Effect of Porosity

Purposely put porosity into UO_2

- ***Good - Reduces gross swelling***
- ***Bad - Reduces thermal conductivity***

Define;

$$P = \frac{\text{Volume of Pores}}{\text{Volume of Pores + Volume of Solid}}$$

Model of Heat Conductivity

(Figure 10.15)

Fraction of xsection area occupied by pore

$$k = P_c\, k_{(\text{ Pore Tube})} + (1 - P_c)\, k_s$$

Fraction length occupied by pore tube

$$\frac{1}{k_{(poretube)}} = \frac{P_L}{k_p} + \frac{1 - P_L}{k_S}$$

This produces ;

$$k = k_s\,(1 - P^{2/3}) \qquad \text{where } P = P_c P_L$$

Radiation Transport (see figure 10.16)

$$k_p = k_g + 4\varepsilon\sigma l_y T^3$$

thermal conductivity of gas

Empirical (to account for the minimum)

$$k = (3.11 + 0.0272\ T)^{-1} + 5.39 \times 10^{-13}\ T^3$$

10.4
Temperature Profiles in Cylindrical Fuel Rods

$$T - T_s = \left(\frac{HR^2}{4k}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]$$

both a function of r, t

Previously we may have used ,

$$\frac{(T - T_s)}{(T_0 - T_s)} = 1 - \left(\frac{r}{R}\right)^2$$

but this is not good enough when H and k are varying

===

10.4.1
Volumetric Heating Rates

$$P = \frac{\text{Power}}{\text{Unit Length of Rod}} \quad \left(\frac{W}{cm}\right)$$

This is called the thermal rating

H Can Vary for at least 4 Reasons

Not normal 1.) **Gradient in initial enrichment**

Fast reactors 2.) **Pu migration**

Both LWR & FBR 3.) **Burnout or breeding effects**

Thermal reactors 4.) **Flux effects**

$$\frac{P}{\pi(R_o^2 - r_o^2)} = \frac{2\pi}{\pi(R_o^2 - r_o^2)} \bullet \int_{r_0}^{R} rH(r)dr = \overline{H}$$

10.4.2

Thermal Conductivity Integral

integrating,

$$\frac{1}{r}\frac{d\left[rk\frac{dT}{dr}\right]}{dr} + H = 0 \qquad \text{if H is constant}$$

first time,

$$rk\frac{dT}{dr} = -\frac{Hr^2}{2} \quad \text{(solid rod)}$$

again,

(conductivity integral)

$$\int_{T_S}^{T_0} kdT = \frac{HR^2}{4} = \frac{P}{4\pi}$$

Major use of conductivity integral is that when T_s and P are known, it can be used to estimate T_o

$$\int_{T_s}^{T_0} k\,dT = \int_0^{T_0} k\,dT - \int_0^{T_s} k\,dT = \frac{P}{4\pi}$$

This integral has been measured for a mixed oxide fuel

(See figure)

Value for T_m

$$= \int_0^{T_m} kdT = 93 \pm 4\,\frac{W}{cm}$$

If we know T_s, P (from experimental measurements), then we can get a value of,

$$\int_0^{T_0} kdT$$

which in turn will give T_o from figure 10.20

____

example; It is shown that as a result of self shielding, the heat generation rate in a thermal reactor fuel pin is ,

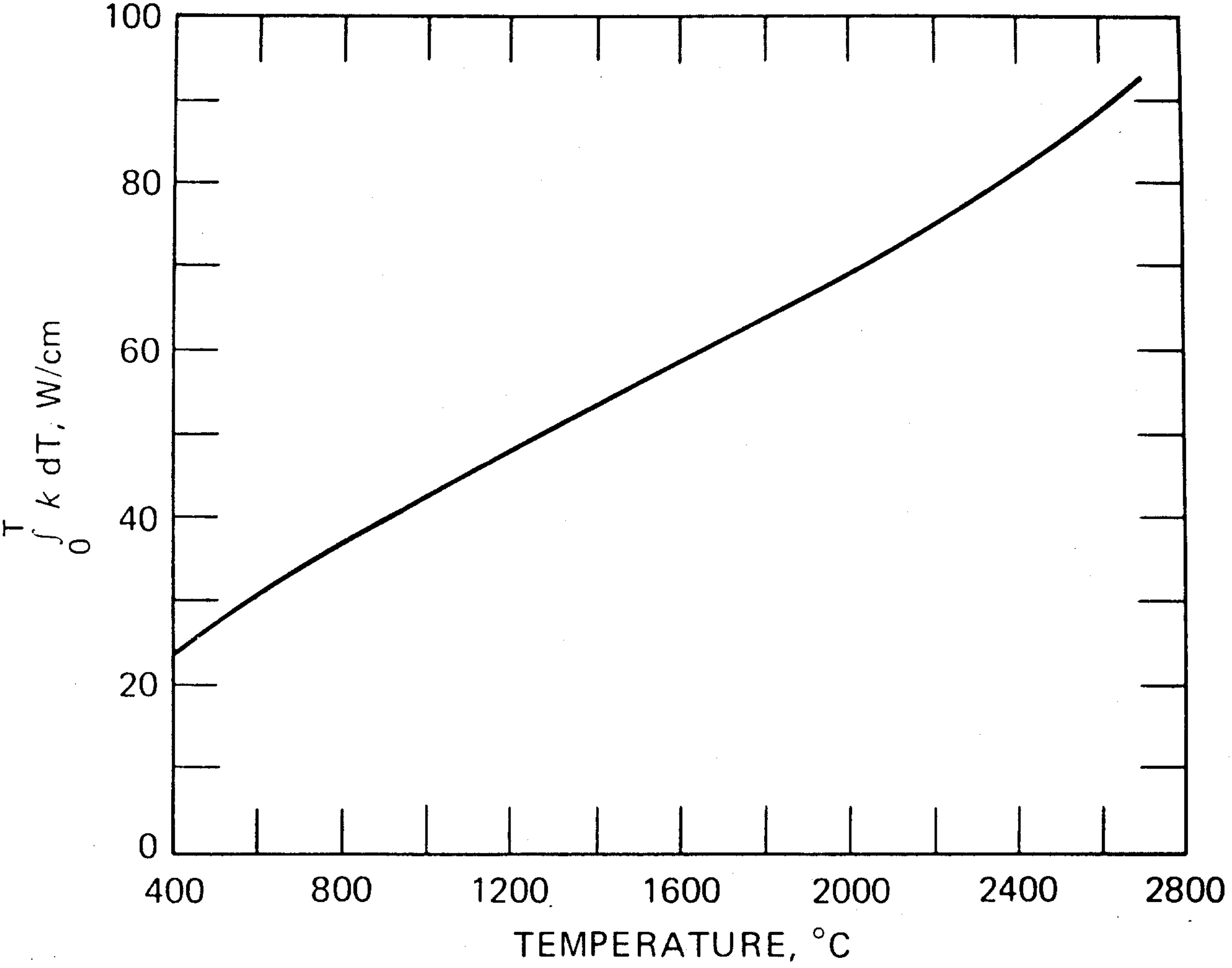


Fig. 10.20 Recommended minimum thermal conductivity integral for $(U_{0.8}Pu_{0.2})O_2$. [From M. J. McNelly, Liquid Metal Fast Breeder Reactor Design Study (1000-MWe UO_2—PuO_2 Fueled Plant), 2 vols., USAEC Report GEAP-4418, General Electric Company, January 1963.]

κ **is reciprocal of neutron diffusion length**

$$\kappa = \frac{1}{L} \equiv \frac{\sqrt{6}}{\sqrt{\overline{r^2}}}$$

$\overline{r^2}$ **=** mean Vector distance that a monoenergetic neutron travels fom its source to where it is absorbed

$$\mathbf{H(r)} = \frac{\mathbf{P}}{\pi\mathbf{R}^2}\left\{\frac{(\kappa\mathbf{R})}{2\mathbf{I}_1(\kappa\mathbf{R})}\right\}\mathbf{I}_0(\kappa\mathbf{R})$$

Bessel Functions

===

Using H(r) in equation above

$$\int_{T_s}^{T_0}\mathbf{k\,dT} = \frac{\mathbf{P}}{4\pi}\left[\frac{\mathbf{I}_0(\kappa\mathbf{R}) - 1}{\left(\frac{\kappa\mathbf{R}}{2}\right)\bullet\mathbf{I}_1(\kappa\mathbf{R})}\right]$$

See figure 10.21.

Note that reduced Conductivity Integral means lower T_O, ----same as moving the heat source to the outside

===

- **Note flux depression in fast reactors is absent**

10.4.3

Effect of Fuel Restructuring

***4 Main Regions** -(Figure 10.23)*

	Radius	T °C	$\frac{\rho_i}{\rho_s}$,%
1.) Central Void	r_o	≈ 2800	0
2.) Columnar Region	r_o to r_1	1700-2200	98-99
3.) Equiaxed Grains	r_1 to r_2	1600-1700	95-97
4.) As Fabricated	r_2 to r_3	≈ 1000	as fab.

Can determine the central void radius by;

$$r_o^2 = \left\{\frac{(\rho_1 - \rho_2)}{\rho_1}\right\} r_1^2 + \left\{\frac{(\rho_2 - \rho_3)}{\rho_1}\right\} r_2^2 \qquad \textit{(10.70}$$

Note ; we have to correct volumetric heating rates by density differences

$$H_3 = \frac{P}{\pi R^2}, \qquad H_2 = \left(\frac{P}{\pi R^2}\right) \bullet \left[\frac{\rho_2}{\rho_3}\right] \ldots\ldots\ldots$$

Using these values with the heat conduction eq., we find that things can be expressed as a function of as fabricated porosity

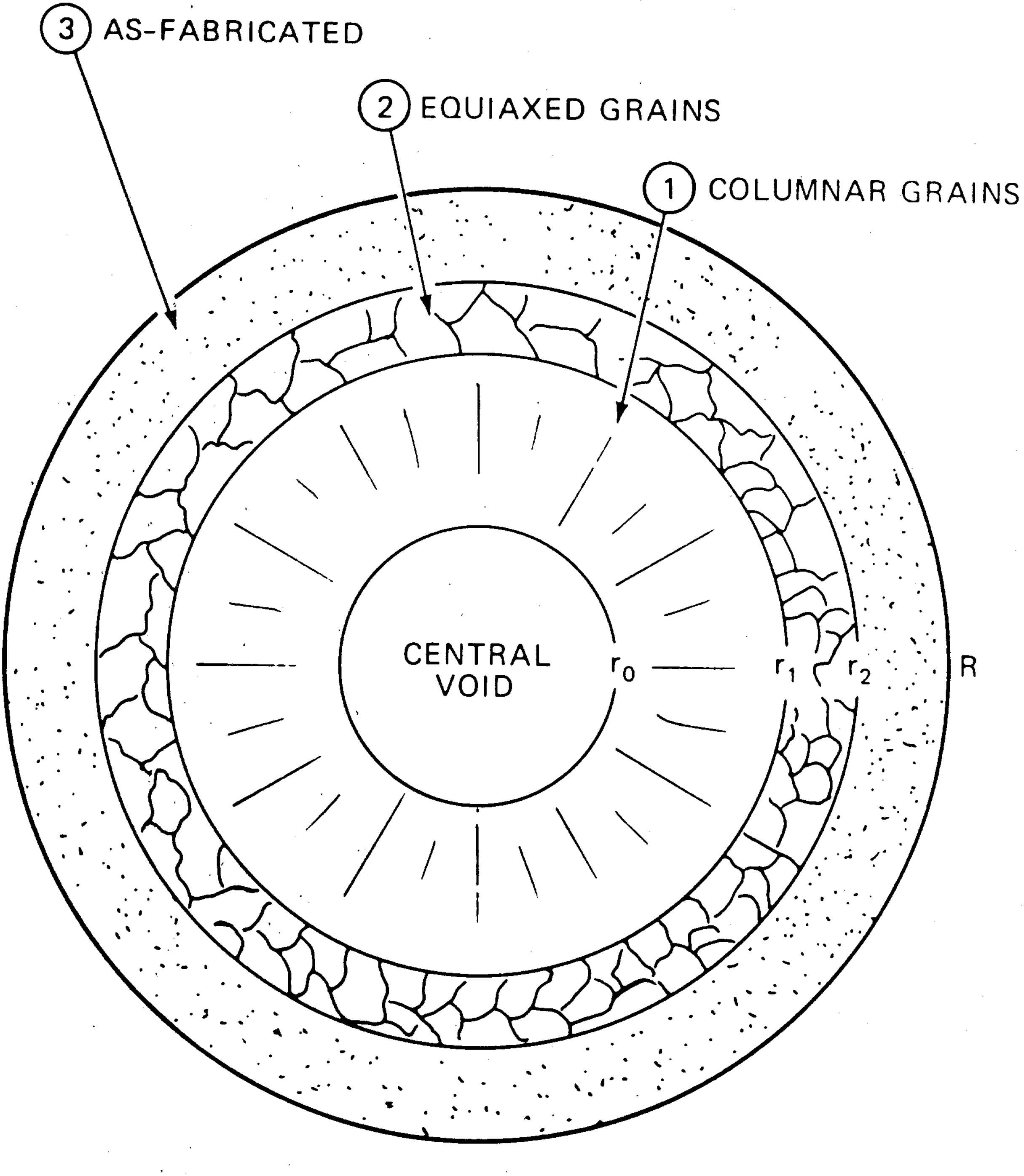
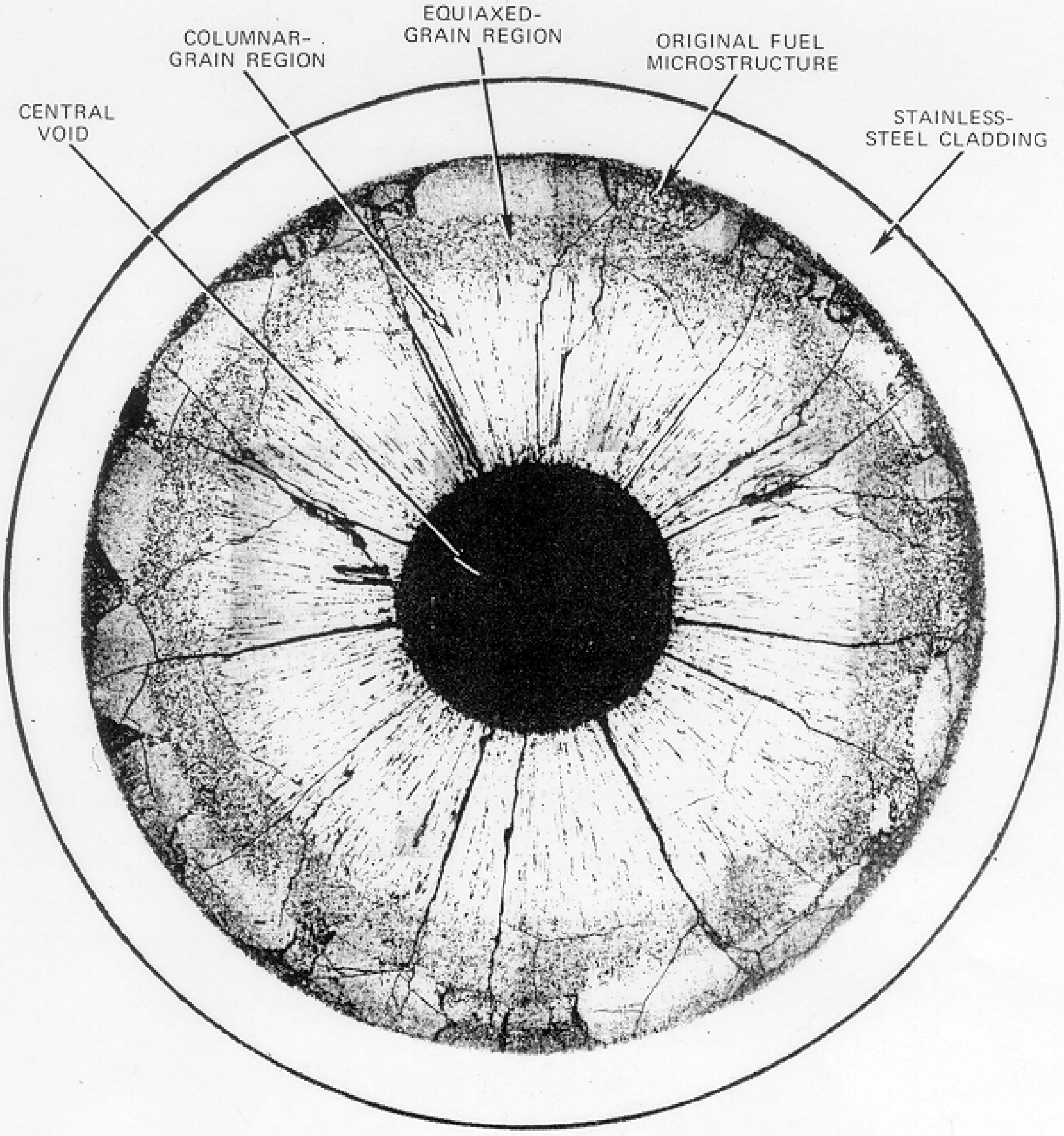


Fig. 10.23 Regions of a restructured fuel rod.



Cross section of mixed-oxide fuel rod irradiated to 2.7% burnup. No melting. [From D. R. O'Boyle et al., *J.*
, 29: 27 (1969).]

As fabricated region;

$$\int_{T_s}^{T_2} k_3\,dT = \left(\frac{P}{4\pi}\right)\left[1-\left(\frac{r_2}{R}\right)^2\right] \qquad 10.88$$

Equiaxed Region; (10.89)

$$\int_{T_2}^{T_1} k_2\,dT = \left(\frac{P}{4\pi}\right)\left(\frac{\rho_2}{\rho_3}\right)\left(\frac{r_2}{R}\right)^2\left[1-\left(\frac{r_1}{r_2}\right)^2-\left(1-\frac{\rho_3}{\rho_2}\right)\ln\left(\frac{r_2}{r_1}\right)^2\right]$$

Columnar Region (10.90)

$$\int_{T_1}^{T_0} k_1\,dT = \left(\frac{P}{4\pi}\right)\left(\frac{\rho_1}{\rho_3}\right)\left(\frac{r_1}{R}\right)^2\left[1-\left(\frac{r_0}{r_1}\right)^2-\left(\frac{r_o}{r_1}\right)^2\ln\left(\frac{r_1}{r_o}\right)^2\right]$$

Procedure to Find Temperature Distribution

$T_s, P, \frac{\rho 3}{\rho s}$ ---------> $T_1, \frac{\rho 1}{\rho s}$ -----------> **Determine**

known $T_2, \frac{\rho 2}{\rho s}$ $\int_{T_i}^{T_{i-1}} k_i\,dT$

Experiment

i.e., for 95% dense material (10.91)

$$\int_{T_i}^{T_{i-1}} k_i\,dT = \left[\left(\int_0^{T_{i-1}} k\,dT\right)_{P=0.05} - \left(\int_0^{T_i} k\,dT\right)_{P=0.05}\right] \bullet \left\{\frac{f(P_i)}{f(0.05)}\right\}$$

where $f(P_i)$ = reduction in thermal conductivity
in the i_{th} zone with P_i porosity
(see eq. 10.40, 10.42, 10.44)

once $\int k_i dT$ is determined for as fabricated and equiaxed zones by eq. 10.91;

then eq. 10.88 is used to find $\frac{r_2}{R}$

then eq. 10.89 is used to find $\frac{r_1}{R}$

then eq. 10.70 is used to find r_o

then eq. 10.90 is used to find $\int_{T_1}^{T_o} k_1 dT$

then eq. 10.91 is used to find $\left(\int_0^{T_o} k\ dT\right)_P$

then fig. 10.20 gives T_o (fig.10.24)

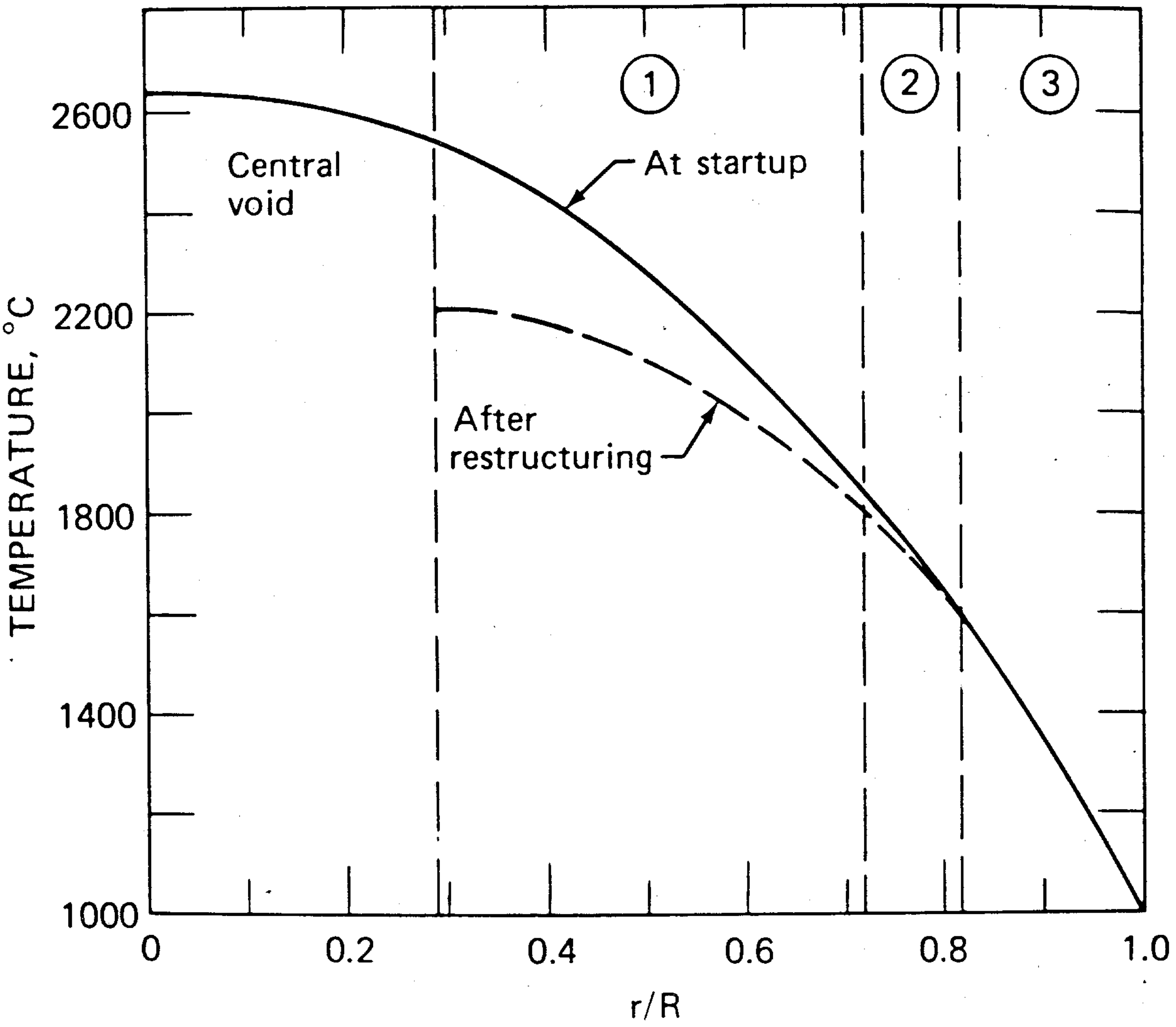


Fig. 10.24 Temperature distribution in a mixed-oxide fuel pin before and after restructuring. $\mathscr{P} = 500$ W/cm; $T_s = 1000°C$; initial density = 85% theoretical density; $T_1 = 1800°C$, $\rho_1/\rho_s = 98\%$ TD; $T_2 = 1600°C$, $\rho_2/\rho_s = 95\%$ TD; $f(P) = 1 - P^{2/3}$.

10.4.4
Fuel Surface Temperature

So far we have assumed we know T_S, but T_S is a function of axial position (z)

From heat transfer considerations;

changes with z

$$T_s = T_{coolant} + \frac{P}{2\pi R_c U}$$

$$T_{inlet} + \frac{\#rods}{QC_{pc}} \int_0^z P(z')dz' \qquad \frac{1}{U} = \frac{1}{h_{gap}} + \frac{t_c}{k_c} + \frac{1}{h_{coolant}}$$

==

Table 10.5 - Note Importance of Gap

———————————————————————————

10.4.5
Conductance of Fuel Cladding -Gap

Initial Conditions	**End of life**
gap thickness ≈ 100 μm	*0 to few μm*
gap gas.......He	*He + Kr+ Xe*
gas pressure ≈1bar	*50-100 bar*

Table 10.5 Heat-Transfer Resistances Exterior to the Fuel in a Fast Reactor*

	Typical conductance, W cm^{-2} $^{\circ}C^{-1}$	Temperature drop, °C
Fuel—cladding gap	1	290
Cladding	9	32
Coolant film	12	24
Overall	0.84	346

*Linear power, 550 W/cm; fuel radius, 3 mm; cladding: stainless steel, 0.25 mm thick, $k_c = 0.22$ W cm^{-1} $^{\circ}C^{-1}$; coolant: sodium.

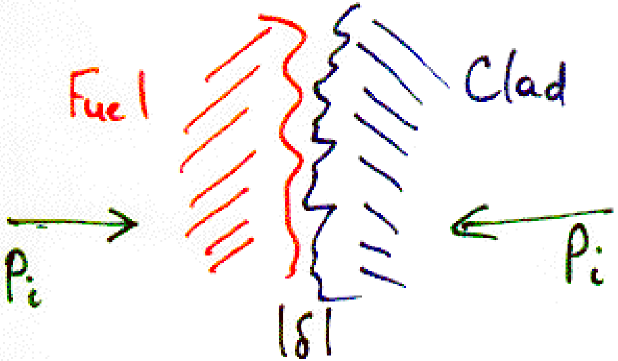
Closed Gap

When fuel expands to contact the cladding, the heat transfer is across the various contact points and the gas in between

$$h_{gap} = \underbrace{\left[\frac{k_g}{(\delta + g_c + g_f)}\right]}_{\text{gas}} + \underbrace{C\left\{\frac{2k_f k_c}{k_f + k_c}\right\}\left\{\frac{P_i}{\delta^{\frac{1}{2}}H}\right\}}_{\text{conduction}}$$

g = Temp jump distance

$$h_{gap} \approx \frac{1\ \text{Watt}}{\text{cm}^2\,{}^\circ\text{C}}$$



Conductance of open gap

$$h_{gap} = \frac{k_g}{t_{gap}} + \frac{4\sigma T^3}{\left(\frac{1}{\varepsilon_c}\right) + \left(\frac{1}{\varepsilon_f}\right) - 1}$$

Thermal Conductivity of Gases

$$k_g \text{ (pure gas)} = A \times 10^{-6}\ T^{0.79} \qquad \text{W/cm°C}$$

15.8 He
1.15 Kr
0.72 Xe

Thermal Conductivity of Gas Mixture

$$k_g = \left(k_{He}\right)^{x_{He}} \left(k_{Xe}\right)^{1-x_{He}}$$

At normally small gaps during operation, the gas conduction term dominates

---

Problem 10.2

Assume:

- *Fast reactor fuel pin characteristics as in Table 10.2*
- *Gap closure*
- *Only He fill gas*
- *linear power, P, is symmetric* (midplane)
- $\int$ **kdT** *from eq. 10.20*
- $h_{coolant} = \dfrac{12\,\text{Watts}}{\text{cm}^2\,{}^\circ\text{C}}$
- $k_c = 0.22\,\dfrac{\text{Watts}}{\text{cm}\,{}^\circ\text{C}}$

Wanted: *T_0 and T_S @ midplane (before core restructuring)*

Schematic