

# **Experimental and Theoretical Studies of Electrostatic Confinement**

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# **Outline**

**I. Electrostatic Basics**

**II. Motivation**

**III. Review of Periodically Oscillating  
Plasma Sphere physics**

**IV. Potential "Show Stoppers"**

**V. Equilibrium and Stability of Virtual  
Cathodes**

**VI. Experiment**

**VII. Spherically Convergent Ion Focus  
(SCIF) Systems**

**VIII. INS as a neutron Source**

**IX. Conclusions**

# Electrostatic Basics

## I. $T \sim V_{\text{applied}}$ (easy)

- A. Inexpensive machines give lots of neutrons
- B. Near-term applications as neutron sources
- C. Advanced fuels are easier than in conventional systems

## II. $\tau_{\text{classical}} \rightarrow \infty$ (not too bad)

- A. Confine plasma in deep potential wells
- B. No cross-field diffusion size limit

## III. Density limited by $\lambda_{\text{Deff}}/a \sim 1$ (difficult)

- A. Low density  $\Rightarrow$  Low power density
- B. Power density  $\sim 1/r^4$ , Power out  $\sim 1/r$

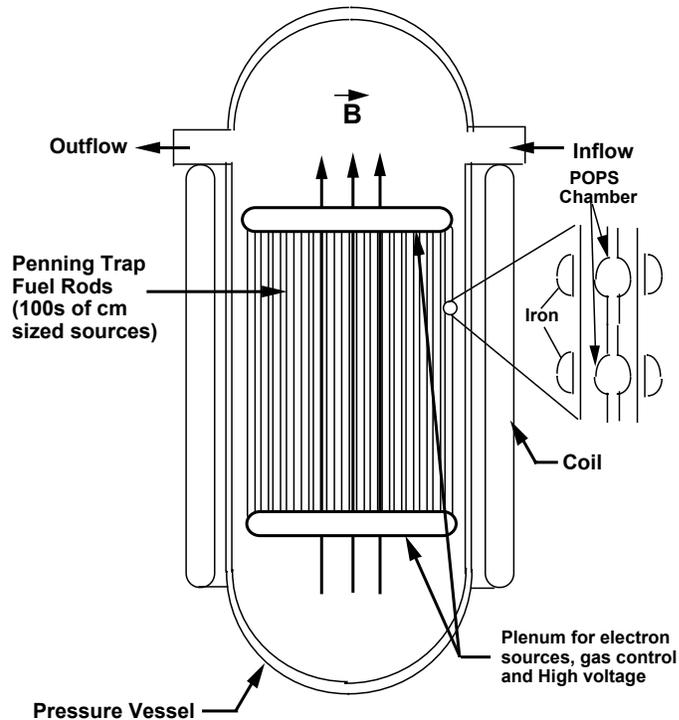
## IV. Summary

- A. Small size, high voltage (difficult technology problem)
- B. Density enhancement scheme required
  1. Focused ion plasma ( subject to Nevins problem)
  2. Oscillating thermal plasma

# Why IEC?

- \* **Massively Modular Penning Trap Reactor**

## Penning Trap Reactor Vessel



- \* **Mass Power Density for Modular Reactors is a paradigm shift from conventional systems.**

$$\text{MPD} = \frac{2 \pi \eta P_{\text{wall}}}{\rho} \frac{a^2}{a (2 a t + t^2)} \frac{F a}{r_{\text{tube}}}$$

- \* **High MPD (~ LWR) can be achieved with conventional wall loads.**

- \* **Why can we do this with IECs?**

- **Confinement doesn't depend on size**
- **Power out ~ 1/r<sub>tube</sub>**

- \* **Problem: Beam Systems have Trouble getting Q > 1.**

# Power Scaling

## I. Poisson's Equation

$$n_e \sim \nabla^2 \phi \sim \Delta \phi / a^2 \text{ (note: here "a" is the } r_{\text{tube}})$$

## II. Average ion density $\sim 10\%$ electron density

$$n_i \sim .1 n_e$$

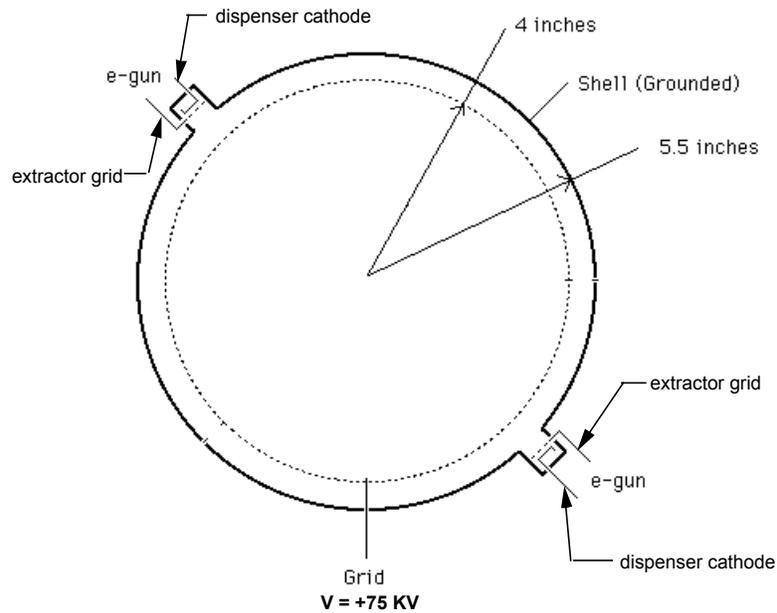
## III. Fusion power density

$$p = 1/2(n_i)^2 \langle \sigma v \rangle \sim 1/a^4$$

## IV. Total power

$$P = 4/3 \pi a^3 p \sim 1/a$$

# POPS Ion Physics<sup>2,3</sup>

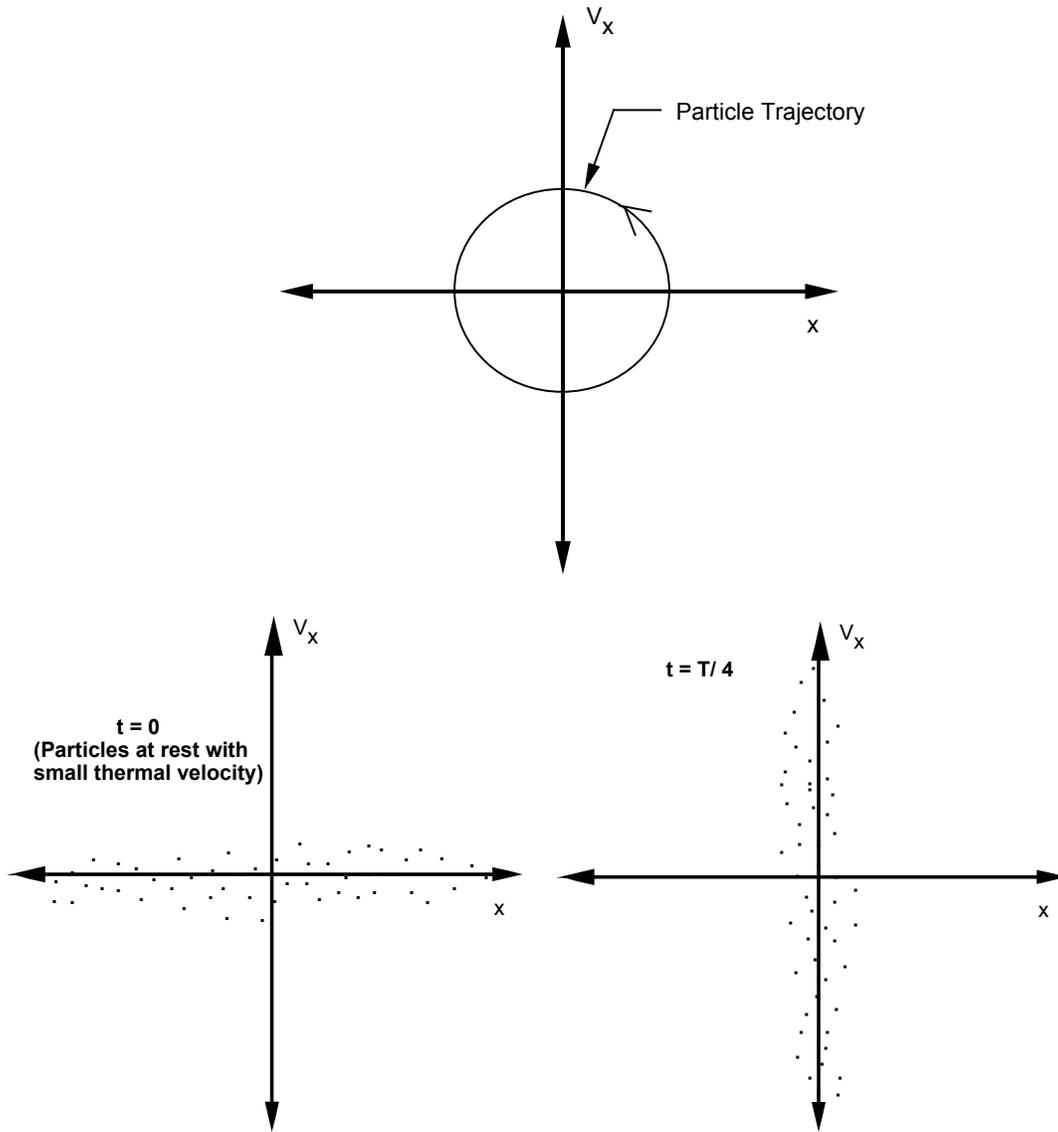


1. **1-D Time and space separable solutions exist.**
2. **Solutions are stable.**
3. **Stable spectrum has an infinite number of discrete modes with accumulation point at  $\omega=0$ .**
4. **Density profile is Gaussian in radius, Maxwellian in velocity.**
5. **Profiles remain in l.t.e. throughout oscillation (eliminates "Nevins Problem")**
6. **Solutions likely to be attractors.**

2. D. C. Barnes, R. A. Nebel, *Physics of Plasmas* 5, 2498 (1998).

3. R. A. Nebel, D. C. Barnes, *Fusion Technology* 38, 28 (1998).

# Ion Phase Space Motion in a Harmonic Oscillator



- **Distribution Functions Move as a Rigid Rotor.**
- **Density and Velocity Profiles Exchange Every 1/4 Period.**
- **Maxwellian velocity distribution requires Gaussian density profile**

# Potential Show Stoppers

- **High Voltage, Small Size**
  - **How bad is it?**
  - **Stability limits on Virtual Cathode (Electron Cloud).**
- **Electron Cloud Uniformity**
  - **Attractor  $\Rightarrow$  Energy Flow-through Issue**
  - **How fast do the ions relax and thermalize?**
- **Space Charge Neutralization During Ion Collapse**
- **Impurity Control**
- **Oscillation Phase Locking and Control**
- **Insulator Integrity**

# Virtual Cathode Equilibrium and Stability

## Equilibrium

Pressure balance and Poisson's equation lead to

$$\phi_{0\text{eff}}(\mathbf{r}) = \phi_{00\text{eff}}(1 - (r/a)^2)$$

where

$$\phi_{00\text{eff}} = -e(n_0 - n_b)a^2/(6\epsilon_0)$$

and

$$p_0(\mathbf{r}) = p_{00} + en_0(\phi_{0\text{eff}}(\mathbf{r}) - \phi_{00\text{eff}})$$

## Stability

Energy principle leads to sufficient condition for stability

$$ds/d\phi_{0\text{eff}} < 0. \text{ (Rayleigh-Taylor criterion)}$$

where  $s = p/(m_e n)^{\gamma}$  is the entropy density. But for constant density

$$ds/d\phi_{0\text{eff}} = (dp/dr)/(d\phi_{0\text{eff}}/dr) = en_0 > 0$$

which is always violated everywhere in the plasma.

## Dimensionless Linear Eigenvalue Equations

$$\phi'' + 2/x\phi' - 1(1+1)/x^2\phi = 6n$$

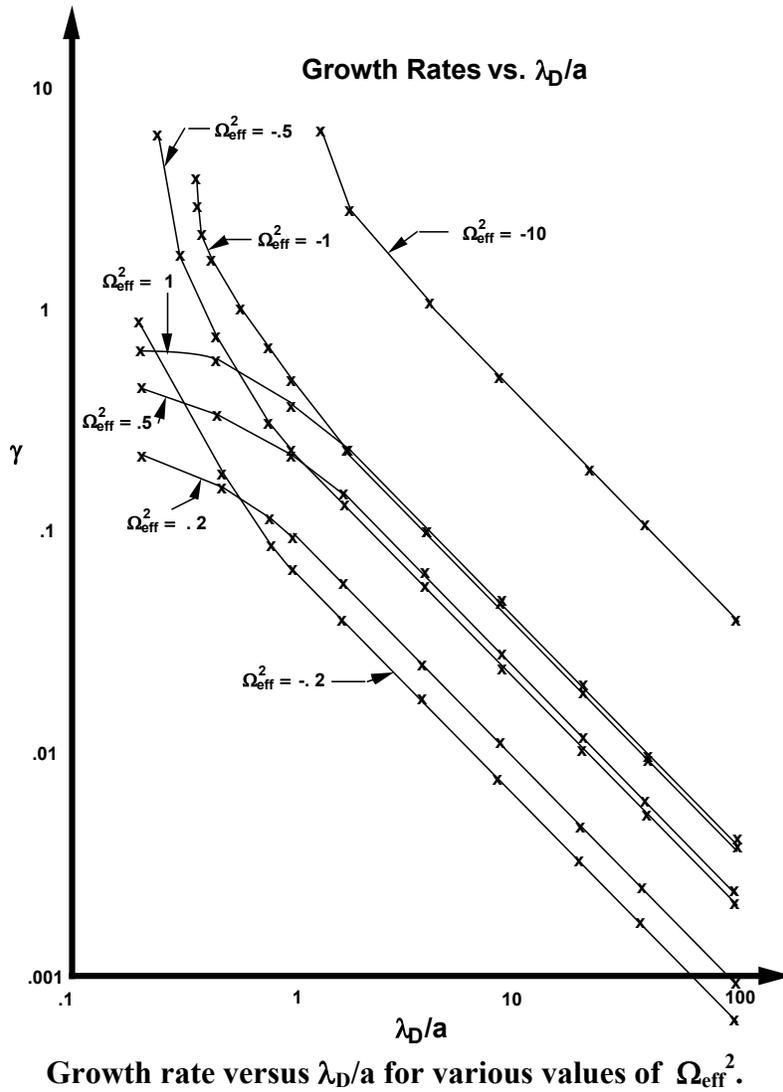
$$(-\Omega^2 + \Omega_{\text{eff}}^2 + 1)n - (\lambda_D/a)^2[p'' + 2/x p' - 1(1+1)/x^2 p] + (\Omega_{\text{eff}}^2 x/3)n' = 0$$

$$-\Omega^2(\lambda_D/a)^2 p + \Omega_{\text{eff}}^2 x/3[-(\lambda_D/a)^2 p' + \phi'/6 + (\Omega_{\text{eff}}^2 x/3)n] + \Gamma[(\lambda_D/a)^2 + \Omega_{\text{eff}}^2 x^2/6]\Omega^2 n = 0$$

where  $\omega_{pe}^2 \equiv e^2 n_0 / (\epsilon_0 m_e)$ ,  $\omega_{\text{eff}}^2 \equiv \omega_{pe}^2 - e^2 n_b / (\epsilon_0 m_e)$ ,  $\Omega^2 \equiv \omega^2 / \omega_{pe}^2$ ,  $\Omega_{\text{eff}}^2 \equiv \omega_{\text{eff}}^2 / \omega_{pe}^2$ ,  
 $\lambda_D \equiv [\epsilon_0 k_B T_{00} / (n_0 e^2)]^{1/2}$ ,  $x \equiv r/a$ ,  $p \equiv \delta p / p_{00}$ ,  $n \equiv \delta n / n_0$ ,  $\phi \equiv -\delta\phi / \phi_{00}$ .

The equations form a fourth order, self-adjoint system of equations. The only dependences are on  $\lambda_D/a$ ,  $\omega_{\text{eff}}^2$ ,  $\Gamma$ , and  $\omega_{pe}^2$ .

# Growth Rates



- \* **No window of absolute stability**
- \* **Marginal points at Brillouin limit and  $\lambda_D/a \rightarrow \text{infinity}$ .**
- \*  **$\gamma \sim 1 / (\lambda_D/a)$  for large  $\lambda_D/a$  (incompressible Rayleigh-Taylor limit).**

# Stable Virtual Cathodes

**Do stable profiles exist that are sufficiently close to the desired harmonic oscillator potential that the POPS scheme will work?**

**Combine the marginally stable compressible Rayleigh-Taylor profile:**

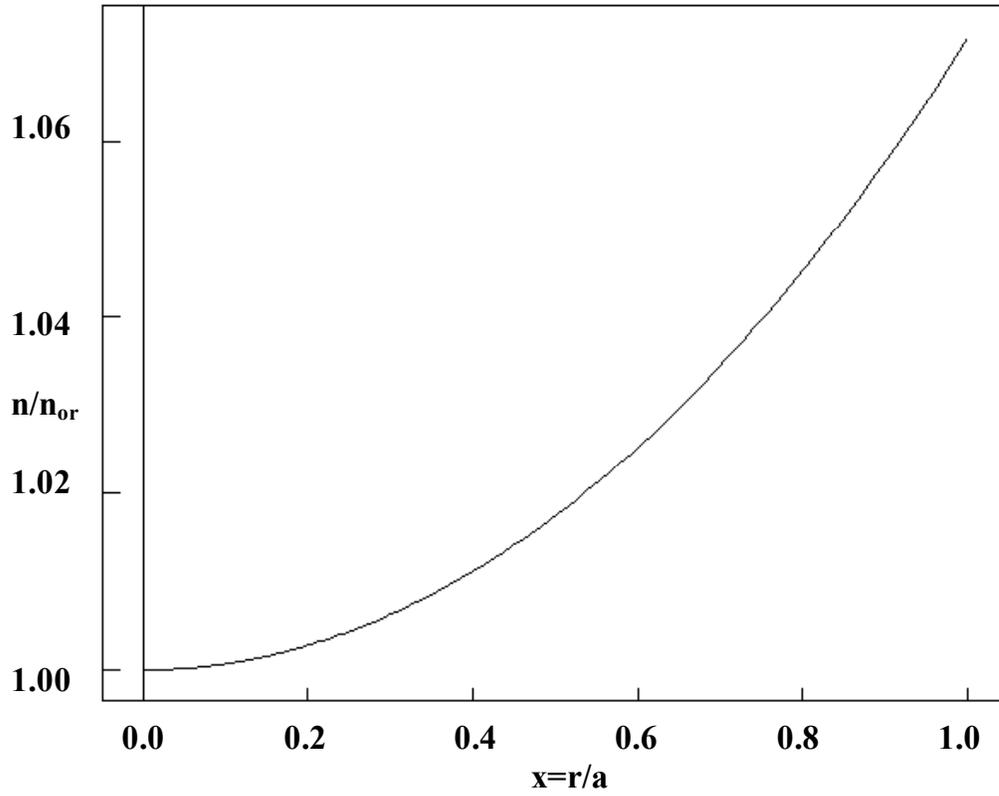
$$d(p/n^\Gamma)/dr = 0$$

**with pressure balance and Poisson's equation and write in dimensionless form:**

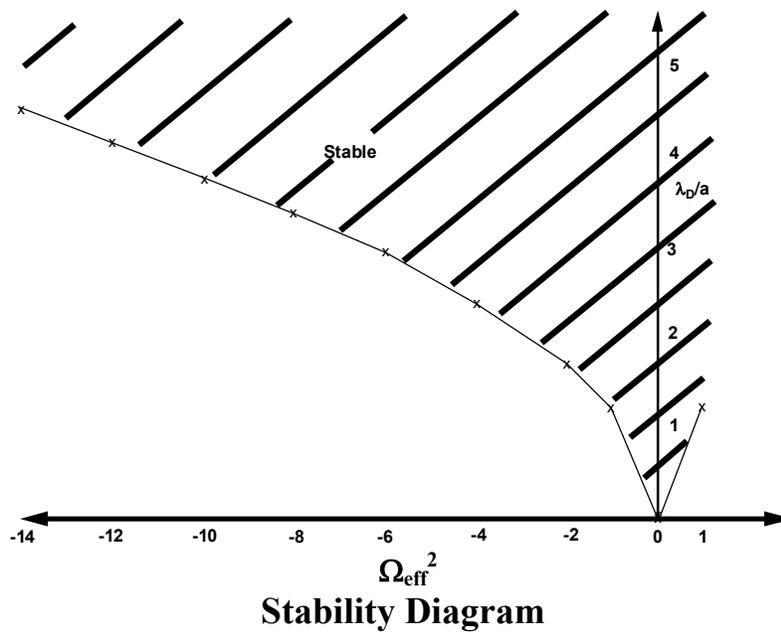
$$v'' + 2/x v' + [(\Gamma-2)/v](v)^2 - \Gamma/(\lambda_D/a)^2 v^{(2-\Gamma)}(v - v_b) = 0$$

**where  $v \equiv n/n_{or}$  and  $v_b \equiv n_b/n_{or}$ .**

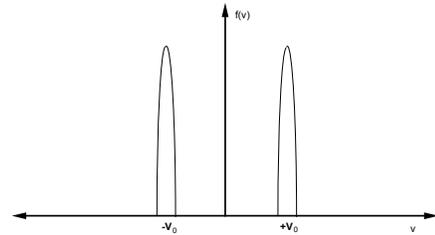
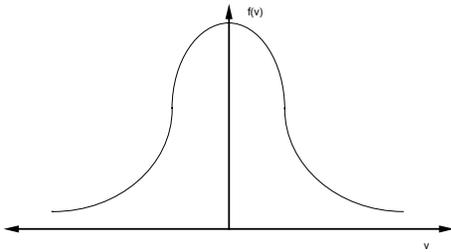
# Marginal Stability



Marginally stable density profile for  $\lambda_D/a = 1.2$ ,  $\delta\phi/\phi = .1\%$



# Kinetic Effects



Compressible Rayleigh-Taylor



Electron-Electron Two-Stream

\* **Approximations**

- Cold beam
- Slab geometry

\* **Dispersion relation**

$$1 = \omega_{pe}^2 [1/(\omega + kV_0)^2 + 1/(\omega - kV_0)^2]$$

\* **Marginal Limit**

$$\omega_{pe}^2 = k^2 V_0^2 / 2$$

or

$$(\lambda_{Deff}/a)^2 = 1$$

where

$$\lambda_{Deff} \equiv [\epsilon_0 k_B V_0^2 / (2n_0 e^2)]^{1/2}$$

\* **Conclusion:**

$1 \leq (\lambda_{Deff}/a)_{crit}$  for two-stream stability

\* **Does a critical value for  $\lambda_{Deff}/a$  exist?**

$$(\lambda_{\text{Deff}}/a)_{\text{crit}}$$

**Fluid results (thermal electron distribution):**

Stable virtual cathodes exist which are "close enough" to the desired harmonic oscillator potential if:

$$1.2 < \lambda_{\text{Deff}}/a.$$

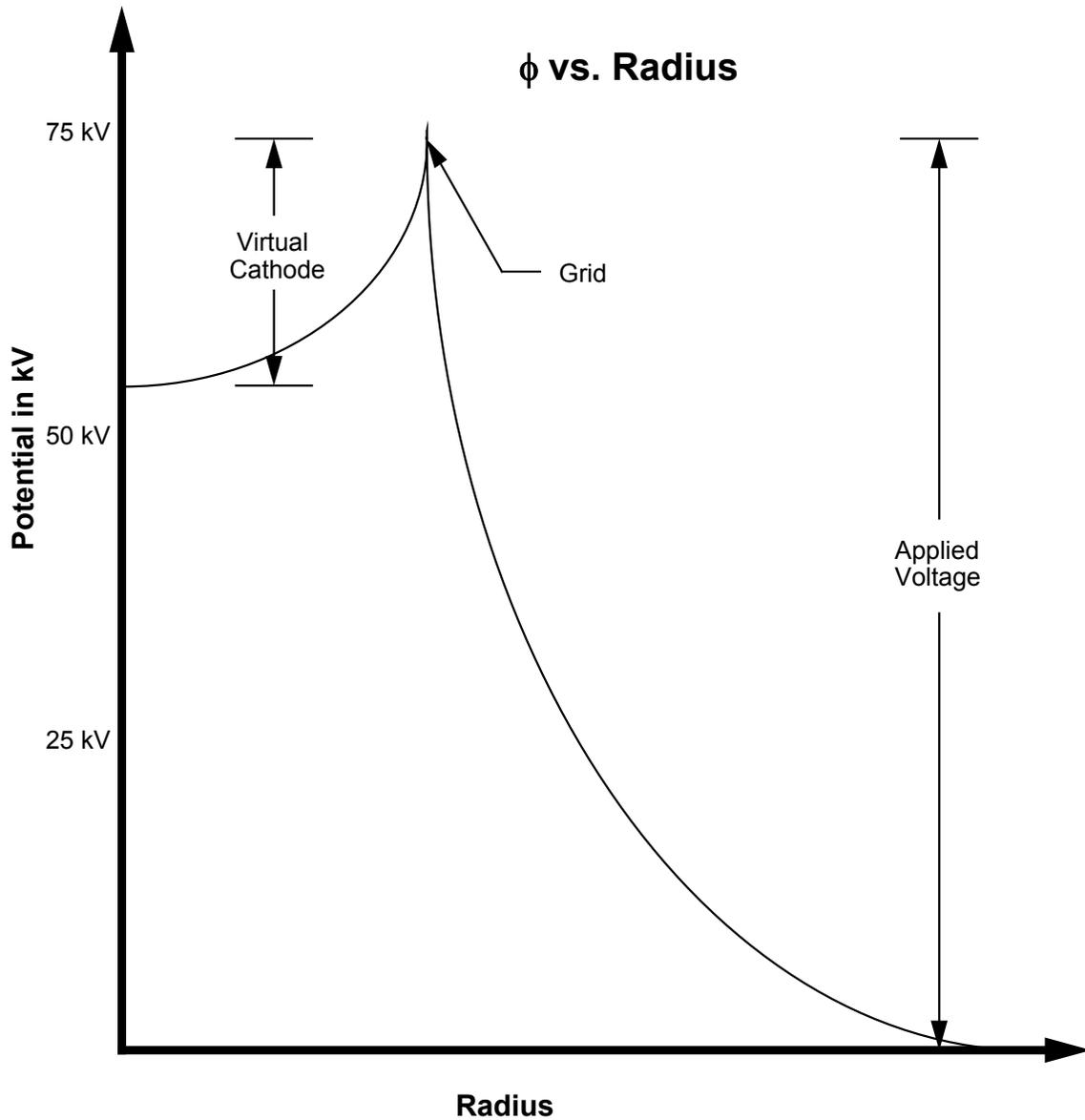
**Kinetic results (self-colliding beam distribution):**

Stable virtual cathodes exist if:

$$1 < \lambda_{\text{Deff}}/a.$$

**We propose to find  $(\lambda_{\text{Deff}}/a)_{\text{crit}}$  experimentally.**

# Why is $(\lambda_{\text{Deff}}/a)_{\text{crit}}$ important?



$$|\phi_{\text{applied}}| / |\phi_{\text{virtual}}| \sim (\lambda_{\text{Deff}}/a)_{\text{crit}}^2$$

**Theoretical Prediction:**

$$\text{If } (\lambda_{\text{Deff}}/a)_{\text{crit}}^2 = 1 \text{ then } |\phi_{\text{applied}}| / |\phi_{\text{virtual}}| = 7$$

# Virtual Cathode Stability Summary

- \* **POPS virtual cathodes violate a compressible Rayleigh-Taylor stability criterion for electrons.**
- \* **Growth rates fall at large  $\lambda_{\text{Deff}}/a$  and near Brillouin limit, but no window of absolute stability exists in the fluid model.**
- \* **Stable profiles that are "close enough" exist for  $\lambda_{\text{Deff}}/a \geq 1.2$**
- \* **Kinetic 2 stream limit suggests that a critical value of  $\lambda_{\text{Deff}}/a$  for stability exists and that this value is  $\sim 1$ .**
- \* **Since  $\phi_{\text{applied}} \sim (\lambda_{\text{Deff}}/a)_{\text{crit}}^2$  determining  $(\lambda_{\text{Deff}}/a)_{\text{crit}}$  is critical to POPS performance.**

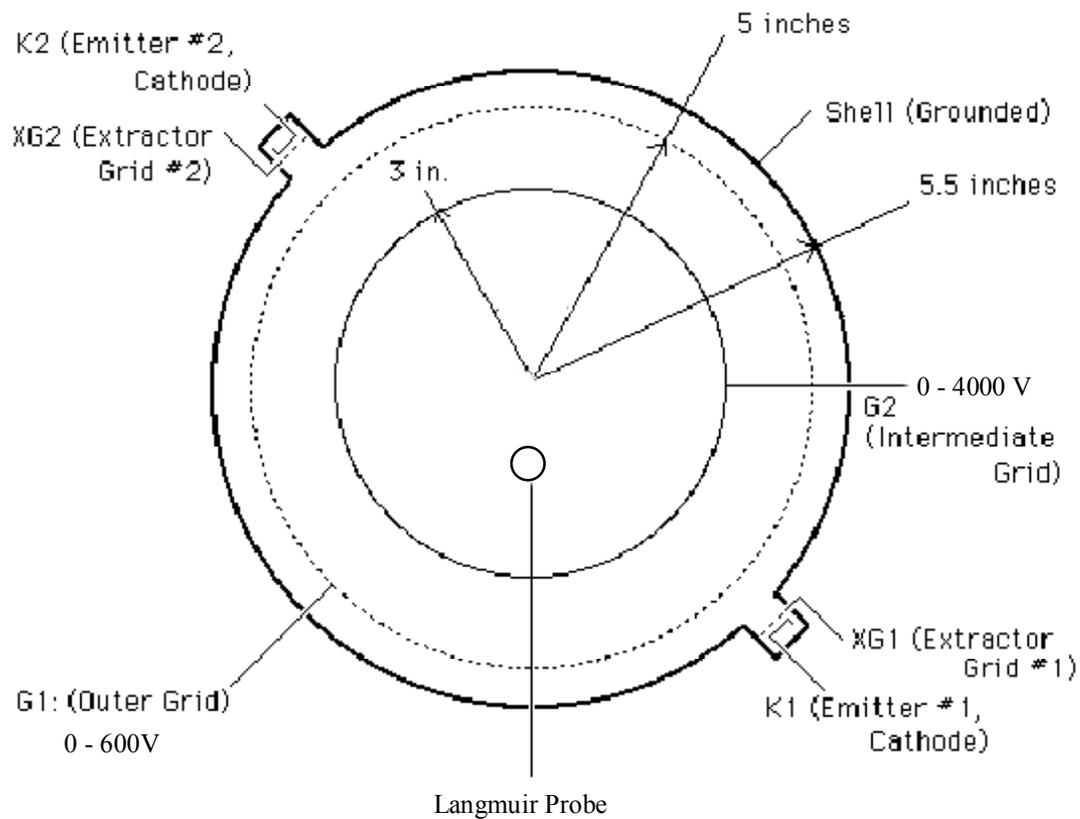
# Experimental Program

- \* **Equilibrium and Stability of Virtual Cathodes.**



**INS Device**

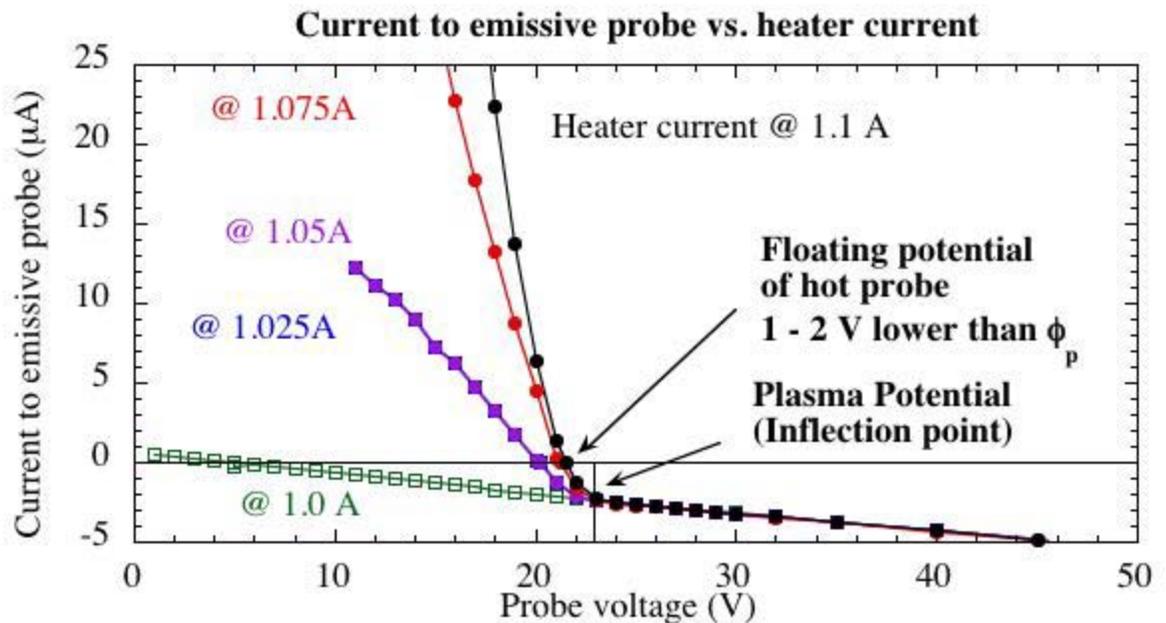
## INS Device Configured for Virtual Cathode and POPS



- **Emissive Probe Used to Measure Plasma Potential**

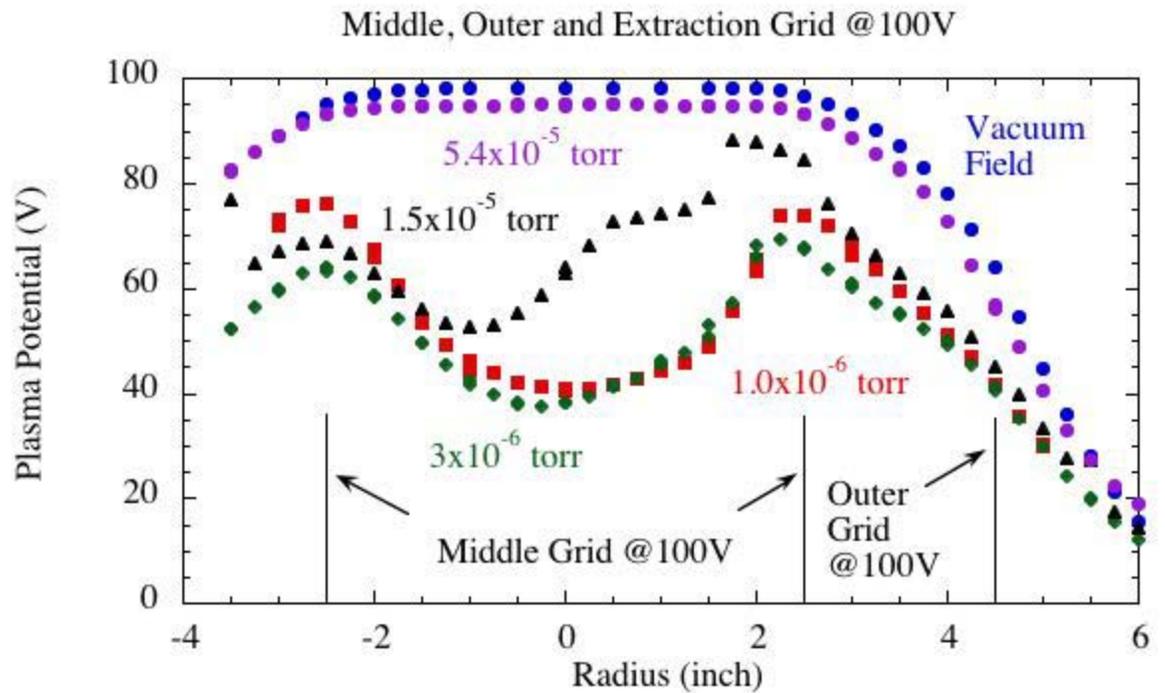
## Emissive probe for plasma potential measurement

- Made of thoriated tungsten (0.076 mm x 1 cm) in alumina tube (3.2 mm O.D.)
- Better suited than Langmuir probe due to **electron beam**
- Floating potential of hot probe  $\sim$  plasma potential (within 1-2 V)
  - following results are taken from floating potential of hot probe
- May disturb plasma: via ion loss to alumina tube



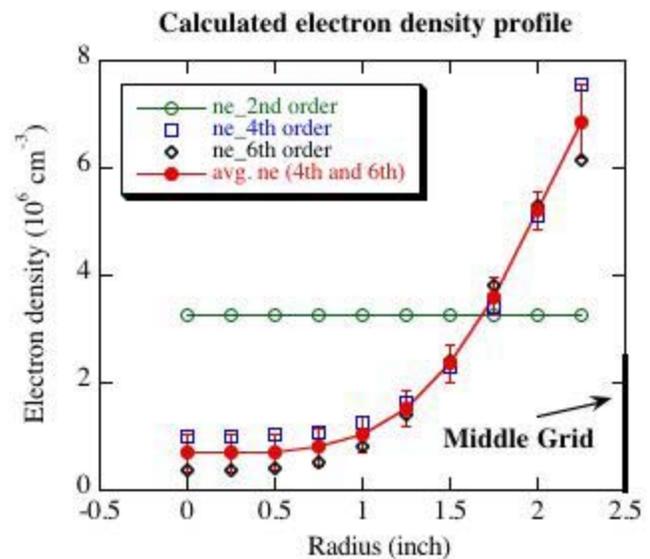
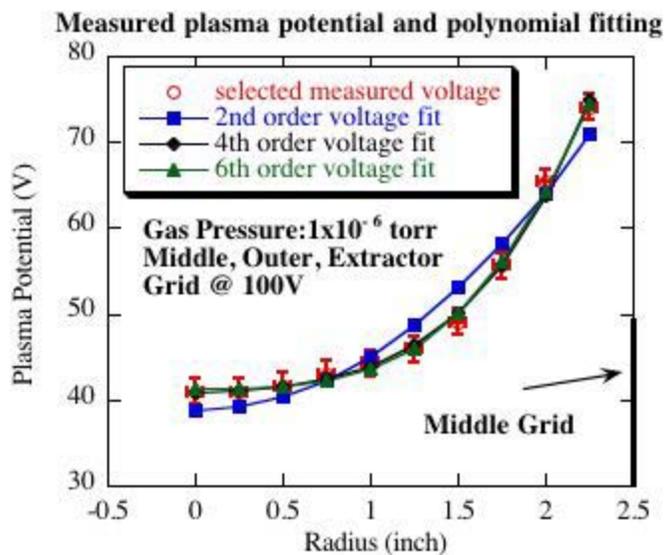
## Radial Plasma Profile as a function of Gas Pressure

- Low Gas pressure ~ deep potential well (up to 60% of bias voltage)
- Increasing gas pressure ~ smaller well depth, radial asymmetry, bifurcation
- High Gas pressure ~ no well formation



## Electron density profile from plasma potential

- Low pressure case  $\sim$  low ion density, compared to injected electrons due to low background ionization
- Ignoring  $n_i$ ,  $n_e$  can be solved from Poisson equation (low  $n_i$  verified later)
- $n_e$  profile from average of 4th order and 6th order polynomial fit of  $\phi_p$
- Off-peak radial density profile: stable profile from fluid dynamics standpoint
- Average electron density in the well  $\sim 3.3 \times 10^6 \text{ cm}^{-3}$ , consistent with electron density calculation from circulating current inside middle grid.



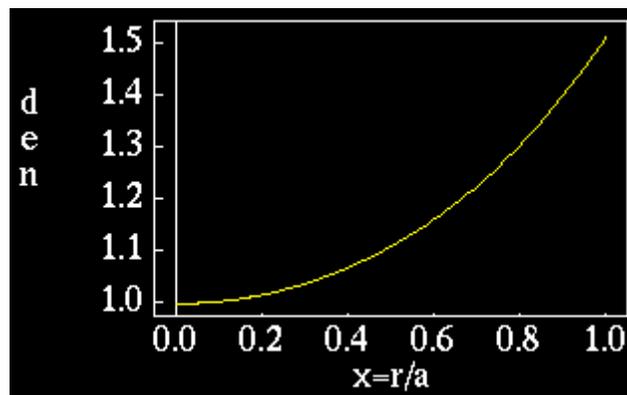
# Profile Results

## Theoretical Prediction:

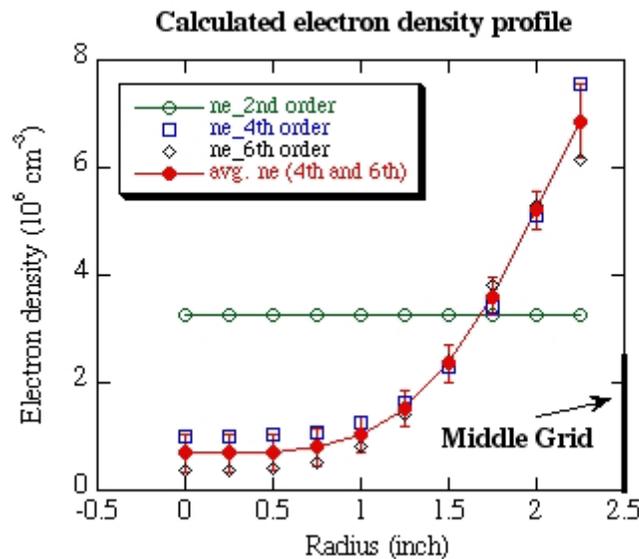
If  $(\lambda_{\text{Deff}}/a)_{\text{crit}}^2 = 1$  and  $n_e = n_{e0}$  then  $|\phi_{\text{virtual}}| / |\phi_{\text{applied}}| = .143$

## Experimental Observation:

$|\phi_{\text{virtual}}| / |\phi_{\text{applied}}| \sim .6$



Theoretical Electron Density Profile at Marginal Limit

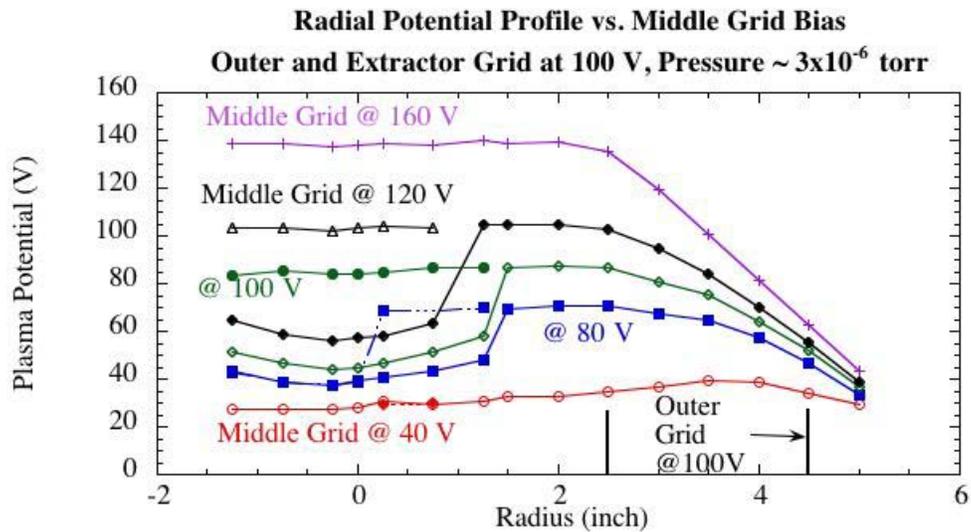


Experimental Electron Density Profile

- Experimental Profile should be stable

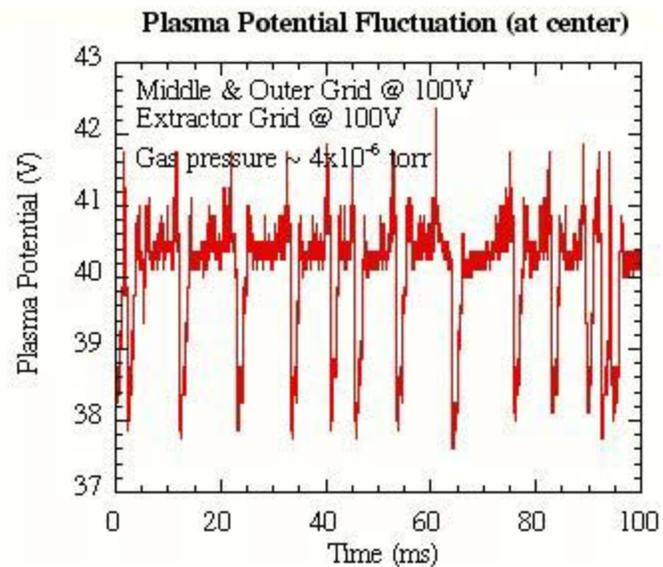
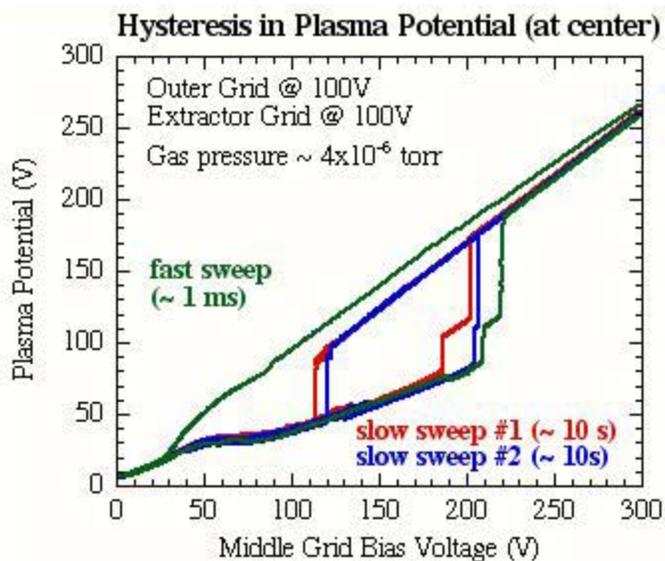
## Bifurcation of Radial Plasma Potential Profile

- Two stable equilibria for radial plasma potential profile
- Deep well: well depth of 50-70 V @120 V bias (note the radial inhomogeneity)
- No well: well depth of less than a few voltage
- No bifurcation: low grid voltage (e.g. < 50V) or high grid voltage (e.g. > 120V)



## Hysteresis and Fluctuation of Plasma Potential

- Hysteresis of plasma potential
- Different hysteresis path for different rate of voltage sweep
- Fast process for well disappearance vs. slow process for well formation
- Important time scale  $\sim 0.1$ -10 ms
- $\phi_p$  slow fluctuation  $\sim 100$  Hz
- Typically observed in deep well phase but much smaller in no well phase.
- Slow time scale  $\rightarrow$  ion motion & ionization rate.



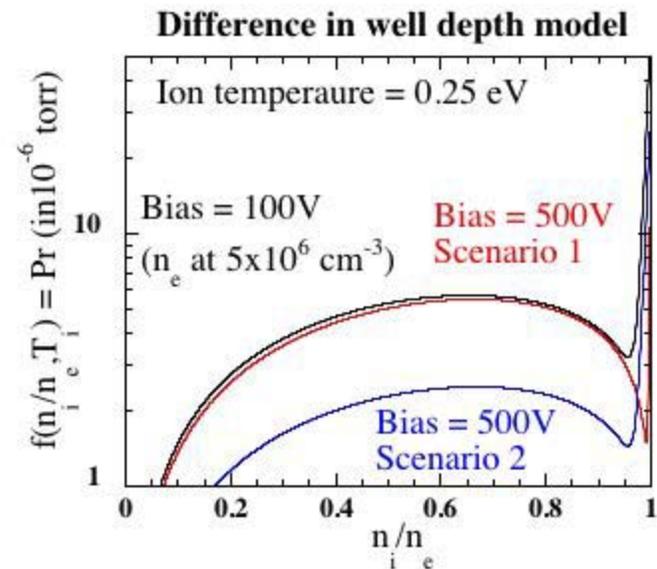
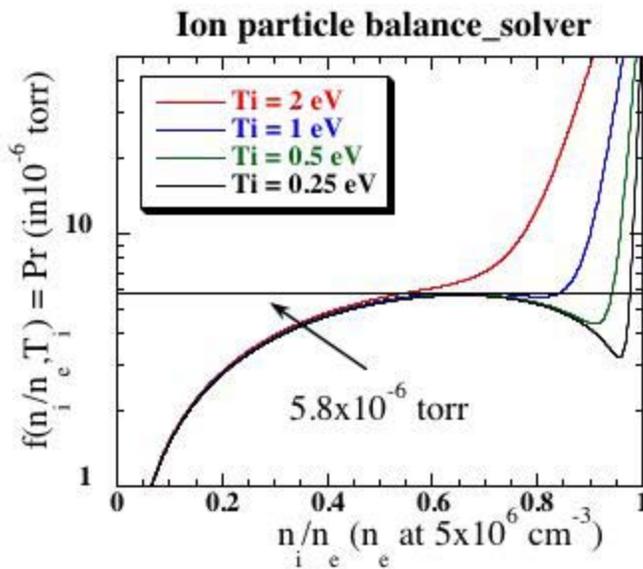
## Ion Particle Balance Model (0D)

- Ion source in the well = Ion loss out of the well
- Ion source: ionization of background neutrals (gas pressure dependent)
  - $S_{\text{ion}} = n_e n_n \langle \sigma v_e \rangle V_{\text{well}}$ ,  $\sigma$ : ionization cross section,  $v_e$ : electron velocity,  $V_{\text{well}}$ : well volume
- Ion loss: loss to the probe structure + loss out of well due to random thermal motion and well anisotropy
  - $L_{\text{ion}} = 0.25 * n_i v_i A_{\text{probe}} + 0.5 * n_i v_i A_{\text{well}} \exp(-\Delta V/T_i)$ ,  $v_i$ : ion velocity,  $A_{\text{probe}}$ : probe surface area (alumina tube),  $A_{\text{well}}$ : well surface area,  $\Delta V$ : well depth
- Poission equation:  $6 \Delta V/a^2 \sim q/\epsilon_0 (n_e - n_i)$ ,  $a$  = well radius  $\sim 1.5$  inch
- Well depth: function of bias voltage and electron injection
- $S_{\text{ion}} = L_{\text{ion}} \rightarrow f(n_i/n_e, n_e, T_i) = n_n$  (after rearranging terms)
- Relevant numbers:
  - $\sigma \sim 1 \times 10^{-16} \text{ cm}^2$  for 40 - 100 eV,  $n_n = 3.5 \times 10^{10} \text{ cm}^{-3} \times \text{Pr}$  (Pressure in  $10^{-6}$  torr)
  - $v_e \sim 4.2 \times 10^7 * (V_0 - \Delta V/2)^{0.5}$ ,  $V_0$ : bias voltage (100V)
  - $\Delta V \sim 7.8 * (1 - n_i/n_e) * n_e$  (in  $10^6 \text{ cm}^{-3}$ ),  $n_e \sim 5 \times 10^6 \text{ cm}^{-3}$  (from measurement)
  - $v_i \sim 4.9 \times 10^5 * (\Delta V + T_i)^{0.5}$
  - $A_{\text{probe}} = 6.3 \text{ cm}^2$  (at the center),  $V_{\text{well}} = 232 \text{ cm}^3$ ,  $A_{\text{well}} = 182 \text{ cm}^2$

## Solutions from ion particle balance model

- Graphically solving  $f(n_i/n_e, n_e, T_i) = n_n$
- Low pressure: one solution (deep well)
- Medium pressure: 3 solutions (2 stable, 1 unstable), bifurcation between deep well and no well
- High pressure: one solution (no well)
- Consistent with experiments

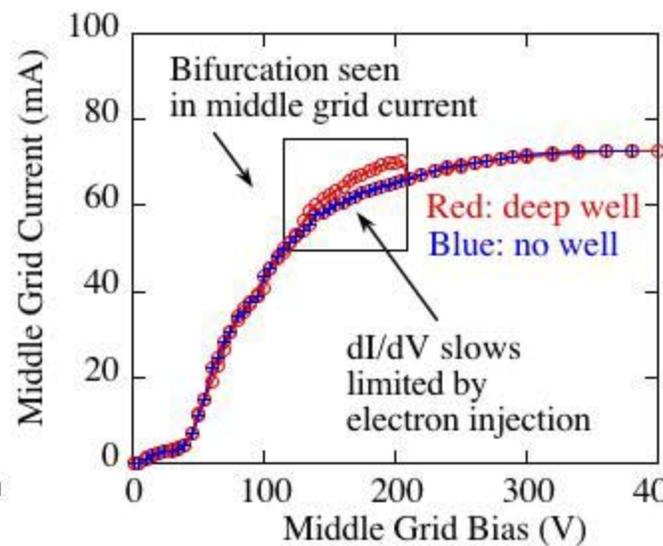
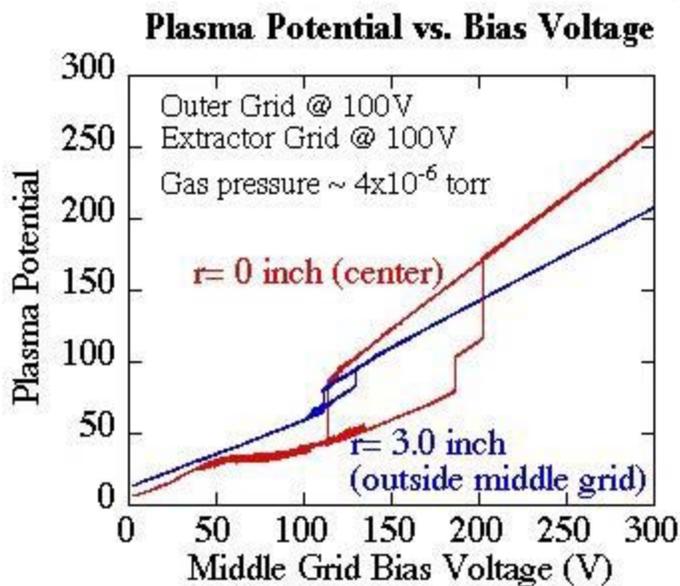
- Scenario 1: well depth proportional to bias voltage ( $n_e$  increase with bias, not limited by electron injection)
- Scenario 2: well depth limited by electron injections ( $n_e = \text{fixed}$ )
- Experiments indicate #2 case --> need to enhance electron injection



## Comparison with Experiments

- Bifurcation or 3 solutions are seen at center
- Potential well exist without ion loss to the probe ( $r=3.0$  inch, outside the well)
- Potential well disappears with increasing bias voltage --> electron density limited by injection

- Middle grid current  $\sim$  electron density (assuming const. grid transparency)
- Bifurcation seen by middle grid current
- $dI/dV$  (middle grid) slow above 150V, electron density limited by injection
- Enhanced electron injection --> deeper potential well



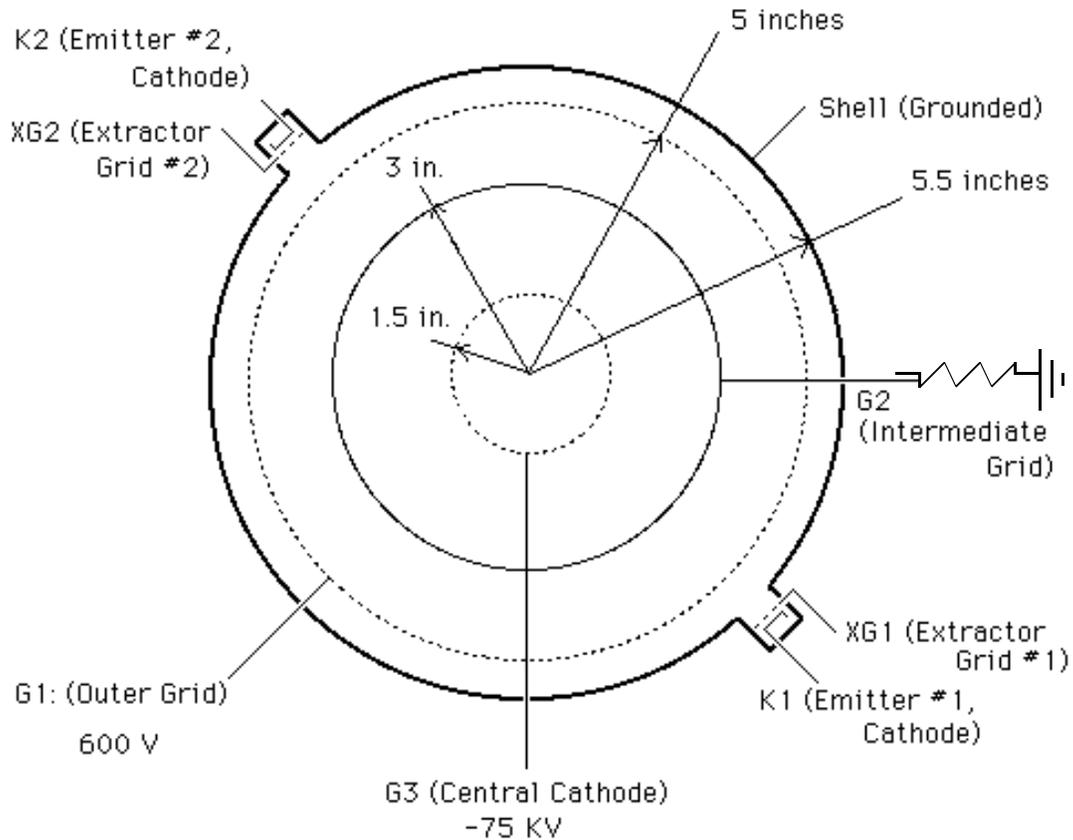
# **INS Experiment Summary**

- \* Potential profiles have been measured with an emissive probe**
- \* Bifurcated equilibria have been observed (three different states)**
- \* Bifurcated equilibria can be understood qualitatively and quantitatively with a simple ionization model.**
- \* Wells as deep as 60% of the applied voltage have been observed.**
- \* Experimental density profiles are strongly peaked off-axis.**
- \* Even though the potential wells are much deeper than expected, theory predicts that the observed density profiles should be stable.**
- \* Stability limits of virtual cathode have not yet been tested.**

# The Next Steps

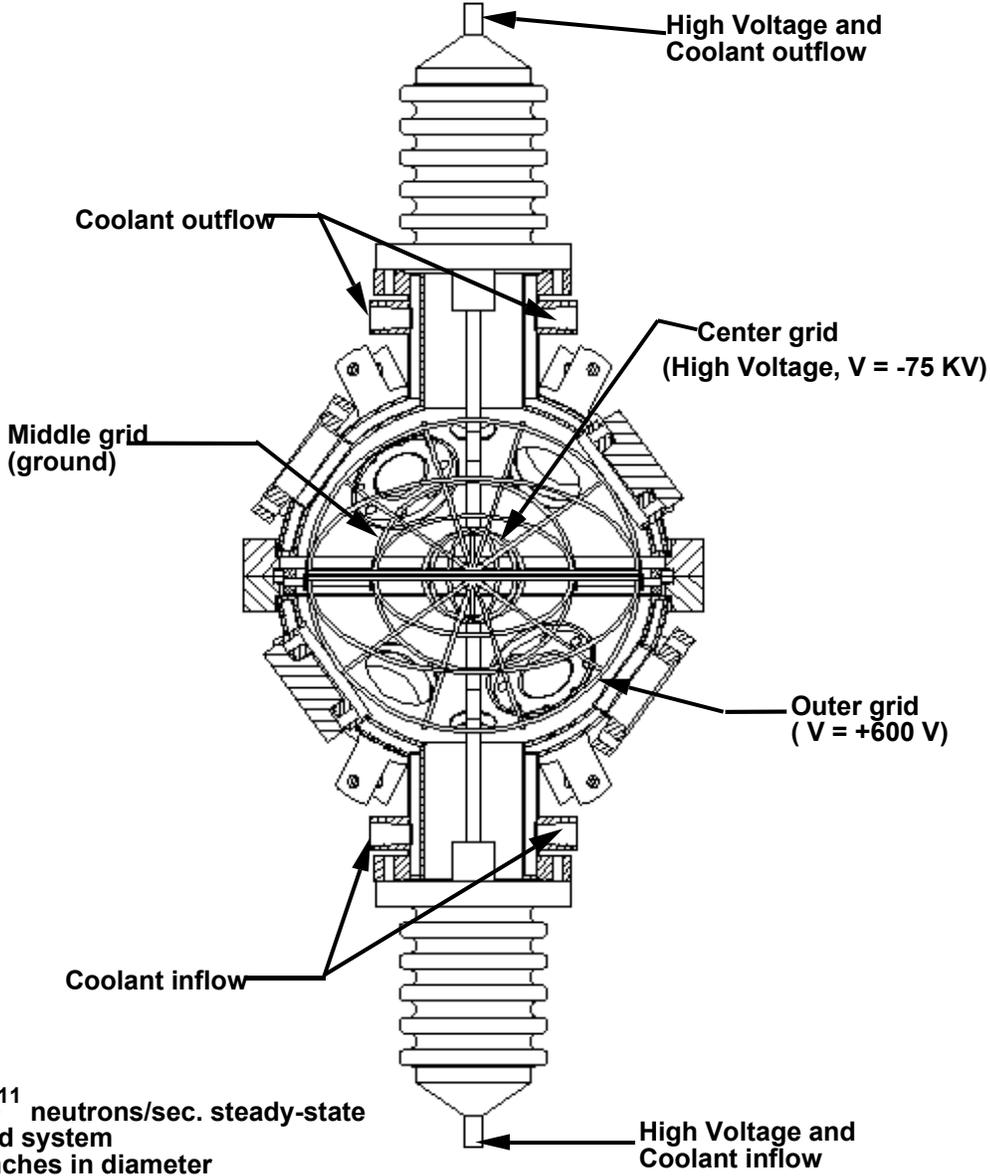
- **Flatten Density Profiles.**
  - **More focused electron beam from dispenser cathodes.**
- **Operate with deep well at higher voltages.**
  - **Pulsed discharges.**
  - **Ion removal techniques like POPS dumping?**
  - **More electron density in the well.**
- **Look for stability limit.**
- **Look at fluctuation data.**
- **Look for ion current resonant response at POPS frequency.**

# INS as Neutron Source



Parameter	Present	IEC Target or Already Proven
Neutron Yield (n/s)	$10^8$	$10^{11}$ D-T or $5 \times 10^8$ D-D
Lifetime (hours)	500	10,000
Operation	Pulsed	Pulsed or steady state
Nominal cost \$k	\$100k	Same
Power	1kW	25 kW

# Cutaway view of INS showing grids

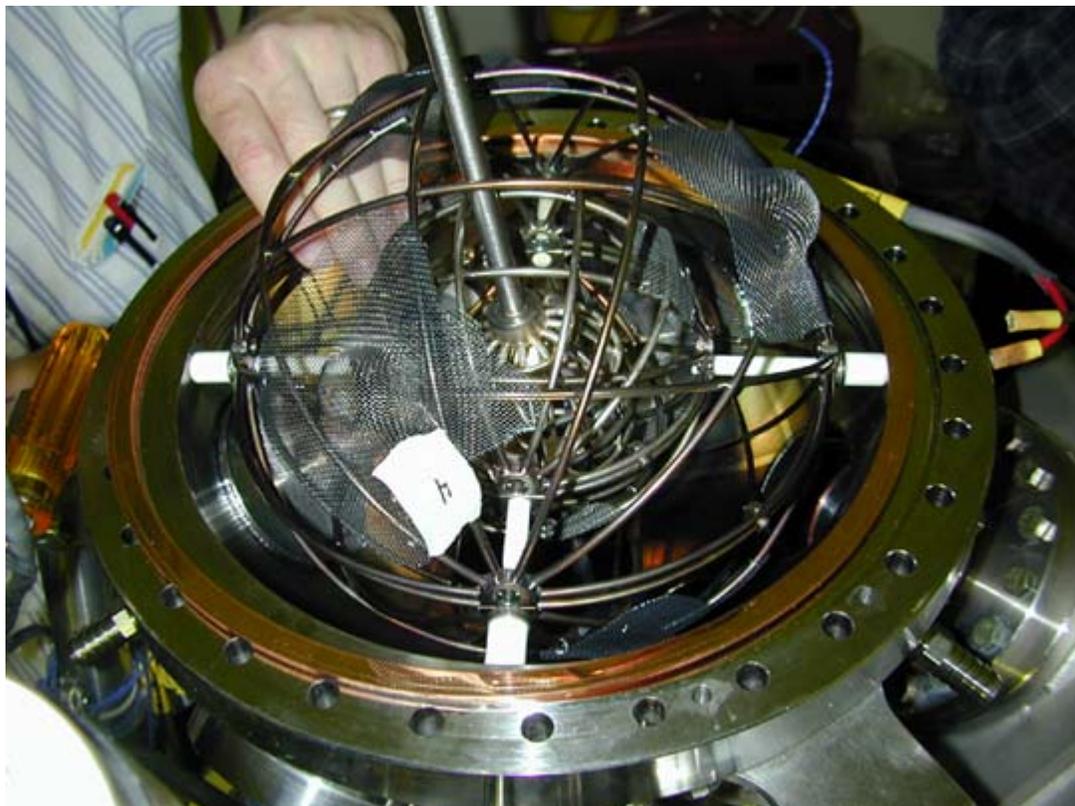


$1 \times 10^{11}$  neutrons/sec. steady-state  
3 grid system  
12 inches in diameter  
Actively cooled  
6 e-guns to ionize gas  
25 KW of power (75 KV @ 335 mA)

# INS Experiment



# INS Device



# **Near-Term Applications**

- **Neutron source for "real-time" assay**
  - \* **HEU detection**
  - \* **Nuclear waste assay**
  - \* **Landmine detection**
  - \* **High explosive detection (Unexploded Ordnance)**
  - \* **Drug detection**
  - \* **Chemical and Biological Weapons**
  
- **Higher gain applications**
  - \* **Neutron tomography (imaging for the above)**
  - \* **Isotope production**
  - \* **Transmutation of waste**
  - \* **Power production (POPS required)**

# Nuclear Assay Applications

## \* High Explosives Detection (UXO, Landmines, Chemical Weapons Dispersant, "Suitcase Sniffers", Truck Bombs, etc.)

- $H^2 + H^3 \rightarrow He^4 + n$  (14.1 MeV) (fusion reaction)
- $N^{14} + n \rightarrow N^{15} + \gamma$  (10.8 MeV)

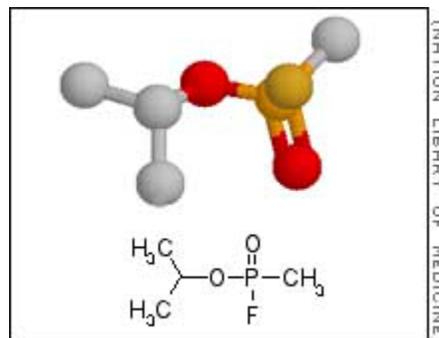
## \* Special Nuclear Materials Detection

- $H^2 + H^3 \rightarrow He^4 + n$  (14.1 MeV) (fusion reaction)
- $U^{235} + n \rightarrow fp + 3n$  (fission and neutron multiplication or delayed neutrons)

## \* Spent Fuel Assay

- $H^2 + H^3 \rightarrow He^4 + n$  (14.1 MeV) (fusion reaction)
- $U^{235} + n \rightarrow fp + 3n$  (fission and neutron multiplication)
- $Pu^{239} + n \rightarrow fp + 3n$  (fission and neutron multiplication)

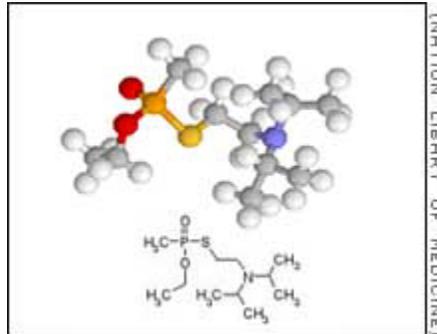
## \* Sarin Detection



- $H^2 + H^3 \rightarrow He^4 + n$  (14.1 MeV) (fusion reaction)
- $P^{31} + n \rightarrow P^{32} + \gamma$  (7.93 MeV)
- $F^{19} + n \rightarrow F^{20} + \gamma$  (6.60 MeV)
- $P^{32} \rightarrow S^{32} + e^-$  (1.17 MeV)
- $F^{20} \rightarrow Ne^{20} + e^-$  (5.40 MeV)

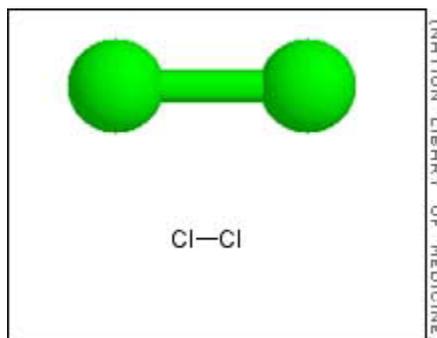
# Nuclear Assay Applications cont.

## \* VX



- $H^2 + H^3 \rightarrow He^4 + n$  (14.1 MeV) (fusion reaction)
- $N^{14} + n \rightarrow N^{15} + \gamma$  (10.8 MeV)
- $P^{31} + n \rightarrow P^{32} + \gamma$  (7.93 MeV)
- $S^{32} + n \rightarrow S^{33} + \gamma$  (8.64 MeV)
- $P^{32} \rightarrow S^{32} + e^-$  (1.17 MeV)

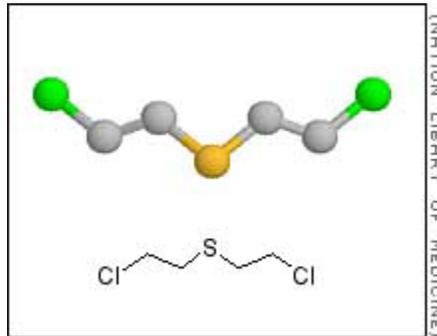
## \* Chlorine



- $H^2 + H^3 \rightarrow He^4 + n$  (14.1 MeV) (fusion reaction)
- $Cl^{35} + n \rightarrow Cl^{36} + \gamma$  (7.97 MeV)

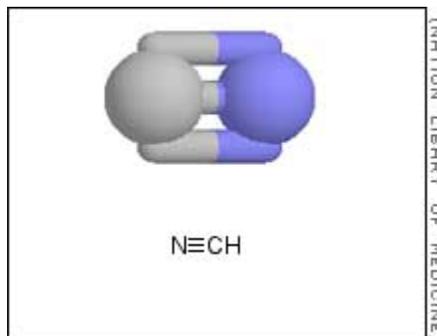
# Nuclear Assay Applications cont.

## \* Mustard gas



- $H^2 + H^3 \rightarrow He^4 + n$  (14.1 MeV) (fusion reaction)
- $S^{32} + n \rightarrow S^{33} + \gamma$  (8.64 MeV)
- $Cl^{35} + n \rightarrow Cl^{36} + \gamma$  (7.97 MeV)

## \* Cyanide



- $H^2 + H^3 \rightarrow He^4 + n$  (14.1 MeV) (fusion reaction)
- $N^{14} + n \rightarrow N^{15} + \gamma$  (10.8 MeV)

## \* Biological Weapons (Anthrax, Small Pox, etc.)

- $H^2 + H^3 \rightarrow He^4 + n$  (14.1 MeV) (fusion reaction)
- $n + \text{processing chemicals} \rightarrow \text{processing chemicals} + \gamma$

# Conclusions

- **Potential profiles have been measured on INS-e with an emissive probe and compared with theoretical stability predictions.**
- **So far there is good agreement between theory and experiment, but the stability limits have not yet been accessed experimentally.**
- **Next Steps**
  - **Flatter Density Profiles.**
  - **Increase Voltage Operating Window.**
  - **Look for stability limit.**
  - **Look at fluctuation data.**
  - **Look for ion current resonant response at POPS frequency.**
- **Build a  $1 \times 10^{11}$  Ion-based D-T Source.**