text{ }^\circ\text{C}$$

$$\text{and } t_g^{\text{hot}} = t_g^{\text{cold}} + 2.9 \times 10^{-3} - 3.22 \times 10^{-6} (1740 - 25)$$

$$= 4.525 \times 10^{-3} \text{ cm}$$

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Repeat this until convergence occurs

$$T_0 = 2530 \text{ }^\circ\text{C}$$

$$T_S = 1050 \text{ }^\circ$$

Problem 10.6

Slab fuel element between two thin plates of cladding

- **Initial porosity = P_0**
- **Power density before restructuring is H_0**

a.) **Find the value of the heat generation rate such that the center temperature just equals the melting temperature. Assume,**

$$k_0 = k_s \left(1 - P^{\frac{2}{3}}\right)$$

using

$$k_0 \frac{d^2 T}{dx^2} + H_0 = 0$$

or,

$$\frac{dT}{dx} = - \frac{H_0}{k_0} x + A \quad \text{but } \frac{dT}{dx} = 0$$

$$\text{@ } x=0, \text{ so } A=0$$

then,

$$T = - \frac{H_0}{2k_0} x^2 + B \quad \text{@ } x = L, T = T_s$$

$$\text{so } B = T_s + \frac{H_0}{2k_0} L^2$$

this gives;

$$T - T_s = \frac{H_0}{2k_0} L^2 \left(1 - \frac{x}{L}\right)^2$$

but at $x=0$, $T(0) = T_m$

then,

$$H_0 = 2k_0 \frac{(T_m - T_s)}{L^2}$$

or,

$$H_0 = \frac{2k_s \left(1 - P_0^{\frac{2}{3}}\right) (T_m - T_s)}{L^2}$$

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b.) After restructuring, fuel is converted to a columnar grain structure with a new porosity, P_1

- **What is the size of the central void, x_0 ?**
- **What is the new power density, H_1 , if the temperature at $x_0 = T_m$?**

$$k_1 = k_s \left(1 - P_1^{\frac{2}{3}}\right)$$

Hole size; $oL = {}_1(L - x_o)$

$$\frac{x_o}{L} = 1 - \frac{o}{1}$$

$$\frac{x_o}{L} = 1 - \frac{(1 - P_o)}{(1 - P_1)}$$

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$$\frac{dT}{dx} = -\frac{H_1}{k_1} x + A$$

$$T = -\frac{H_1}{2k_1} x^2 + Ax + B$$

now $\frac{dT}{dx} = 0$ @ $x = x_o$

so, $A = \frac{H_1 x_o}{k_1}$

and @ $x = L$, $T = T_s$

$$B = T_s + \frac{H_1 L^2}{2k_1} - \frac{H_1 x_o L}{k_1}$$

and $T = T_m$ @ $x = x_o$

substituting;

$$T_m - T_s = \frac{H_1 x_0^2}{2k_1} + \frac{H_1 L^2}{2k_1} - \frac{H_1 L x_0}{k_1}$$

Solving for H_1

$$H_1 = \frac{T_m - T_s}{\frac{L^2}{2k_1} \left(1 - \frac{2x_0}{L} + \frac{x_0^2}{L^2}\right)}$$

$$H_1 = \frac{2k_1 (T_m - T_s)}{L^2 \left(1 - \frac{x_0}{L}\right)^2}$$

but $\frac{o}{1} = 1 - \frac{x_o}{L}$ **or,**

$$H_1 = \frac{2k_0 [T_m - T_s]}{L^2} \frac{k_1}{k_0} \frac{L^2}{L^2 \left(1 - \frac{x_0}{L}\right)^2}$$

finally,

$$H_1 = H_0 \frac{1 - P_1^{2/3}}{1 - P_0^{2/3}} \frac{1 - P_1}{1 - P_0}^2$$

If $P_1 < P_0$ then $H_1 > H_0$