

10.3 Thermal Conductivity

Problem: *To solve for the temperature distribution in a cylindrical fuel rod.*

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$$\frac{1}{r} \frac{d}{dr} r k \frac{dT}{dr} + H = 0$$

**Materials
Properties**

**Nuclear
Properties**

With proper boundary conditions;

$$T - T_s = \frac{HR^2}{4k} \left(1 - \frac{r^2}{R^2} \right)$$

Normally assume that H, k independent of r,z

Not True !

Things that affect k

- **Stoichiometry**
 - **Pu production and redistribution**
 - **Burn up**
 - **Porosity**
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Figure - Relative Thermal Conductivities

General Features of Thermal Conductivity

From chapt. 2 and eq. 10.18

$$k_s = \frac{1}{3} \cdot \quad \cdot C_v \cdot u \cdot$$

phonon speed mean free path
between collisions
Main effect of irrad.

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The mean free path can be derived from the kinetic theory of gases;

$$= \frac{1}{p \ n_p}$$

Collision cross section density of phonons, T

SO

$$= \frac{1}{T}$$

Constraints;

T--> cannot be smaller than a_0

T-->0 cannot be larger than distance
between defects

- **The more deviation from stoichiometry,**
- **The more foreign atoms,**
- **The more porosity,**

The smaller is !

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Typical fix;

$$= \frac{1}{A' + B' T}$$

Found that one could explain high temperature behavior by the generation of defects at high T, which modifies the heat capacity as follows

$$C_v = C_{vo} + \frac{dH_{ex}}{dT}$$

Which leads to the temperature dependence of the thermal conductivity;

$$k_s = \frac{1 + \left(\frac{1}{C_{vo}}\right)\left(\frac{dH_{ex}}{dT}\right)}{A + BT}$$

10.3.2 Stoichiometric Effect

When T is low enough to neglect thermal generation of defects;

$$k_s \propto \frac{1}{A(x,q) + B(x,q)T}$$

mainly
stoichiometry

mainly
Pu content

Coefficient A,

$$A = A_0 + \Delta A(x)$$

from interactions with point defects

$$\Delta A = A^* \sum_i \left\{ Y_i \left(\frac{(r_i - r)}{r} \right)^2 \right\}$$

Const. Conc. of
Point defects

r = radius of host lattice
r_i = radius of defect

Comment about r_i ;

in metals, r_i < r for vacancies

in ceramics, r_i > r for vacancies

(because of charge repulsion)



**For hypostoichiometric UO_2 ,
 replace 2 Pu^{+4} with 2 Pu^{+3} when
 we lose an oxygen atom**

$$\Delta A = A^* [2(0.15)^2 + (0.15)^2] x$$

$$\begin{array}{l} \text{Pu}^{+3} \rightarrow \text{Pu}^{+4} \\ 1.04 \rightarrow 0.93 \end{array}$$

$$= 0.068 A^* x$$

or, $\Delta A/x \approx 400$

Finally,

$$k_s = \frac{1}{A_o + 400x + B(q)T} \quad \text{See Figure}$$

Effect of Pu Content

Generally, B increases with Pu content

$$B = B_o \left\{ \left[\frac{a_o}{(a_o)_{UO_2}} \right]^2 \sum \left[\frac{(T_m)_{UO_2}}{T_m} \right]^{\frac{3}{2}} \right\}$$

- No real good theory, but B depends on molar volume and Debye Temperature (a T_{mp})
- Note that stoichiometry is more effective than Pu content (see figure)

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Burn Up Effects

Should lower k_s but no good experimental conformation of that

The Low thermal Conductivity of UO2 is a Major Disadvantage

