Chapter 14
Pore Migration and Fuel Restructuring Kinetics

Major Differences Between Pores and Bubbles

Parameter	Pores	Bubbles
Size	Large, >1 mm	Small ≈ 0.01 mm
Gas	He, CO, CO_2	Fission Gases
Gas Pressure	Low, Few Atm.	High, 100's ATM.
Densification	Important	Not Applicable
Shape	Lenticular	Spherical
Movement Diffusion	Vapor Transport	Surface or Volume Diffusion

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Bubble Migration

Surface Diffusion α 1/R
Volume Diffusion α constant

Pore Migration

Vapor Transport a R (mech. equilibrium.)
See Figures 14.1, 14.2

14.2 Pore Migration by Vapor Transport

Figure 14.5

Set heat transfer parameters equal

$$k_p \left(\frac{dT}{dx}\right)_p = k_s \left(\frac{dT}{dx}\right)$$

and $\frac{k_s}{k_p} \approx 5$

Note ; $v_p \quad J\Omega$

$$J = \left\{\frac{D_g}{kT\delta}\right\}\left[P^\circ(x+\delta) - P(x)\right]$$

$$\left(\frac{dP^\circ}{dT}\right)\left(\frac{dT}{dx}\right)_p \delta$$

Diff. Coeff. for matrix molecules in gas

$$D_g = \left(\frac{Const.T^{\frac{3}{2}}}{\sigma_{12}{}^2\Omega_D p}\right)\sqrt{\left(\frac{M_1 + M_2}{M_1 M_2}\right)}$$

Collision diam. for 2 species in gas

Parameter from theory $\frac{kT}{\varepsilon_{12}}$

Force constant

Two things to consider;

1.) Impurities (Fig. 14.6)
They depress vapor pressure on the hot side.

2.) Cold Side Condensation Limitations
Increases vapor pressure

14.3 Porosity Redistribution Kinetics

Read section for model description
(Figures 14.8, 14.9)

14.4 Columnar Grain Growth

Using the approach by Nicols, get pore velocity as a function of fuel radius. See figure 14.10.

Define d= distance a pore at outer edge of columnar zone moves into zone.

$$t = -\int_{r_1}^{r_1-d} \frac{dr}{v_p(r)} = -\int_{T_1}^{T_1^d} \frac{dT}{\left[v_p \left(\frac{dT}{dr} \right) \right]}$$

outside of columnar grain region

Rest of section involved with above equations for t with appropriate values of v_p

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Problem: Define the fractional radius of central void in terms of initial porosity

14.5 Equiaxed Grain Growth Region

- **Curved grain boundaries cause large grains to grow at the expense of small grains**

Figure 14.12

Atoms like to be on the concave side instead of convex side because they are surrounded by more matrix atoms

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- **Kinetics**

$$d^m - d_o^m = k_o t \exp\left(-\frac{Q}{kT}\right)$$

$$d^2 - d_o^2 = k_o t^a \exp\left(-\frac{Q'}{kT}\right)$$

Problem 14.1

During operation of a fresh fuel pin, all pores within 0.8 R migrate toward the center void.

a.) Calculate radius of central void if initial porosity is P_o

b.) Determine T(r) in $r_o < r < 0.8$ R

- assume k independent of T
- assume flat power density appropriate to r_o.

c.) Assume $v_p(T)$ is known, how would you calculate time to form a central void

d.) In $v_p(T)$ of c.) , use a gas pressure p which reflects the collection of gas in the path of the pore.

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a.) Calculate r_{of}

$$\pi(0.8R)^2 P_o = \pi r_{of}^2$$

$$\frac{r_{of}}{R} = 0.8\sqrt{P_o} \qquad 1.)$$

b.) For constant k, the heat conduction in the fuel is;

$$\frac{k}{r}\left[\frac{d}{dr}\left(r\frac{dT}{dr}\right)\right] = -H \qquad 2)$$

use $T(R) = T_s$ and $\left(\frac{dT}{dr}\right)_{r_o} = 0$ 3.) , 4.)

If the rod is operated at constant linear power, the average heat generation rate in the columnar grain region is (if the restructuring takes place in that zone only);

$$\bar{H} = \frac{H_o}{1 - \left(\frac{r_o}{0.8R}\right)^2} \qquad 5.)$$

where H_0 is the initial power density of the fuel

Since r_o starts at 0 and increases to value given above, $\frac{r_o}{R}$ increases with time and therefore $\bar{H}$ increases with time.

Assume that local power density is equal to average power density throughout the region (H in equation 2 is equal to $\bar{H}$ given by eq 5 for all $r_o < r < 0.8\ R$)

$$T_1(r) = -\frac{\left[\bar{H}r^2\right]}{4k} + A\ln r + B \qquad 6.)$$

From boundary condition 4

$$A = \left[\frac{\bar{H}r_o^2}{2k}\right] = \left[\frac{H_o R^2}{2k}\right]\left(\frac{\bar{H}}{H_o}\right)\left(\frac{r_o}{R}\right)^2 \qquad 7.)$$

Heat Generation Rate in $0.8R<r<R$

In the unrestructured region; $(r>0.8R)$

$$T_2(r) = -\left(\frac{H_o r^2}{4k}\right) + C\ln r + D \qquad 8.)$$

Using B. C. (3)

$$T_s = -\left(\frac{H_o R^2}{4k}\right) + C\ln R + D \qquad 9.)$$

If we match the results at $r=0.8\,R$

$$T_1(0.8R) = T_2(0.8R) \qquad 11.)$$

and

$$\left(\frac{dT_1}{dr}\right)_{0.8R} = \left(\frac{dT_2}{dr}\right)_{0.8R} \qquad 12.)$$

Note it is assumed that k is constant throughout

$$r_o < r < R$$

Using (6) and (8) in (11) yields;

$$-\left(\frac{\overline{H}}{4k}\right)(0.8R)^2 + A\ln(0.8R) + B =$$

$$= -\left(\frac{H_o}{4k}\right)(0.8R)^2 + C\ln(0.8R) + D \qquad 13.)$$

using (6) and (8) in (12) yields

$$-\left(\frac{\bar{H}}{2k}\right)(0.8R) + \left(\frac{A}{0.8R}\right) = -\left(\frac{H_o}{2k}\right)(0.8R) + \left(\frac{C}{0.8R}\right) \qquad 14.)$$

Solving (14) for C gives

$$C = A - \left\{\frac{\left(\bar{H} - H_o\right)}{2k}\right\}(0.8R)^2 = A - \left(\frac{H_o R^2}{2k}\right)(0.64)\left(\frac{\bar{H}}{H_o} - 1\right) \qquad 15.)$$

Using eq 7 for A we get

$$C = \left(\frac{H_o R^2}{2k}\right)\left[\left(\frac{r_o}{R}\right)^2\left(\frac{\bar{H}}{H_o}\right) - 0.64\left(\frac{\bar{H}}{H_o} - 1\right)\right] \qquad 16)$$

Solving eq (9) for D

$$D = T_s + \left(\frac{H_o R^2}{4k}\right) - C\ln R \qquad 17.)$$

where C is given by eq (16)

Using eq (13) and (17) to determine B

$$B = \left(\frac{\bar{H} - H_o}{4k}\right)\bullet(0.8R)^2 + (C - A)\ln(0.8R) + T_s + \left(\frac{H_o R^2}{4k}\right)$$
$$-C\ln R$$

Substituting B into (6)

$$T_1(r) - T_s = -\left(\frac{\bar{H} r^2}{4k}\right) + A\ln r + \left(\frac{\bar{H} - H_o}{4k}\right)(0.8R)^2$$

$$+(C - A)\ln(0.8R) + \left(\frac{H_o R^2}{4k}\right) - C\ln R$$

use

$$C\ln R = [A + (C - A)]\ln R = A\ln R + (C - A)\ln R$$

and

$$T_1 - T_s = -\left(\frac{\bar{H} r^2}{4k}\right) + \left(\frac{H_o R^2}{4k}\right) + \left(\frac{\bar{H} - H_o}{4k}\right)(0.8R)^2$$

$$+A\ln\left(\frac{r}{R}\right) + (C - A)\ln(0.8)$$

$$T_1 - T_s = -\left(\frac{H_o R^2}{4k}\right)\left\{0.36 + \left(\frac{\bar{H}}{H_o}\right)\left(0.64 - \left\langle\frac{r^2}{R^2}\right\rangle\right)\right\}$$

$$+\left(\frac{H_o R^2}{2k}\right)\left(\frac{\bar{H}}{H_o}\right)\left(\frac{r_o}{R}\right)^2 \ln\left(\frac{r}{R}\right)$$

$$-\left(\frac{H_o R^2}{2k}\right)[0.64\ln(0.8)]\left(\frac{\bar{H}}{H_o} - 1\right)$$

eq 5 $$\frac{\bar{H}}{H_o} = \frac{1}{\left(1 - \left(\frac{r_o}{0.8R}\right)^2\right)}$$

and $H_o = \frac{\quad}{\pi R^2}$

$$T_1 - T_s = \frac{}{4\pi k}\left\{ 0.36 + \frac{\left[0.64 - \left(\frac{r}{R}\right)^2\right]}{\left[1 - \left(\frac{r_o}{R}\right)^2\right]} + \frac{\left[\left(\frac{r_o}{R}\right)^2\right]}{\left[1 - \left(\frac{r_o}{R}\right)^2\right]} \bullet \left[\ln\left(\frac{r}{R}\right) - 0.64\ln(0.8)\right] \right\} \quad 18)$$

Eq (18) gives the temperature distribution in the columnar grain region as a function of $\left(\frac{r_o}{R}\right)$.
The central void radius starts at r=0 at t=0 and grows to r_{of} ***(eq 1) at the end of restructuring.***
We assume we know $\left(\frac{r_o}{R}\right)$ ***as a function of time***

Therefore the temperature distribution in the columnar grain region is a function of both r and t, or T(r,t).

c.) Time to complete restructuring = Time for a pore at the outer edge of the columnar grain region to migrate to the final central void position $r_o = r_{of}$

The pore velocity is a known function of temperature, but as a result of the solution in b.), the temperature is a function of r and t. Therefore, the velocity of a pore is a function of r and t, or,

$$v_p(r,t)$$

Let r= radial position of a pore which started at r=0.8R at t=0.

$$v_p(r,t) = dr/dt \qquad 19)$$

$$r(0) = 0.8R \qquad 20)$$

Eq. 19 must be integrated numerically, obtaining v_p as a function of T from pore migration theory and $T_1(r,t)$ from (18)

Eq (19) must be integrated from r=0.8R up to $r = r_{of} = 0.8R\sqrt{P}$. The time at which this is reached is t_f.

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d.) The pore velocity is given by a combination of eqs (14.6) and (14.9)

$$v_p(T,p) = \left(\frac{C}{pT^{\frac{3}{2}}}\right)\exp\left(-\frac{\Delta H_{vap}}{RT}\right) \qquad 21)$$

where C is a constant

In the as fabricated (cold) fuel, the number of gas atoms in each pore is given by;

$$P_o\left(\frac{4\pi r_{po}^3}{3}\right) = mkT_a \qquad 22)$$

When heated to a temperature T and subject to the condition of mechanical equilibrium;

$$p\left(\frac{4\pi r_p^3}{3}\right) = mkT \qquad 23)$$

$$p = \frac{2\gamma}{r_p} \qquad 24)$$

dividing (22) by (23) we get

$$\left[\frac{p_o}{p}\right]\bullet\left(\frac{r_{po}}{r_p}\right)^3 = \frac{T_a}{T} \qquad 25)$$

Using r_p from 24 and 25;

$$p = \sqrt{\frac{T_a}{p_o T}} \cdot \left(\frac{2\gamma}{r_{po}} \right)^{\frac{3}{2}} \qquad 26)$$

substituting (26) into (21) gives

$$v_p(T) = C\sqrt{\frac{p_o}{T_a}} \cdot \left(\frac{r_{po}}{2\gamma} \right)^{\frac{3}{2}} \cdot \exp\left(-\frac{\Delta H_{vap}}{RT} \right) \cdot \frac{1}{T}$$