Theoretical Energy Required Per SWU

- **From thermodynamics**

$$\frac{\text{kWh}}{\text{SWU}} = \frac{4RT}{M(1-\alpha)^2}$$

R = Gas Constant= 8.31441 $\frac{\text{J}}{\text{mole °K}}$

Where, T = ° K

M = Molecular Mass

if T= 58° C,

min Energy/SWU = 702 $\frac{\text{kWh}}{\text{SWU}}$

actually, $\approx$ **3000** $\frac{\text{kWh}}{\text{SWU}}$

History

US Capacity in mid 1970's 17,000,000 SWU/y

Cascade Improvement & 10,000,000 $\frac{\text{SWU}}{\text{y}}$

Cascade Uprating Prog (1983)

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total 27,000,000 $\frac{\text{SWU}}{\text{y}}$

Because of plant cancellations in 1970's and 1980's, and because of European built plants,

there is considerable excess capacity

Figure 2.21

Centrifuge Method

Date	Events
1938	*JW Beam, Germany in WWII*
1950's	*Germans built larger gas centrifuges*
1960's	*Centrifuges become competitive*
1970	*FRG and Netherlands build a plant*
1976	*Japanese build a plant*
1977	*Carter Administration announces that the next enrichment plant would use the gas centrifuge process.*
1985	*Almost completed gas centrifuge plant (8,800,000 SWU/y) shut down by Reagan administration for laser isotope plant.*

Principles

- **A rotating drum compresses gas molecules in the cylinder to outer wall**

- **Lighter molecules concentrate in the center**

Figure 3.7

Gas Centrifuge factor;

$$\alpha = 1 + \left\{ \frac{[M_H - M_L]\omega^2 a^2}{2RT} \right\}$$

where; ω **= angular velocity (rad/s)**

a = radius of rotor (inside dimension)

M_H **= Molecular Wt. of** $^{38}UF_6$

M_L **= Molecular Wt. of** $^{35}UF_6$

for 30 cm diameter rotor at 350 $\frac{m}{s}$ **@ 300 °K**

α **= 1.1 to 1.2** ***(Compare to gas diffusion)***

Features

- **Small capacities; a 9,000,000** $\frac{SWU}{y}$ **plant requires 90,000 to 100,000 machines**

- **Reliability - if mean time between failure is 3 years, then more than 3 machines break per h**

- **Lower Power requirements** $\approx 300 \frac{kWh}{SWU}$

  if 1 kWh ≈ 5¢, then;

  Gas centrifuge cost $\approx \frac{\$15}{SWU}$

Gaseous Diffusion $\approx \frac{\$\,150}{SWU}$

Nozzle Method

- **Developed by Becker in FRG, 1960 (later in South Africa, Netherlands)**
- **UF_6 (5%) + H_2 (95%) @ 20 -200 torr forced into orifice (See figure 3.8)**

  $\alpha = 1.015$
- **Must be very precise and clean (≈ 30μ)**
- **Smaller number of stages than gaseous diffusion to get to 3% (500 vs 1200)**
- **Slightly higher electricity consumption**

  $4000\,\frac{kWh}{SWU}$

Atomic Vapor Laser Isotope Separation (AVLIS)

- **First demonstrated in 1975**
- **Principle relies on many absorption lines in U (300,000)**
- **There is a significant separation between ^{35}U and ^{38}U which can be exploited by the use of variable wavelength lasers.**

Figure 3.9

Figure 3.10

1.) Pool of liquid U formed

2.) Photo excite ^{35}U at 5915Å

3.) Use UV light to ionize excited atom

4.) Collect positively charged ^{35}U

Advantage - 300 $\frac{\text{kWh}}{\text{SWU}}$

plus less moving parts and higher reliability

Overall Costs For Enriched U

$$PE = \frac{(PU + PC)F}{P} + PS \sum S$$

Price of Enriched U	**Price of Natural U**	**Conversion Costs**	**SWU Costs**

The price of enriched U can be optimized with respect to waste tails

$$\frac{\delta \, PE}{\delta x_w} = 0$$

gives;

$$\frac{PU + PC}{PS} = \frac{\left(x_f - x_w\right)\Sigma\left(1 - 2x_w\right)}{x_w\left(1 - x_w\right)} + \left(1 - 2x_f\right)\ln\left[\frac{x_w\left(1 - x_f\right)}{x_w\left(1 - x_w\right)}\right]$$

Problem -2

Calculate the cost of enriched U (in $\frac{\$}{kg}$)if the product is 2.8% enriched, the tails assay is 0.25%, and the conversion loss is 0.5%. Assume the price of natural U is \$40/kg, the cost of conversion is \$6/kg, and the price is \$80/SWU.

Answer:

$$\frac{F}{P} = \frac{(2.8-0.25)}{(0.7111-0.25)} = 5.531$$

$$V(x_f) = (2\cdot 0.00711-1)\cdot \ln\left[\frac{0.00711}{(1-0.00711)}\right]$$
$$= 4.869$$

$$V(x_p) = (2\cdot 0.028-1)\cdot \ln\left[\frac{0.028}{(1-0.028)}\right] = 3.348$$

$$V(x_w) = (2\cdot 0.0025-1)\cdot \ln\left[\frac{0.0025}{(1-0.0025)}\right] = 5.959$$

$$S = 3.348+4.531\cdot 5.959-5.531\cdot 4.869 = 3.418\ \frac{SWU}{kg}$$

$$PE = \left[\frac{40}{(1-0.005)}+6\right]\cdot 5.531+80\cdot 3.418 = \$\ 529/kg$$

Total Fabricated Costs

$$FF = \left\{\left[\frac{PU}{\left(1-\ell_f\right)\sum\left(1-\ell_c\right)}\right] + \frac{PC}{\left(1-\ell_f\right)}\right\}\frac{F}{P} + \frac{PS}{\left(1-\ell_f\right)}\sum S + PF$$

ℓ_c = fraction of U lost in conversion
ℓ_f = fraction of U lost in fabrication
PF = Price of Fabricated Fuel

Problem -3

What is the cost of fabricated fuel if the price for fabrication is $200/kg and there is an 0.8% loss during fabrication. Use the numbers from problem 2 for nat. U, conversion, and enrichments costs.

$$FF = \left\{\left[\frac{40}{(1-0.005)\sum(1-0.008)}\right] + \frac{6}{(1-0.008)}\right\}\sum 5.531$$

$$+\frac{80}{(1-0.008)}\sum 3.418 + 200$$

FF= 732 $/kg

Important Points About the Nuclear Fuel Cycle

- **Only 10% of U. S. Uranium requirements are met from domestic sources, 90% comes from outside the U. S. (85% from Canada)**

- **Imported Russian HEU could provide 40% US in future**

- **Worldwide Demand for Enrichment-1996 MSWU/Year**
  - **US-11.2**
  - **Eastern Europe-4.3**
  - **Western Europe-12.0**
  - **Far East-6.2**
  - **Other-0.4**
  - **Total ≈ 34**

Facility	**Countries Involved**	**SWU/y Capacity (12/31/95)**	**% of World Market (1994?)**	**Technology Used**
U. S. Enrichment Corp.	U.S.	19.3	≈40 (70% of US)	Gaseous Diffusion
Eurodif	France, Belgium, Spain, Italy	10.8	≈20	Gaseous Diffusion
ChinaNucl. Energy Ind. Corp.	Peoples Rep. of China	0.5	NA	Gaseous Diffusion
Minatom	Russia	14	≈30	Gas Centrifuge
Urenco	Netherlands, Germany, U. K.	3.4	≈10	Gas Centrifuge
Power Reactor & Nuclear Fuel Dev.Corp.	Japan	0.2	NA	Gas Centrifuge
Japan Nuclear Fuel Industries Company	Japan	0.6	NA	Gas Centrifuge
Atomic Energy Commission	South Africa	0.3	NA	NA
		total-48.7		