Problem 10.1

It is desired to compute the change in the plutonium fraction of a mixed oxide fast reactor fuel element as a function of burnup β. The initial Pu fraction of the fuel is q_o. Assume that ^{239}Pu only undergoes fission but that ^{238}U both fissions and captures neutrons (to produce ^{239}Pu). The ratio of neutron capture to fission is ^{238}U is known and constant.

$$\alpha_U = \frac{\text{rate of neutron capture in } ^{238}U}{\text{rate of fission in } ^{238}U}$$

Obtain an approximate solution to the nuclide balances (on ^{238}U, ^{239}Pu, and fission product pairs) assuming that the conversion ratio defined by

$$C = \frac{\text{rate of neutron capture in } ^{238}U}{\text{rate of fission of } ^{239}Pu}$$

is treated as a constant for the purpose of integrating the differential equation.

Problem 10.1

Wanted -- The change in Pu fraction in mixed oxide fuel elements as a function of burn-up, β.

Assume --

$$q_o = \text{initial fraction of Pu} = \frac{N^0_{Pu}}{N^0_{Pu} + N^0_U}$$

$$\alpha_U = \frac{\text{rate of neutron capture in } ^{238}U}{\text{rate of fission in } ^{238}U}$$

$$C = \frac{\text{rate of neutron capture in } ^{238}U}{\text{rate of fission of } ^{239}Pu}$$

- -

$$^{239}Pu + n \xrightarrow{\sigma^f_{Pu}} 2n + 2FP$$

(neglect all capture by ^{239}Pu to form ^{240}Pu, ^{241}Pu, etc.)

<u>Assume no ^{235}U</u>

$$^{238}U + n \xrightarrow{\sigma^f_U} 2n + 2FP$$

$$^{238}U + n \xrightarrow{\sigma^c_U = \sigma^f_U \alpha_U} {}^{239}U \xrightarrow{\beta^-} {}^{239}Np \xrightarrow{\beta^-} {}^{239}Pu$$

Assume no time lag

Nuclide Balances

For Pu

$$\frac{\mathbf{dN_{Pu}}}{\mathbf{dt}} = -\sigma^{\mathbf{f}}_{\mathbf{Pu}}\mathbf{N_{Pu}}\Phi + \alpha_{\mathbf{U}}\sigma^{\mathbf{f}}_{\mathbf{U}}\mathbf{N_U}\Phi \qquad \textbf{(1)}$$

For ^{238}U

$$\frac{\mathbf{dN_{U}}}{\mathbf{dt}} = -\sigma^{\mathbf{f}}_{\mathbf{U}}\mathbf{N_{U}}\Phi - \alpha_{\mathbf{U}}\sigma^{\mathbf{f}}_{\mathbf{U}}\mathbf{N_U}\Phi \qquad \textbf{(2)}$$

For Fission Products

$$\frac{dN_{FP}}{dt} = 2\sigma^{f}_{Pu}N_{Pu}\Phi + 2\sigma^{f}_{U}N_{U}\Phi \qquad \textbf{(3)}$$

Define Burnup:

$$\beta \equiv \frac{\mathbf{N_{FP}} \bullet \frac{1}{2}}{\mathbf{N^0_U} + \mathbf{N^0_{Pu}}} \qquad \textbf{(4)}$$

Combine Eqs. 3 & 4

$$\frac{\mathbf{d}\beta}{\mathbf{dt}} = \left\{\frac{\sigma^{\mathbf{f}}_{\mathbf{Pu}}\mathbf{N_{Pu}} + \sigma^{\mathbf{f}}_{\mathbf{U}}\mathbf{N_U}}{\mathbf{N^0_U} + \mathbf{N^0_{Pu}}}\right\}\Phi \qquad \textbf{(5)}$$

Now we want to change Eq. 1 from

$$\frac{dN_{Pu}}{dt} \text{ to } \frac{dN_{Pu}}{d\beta}$$

So let

$$\frac{d}{dt} = \frac{d}{d\beta} \bullet \frac{d\beta}{dt} \text{ or } \frac{d}{d\beta} = \frac{d}{dt} \bullet \frac{1}{\frac{d\beta}{dt}}$$

This gives

$$\frac{dN_{Pu}}{d\beta} = \frac{dN_{Pu}}{dt} \bullet \frac{dt}{d\beta} \tag{6}$$

Using Eq. 1 & 5

$$\frac{dN_{Pu}}{d\beta} = \frac{-\sigma^f_{Pu} N_{Pu} + \alpha_U \sigma^f_U N_U}{\sigma^f_{Pu} N_{Pu} + \sigma^f_U N_U} \bullet (N^0_U + N^0_{Pu}) \tag{7}$$

Using Eqs. 2 & 5

$$\frac{dN_U}{d\beta} = \frac{-(1-\alpha_U)\sigma^f_U N_U}{\sigma^f_{Pu} N_{Pu} + \sigma^f_U N_U} \bullet (N^0_U + N^0_{Pu}) \tag{8}$$

The problem now is to solve Eqs. 7 & 8 subject to:

$$\mathbf{N_{Pu}(0) = N_{Pu}^0}$$

$$\mathbf{N_U(0) = N_U^0}$$

= =

Possible solution rewriting 7 & 8 as

$$\frac{dN_{Pu}}{d\beta} = \frac{-1 + \alpha_u\left(\dfrac{\sigma_U^f N_U}{\sigma_{Pu}^f N_{Pu}}\right)}{1 + \left(\dfrac{\sigma_U^f N_U}{\sigma_{Pu}^f N_{Pu}}\right)} \bullet \left(N_U^0 + N_{Pu}^0\right) \qquad (9)$$

$$\frac{dN_U}{d\beta} = \frac{\left(-1 + \alpha_u\right) \bullet \left(\dfrac{\sigma_U^f N_U}{\sigma_{Pu}^f N_{Pu}}\right)}{1 + \left(\dfrac{\sigma_U^f N_U}{\sigma_{Pu}^f N_{Pu}}\right)} \bullet \left(N_U^0 + N_{Pu}^0\right) \qquad (10)$$

Remember that we have defined the conversion ratio as:

$$C = \frac{\alpha_U \sigma_U^f N_U \Phi}{\sigma_{Pu}^f N_{Pu} \Phi} \qquad (11)$$

Then Eq. 9 & 10 become:

$$\frac{dN_{Pu}}{d\beta} = \frac{-(1-C)}{1+\dfrac{C}{\alpha_U}} \bullet \left(N_U^0 + N_{Pu}^0\right) \qquad (12)$$

$$\frac{dN_U}{d\beta} = \frac{-(1+\alpha_U)\bullet\dfrac{C}{\alpha_U}}{1+\dfrac{C}{\alpha_U}} \bullet \left(N_U^0 + N_{Pu}^0\right) \qquad (13)$$

We can now integrate Eqs. 12 & 13 because C, α_u, N_U^0, N_{Pu}^0 are all assumed to be constant

$$\int_{N_0}^{N} dN_U = \int_0^{\beta} d\beta[\ \ ]$$

We then find

$$N_{Pu} = N_{Pu}^0 - \left(\frac{1-C}{1+\dfrac{C}{\alpha_U}}\right) \bullet \left(N_U^0 + N_{Pu}^0\right)\beta \qquad (14)$$

$$\mathbf{N_U} = \mathbf{N_U^0} - \left(\frac{\left(\dfrac{\mathbf{1} + \alpha_\mathbf{U}}{\alpha_\mathbf{U}} \right)}{\mathbf{1} + \dfrac{\mathbf{C}}{\alpha_\mathbf{U}}} \right) \bullet \left(\mathbf{N_U^0} + \mathbf{N_{Pu}^0} \right) \beta \qquad \mathbf{(15)}$$

Since the Pu fraction is defined as:

$$\mathbf{q} = \frac{\mathbf{N_{Pu}}}{\mathbf{N_{Pu}} + \mathbf{N_U}} \qquad \mathbf{(16)}$$

using Eq. 14, 15 and the definition of $\mathbf{q_0}$

$$\mathbf{q} = \frac{\mathbf{N_{Pu}^0} - \mathbf{(A)(N_U^0} + \mathbf{N_{Pu}^0})\beta}{\mathbf{N_{Pu}^0} - \mathbf{A(N_U^0} + \mathbf{N_{Pu}^0})\beta + \mathbf{N_U^0} - \mathbf{(D)(N_U^0} + \mathbf{N_{Pu}^0})\beta}$$

Where A,D are constants

Dividing by $\mathbf{N_U^0} + \mathbf{N_{Pu}^0}$

$$\mathbf{q} = \frac{\mathbf{q_0} - \mathbf{A}\beta}{\mathbf{1} - \mathbf{A}\beta - \mathbf{D}\beta}$$

Note:

$$-\mathbf{(A} + \mathbf{D)}\beta = -\left[\frac{\mathbf{1} - \mathbf{C} + \dfrac{\mathbf{C}}{\alpha_\mathbf{U}} + \mathbf{C}}{\mathbf{1} + \dfrac{\mathbf{C}}{\alpha_\mathbf{U}}} \right] \beta = -\beta$$

Finally

$$\mathbf{q} = \frac{\mathbf{q_0} - \left(\dfrac{\mathbf{1 - C}}{\mathbf{1} + \dfrac{\mathbf{C}}{\alpha_{\mathbf{U}}}}\right)\beta}{\mathbf{1} - \beta} \qquad \textbf{(17)}$$

Example:

q_0 = 0.15

C ≈ 1.1 (0.5 MeV), 0.6 in thermal spectrum

α_u = 400 (1 MeV), ≈ infinity in thermal spectrum

See Figure