PIC Simulation of Polywell

Joel G. Rogers, Ph.D

Abstract - Bussard's landmark 2006 publication[1] showed that the power balance (Q) increases with the size of the Polywell machine as the 5th power of the magnet radius. Break-even radius is, by definition, the radius (R) of the smallest machine that produces more power than it consumes. Practical power machines must be larger than this size, but not much larger because of the steep rise of Q with R. Particle-in-cell simulation[2-3] was used to find the maximum Q for each R by searching the steady-state parameter space defined at startup by knob values. Applying the 5th power scaling law to this optimum Q predicts that the break-even radius for DD fueled Polywell will be 1.3m. This is much smaller than the radius of the planned ITER design, giving Polywell an advantage over the competing magnetic confinement power generation.

[1] R.W.Bussard, "The advent of clean nuclear fusion: superperformance space power and propulsion", 57th IAC, 15 pgs., 2006.

[2] C.K.Birdsall and A.B.Langdon, "Plasma Physics via Computer Simulation", McGraw Hill, 479 pgs., 1985.

[3] E.Kawamura, C.K.Birdsall and V.Vahedi, "Physical and numerical methods of speeding up particle codes…", physics.ucla.edu/icnsp/PDF/kawamura.pdf, 2000.

Pulsed Prototype Polywell WB-7

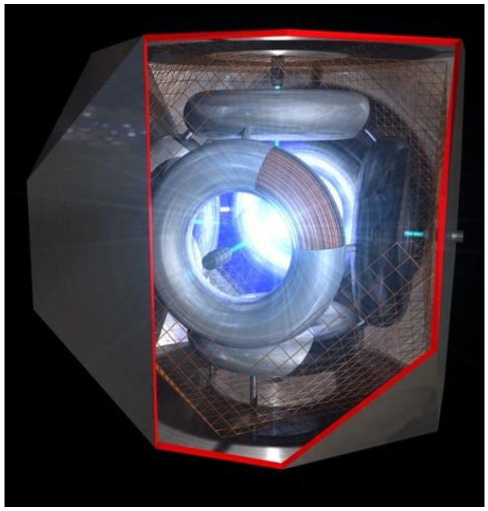


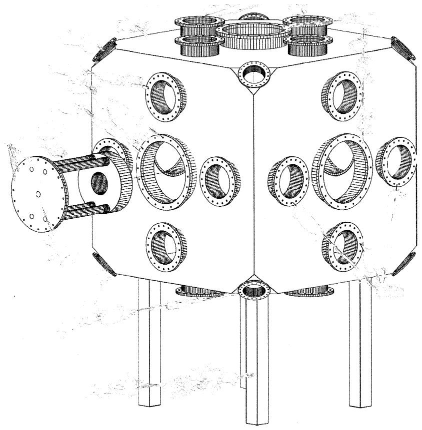
Image from nextbigfuture.com/2009/02/update-on-iec-fusion.html

This shows Dr. Richard Nebel’s Polywell device in Santa Fe. Six magnets in polished vacuum vessels are arranged on the faces of a cube.

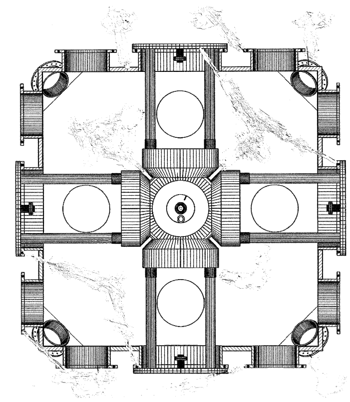
The cube is mounted at the center of a vacuum vessel and biased to high voltage.

Electrons shoot into the interior of the cube along the six magnet axes. Ions are provided by neutral gas ionization by electrons.

Proposed Steady-state Prototype



0.7m and 1.4m Vacuum Tanks



Median Plane Simulated

Six coil magnets are arranged to form a cubic enclosure at the center of the tank. The simulated median plane contains 4 flanges and 4 coil magnets.

Two different tank sizes were simulated, the 2^{nd} scaled up a factor of two from the first. The magnets' sizes were scaled by the same factor.

Deuterium fuel ions were supplied by neutral gas ionization by fast electrons, or alternately by ion guns positioned inside the magnets.

Polywell's Breakeven Radius R_b

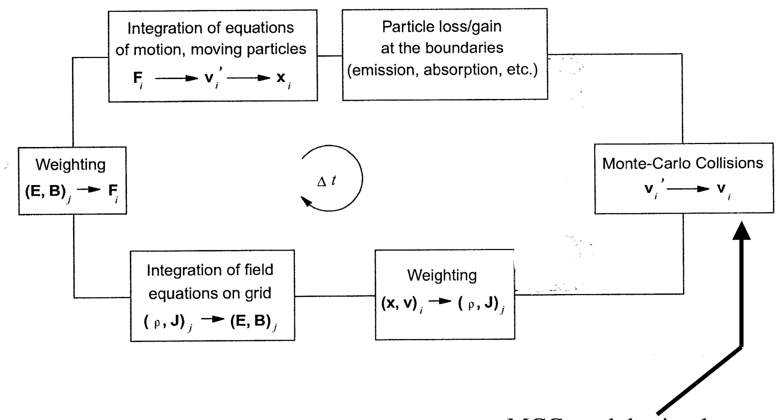
- Fusion power scales as ~ n^2 x Volume.
- At fixed β, $n^2 \sim B^4$
- B ~ R (to maintain magnet cooling).
- Volume ~ R^3.
- Loss ~ magnet surface area ~ R^2.
- **Q ≡ fusion-power/loss-power** ~ $B^4 R^3 / R^2 \sim R^5$.
- Inverting $Q = (R/R_b)^5$ estimates $R_b = R/Q^{1/5}$.
- Therefore maximizing Q minimizes R_b estimate.

This summarizes the scaling law as presented in Bussard's IAC-2006 paper.

The scaling law relates hypothetical performance of a net power machine to performance of an economical, small-scale prototype.

Plasma radius R and Q-factor were determined by simulation. Machine design parameters were varied to maximize Q.

Tech-X Corp. Simulation Software



MCC module simulates elastic/inelastic electron scattering, electron+neutral ionization, and ion+neutral charge exchange.

A median-plane slice of the cube was simulated by tracking $\cong 10^5$ macroparticles in cells of 5-10mm diameter for $\cong 10^6$ time steps..

The "Monte-Carlo Collisions" (MCC) branch was used only for Polywell's fueled by neutral gas ionization.

For Polywell's fueled by ion-guns, the MCC branch was bypassed for speed.

Simulation Knobs Used to Optimize Q

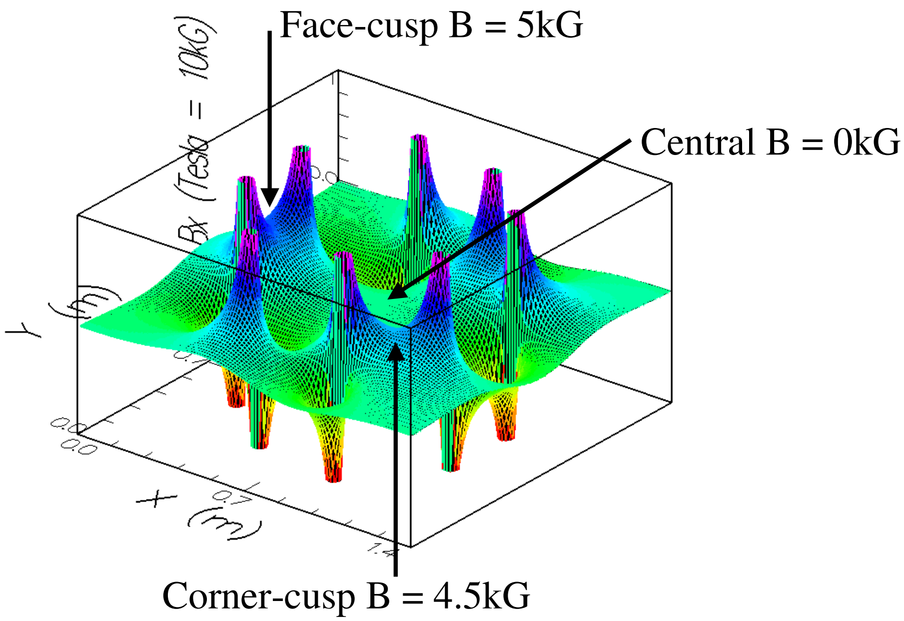
- (1) Electron guns' drive current.
- (2) Electron guns' drive voltage.
- (3) D_2 gas or ion-gun radial position.
- Optimizing performance consisted of searching this 3-dimensional space for points of maximum Q.

The simulation tracked the time evolution of plasma parameters, density, velocity, volume, loss-rates and loss-energies, for 10-20μs simulated time.

At each point in the 3d knob-space Q was computed from simulated parameters.

Q is proportional to the local density-squared times local velocity times local volume and inversely proportional to local lost-power.

Magnetic Field(B_x) Variation in 2d

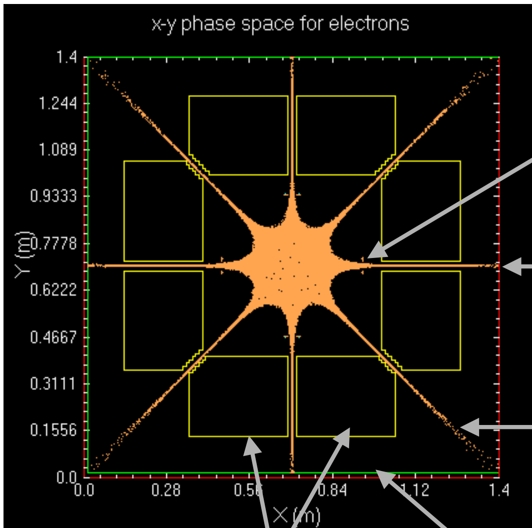


The applied magnetic field was simulated as an analytic expression by adding the contributions of 8 straight wires each carrying a specified current.

Simulated using the electrostatic version of Maxwell's equations, the B-field was constant in time, as shown above.

Rectangular magnet boxes, not shown, prevented particles from approaching the wires and "seeing" the singularities in the B-field .

Simulated Electrons (orange)

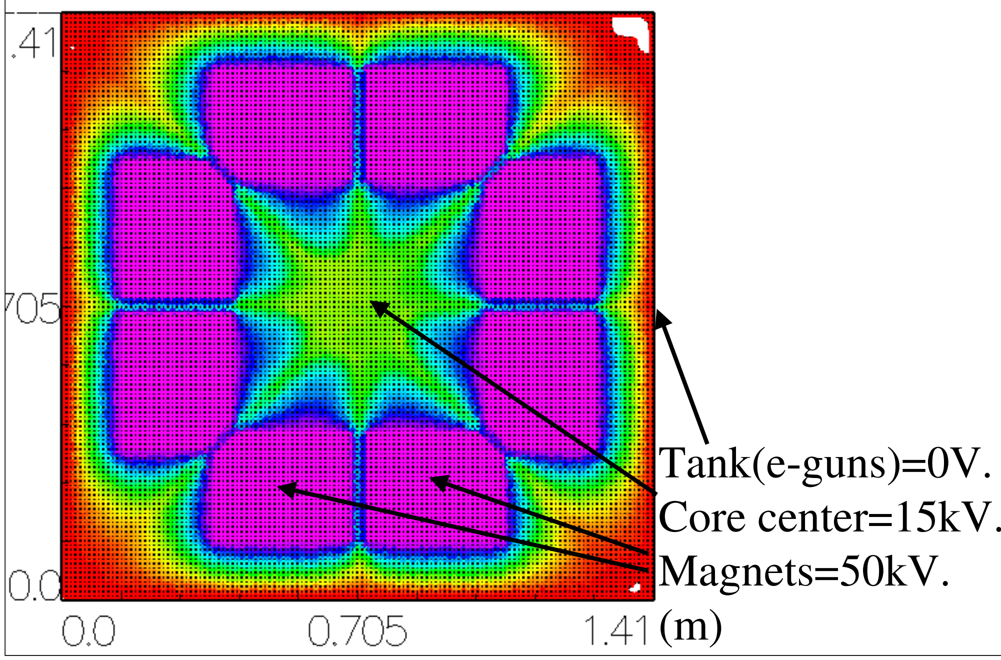


Magnet boxes(yellow) and tank walls(green) were modeled as conductors held at constant voltage.

The tank wall conductor, where the electrons originate, was held at 0V.

Magnet boxes, which attract electrons into the core, were held at high-voltage, 25kV for the 0.7 diameter tank and 50kV for the 1.4m tank.

Electrostatic Potential Well

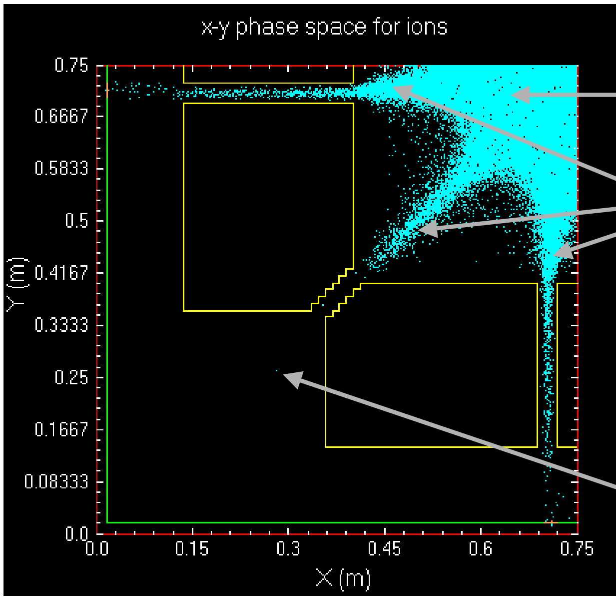


Trapped ions bounce back and forth in the potential well. Each time they come to the edge, they undergo mirror reflection back toward center.

The shape of the potential well is not circular, therefore ions bounce back at different angles each time they cross the well.

The random bounce-angles lead to a uniform density inside the sharp edges of the potential.

Simulated Ion Macroparticles



One of 4 identical quadrants of ions

e-beam + D_2 gas produces ions at 8 points; 3 are shown here.

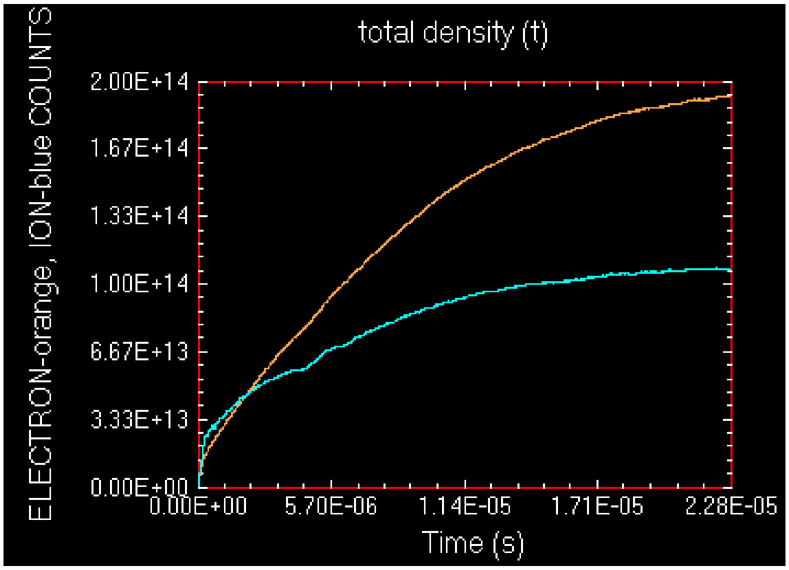
Very few ions are lost through corners.

D_2 gas, not shown, was simulated as a shell 3-cells wide with hollow square center of zero-pressure.

The long narrow channels at the corners and coil centers can support a large pressure differential from inside to outside the magnets, where the pumps are.

The corner channel width simulated, shown above, was ample for electron circulation (shown 2 slides back) and could be reduced for differential pumping.

Defining Steady State Conditions



Ion/electron counts increase to constant values in tens of μs.

At steady state, the total electron count is about 2X the total ion count.

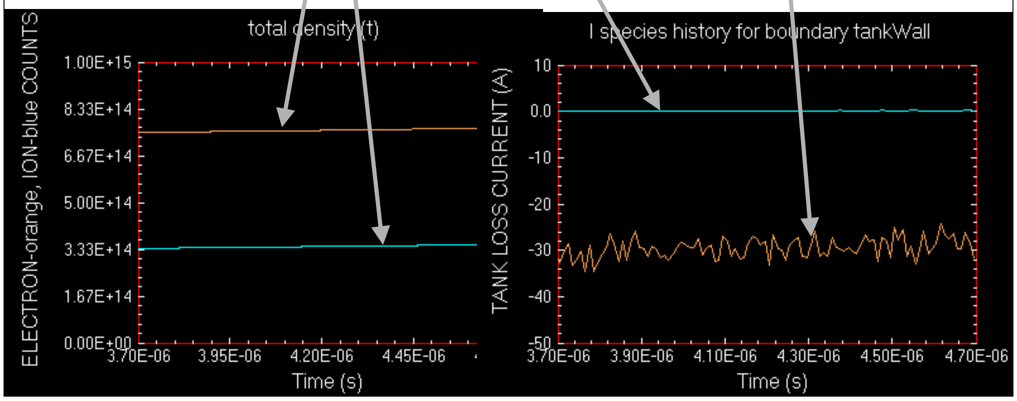
The densities of electrons and ions are equal inside the electrostatic potential well.

About the same number of electrons are trapped in cusps outside the central core as there are inside the well, giving rise to the 2X overall difference in count.

Steady State Density & Loss Current

Definition of steady state ≡ e-current knob(30A) makes both particle counts constant in time.

Loss currents on tank walls: Ions 0A, Electrons -30A

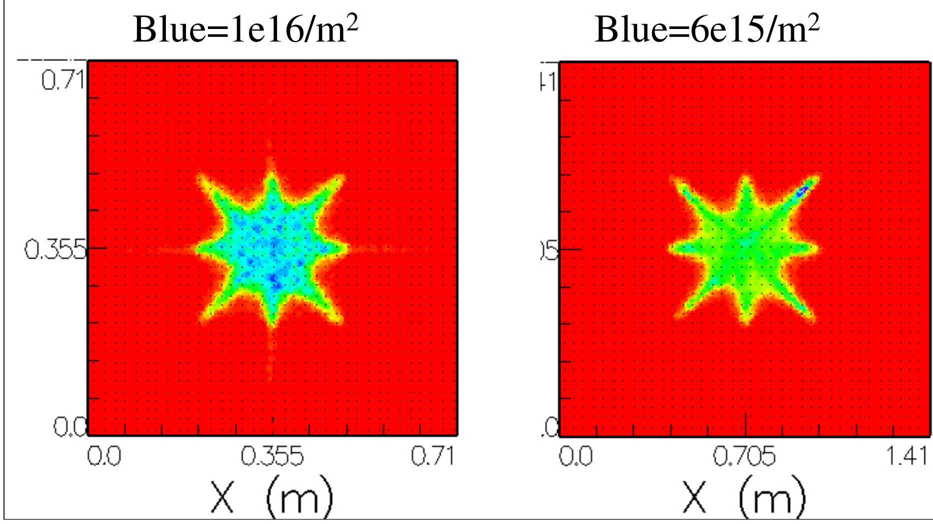


The electron drive current was determined by trial and error to stabilize the electron density. The neutral gas pressure was determined to stabilize the ion density.

Constant electron(orange) and ion(blue) counts are shown on the left.

The right panel shows that the electron loss current equals the electron drive current. This proves that electrons are lost only on the tank, not on the magnets.

Ion-Densities in Median Slices



These simulated areal densities are color coded with blue maximum and red minimum(=0). Green is half-maximum.

Increasing the coil-size and B-field by a factor of 2 produced areal density 2X bigger but plasma diameter expands by less than 2X.

Wiffleball formation would further expand the size of the plasma cloud to be in proportion to the magnet size. Wiffleball formation is required for 5th power scaling.

3d Density from 2d Simulation

- Simulated slab thickness = Debye length λ_D.
- Simulation (previous slide) gives 2d (area) density $\equiv \rho_A = 3e11/cm^2$.
- 3d density: $n = \rho_A / \lambda_D$
- $\lambda_D = (7.4e2)(E_i/n)^{1/2}$ (NRL Formulary pg. 29)
- Combining last 2 lines: $n = (1.2e\text{-}10cm)\rho_A^2$
- n = 1.1e13/cc

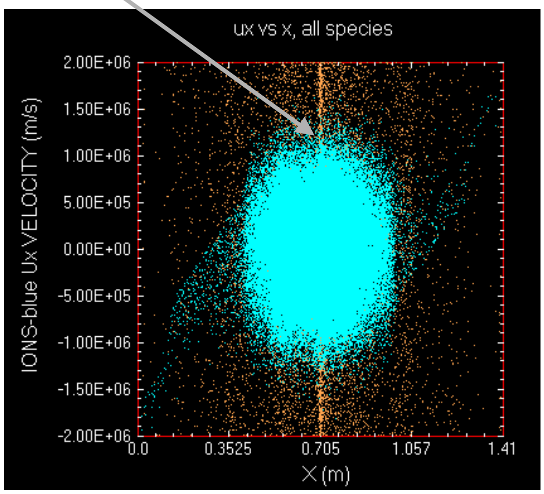
The effective slab thickness is the Debye length of the plasma.

The Debye length is inversely proportional to the square-root of the 3d density.

This makes the 3d density proportional to the square of 2d density.

Velocity, Energy, and β=1 Density

Velocity U_x = 1.2e6 m/s &
Energy $E_i = ½MU^2$ = 15 keV



β=1 Density ~ B^2/E_i
$\equiv n_{\beta 1}$ = 2.7e13/cc

The trapped ion energy was selected to be 15keV. Choosing 15keV maximizes the DD fusion yield, as shown in the yield curve in the next slide.

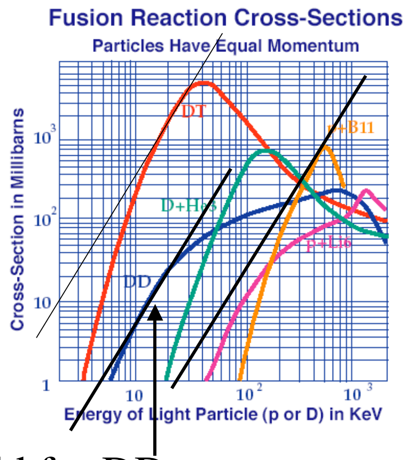
β=1 density was computed from the corner-cusp magnetic field, B=4.5kG from slide#7, and the ion energy, E_i=15keV from this slide.

The formula for β as a function of B and E_i is on pg. 30 of the NRL Formulary.

Optimum Ion Energy for DD = 15keV

From Bussard's 2006 Google Tech Talk

Fusion Type	Fusion Reaction	Reaction Energy
Totally neutron free	$p + {}^{11}B \rightarrow 3\ {}^{4}He$ ${}^{6}Li + {}^{6}Li \rightarrow 3\ {}^{4}He$ (p, 3He cycle)	8.70 MeV 10.44 MeV
Small neutron output (5% to 9%)	${}^{2}H + {}^{2}H - {}^{3}H + p$ ${}^{2}H + {}^{2}H \rightarrow {}^{3}He + n$ (2.45 MeV) ~~${}^{3}He \rightarrow {}^{2}H + {}^{4}He + p$~~	~~10.24 MeV~~ 3.7 MeV NO RECYCLING
High neutron radiation (>80%)	${}^{2}H + {}^{3}H \rightarrow {}^{4}He + n$	14.1 MeV



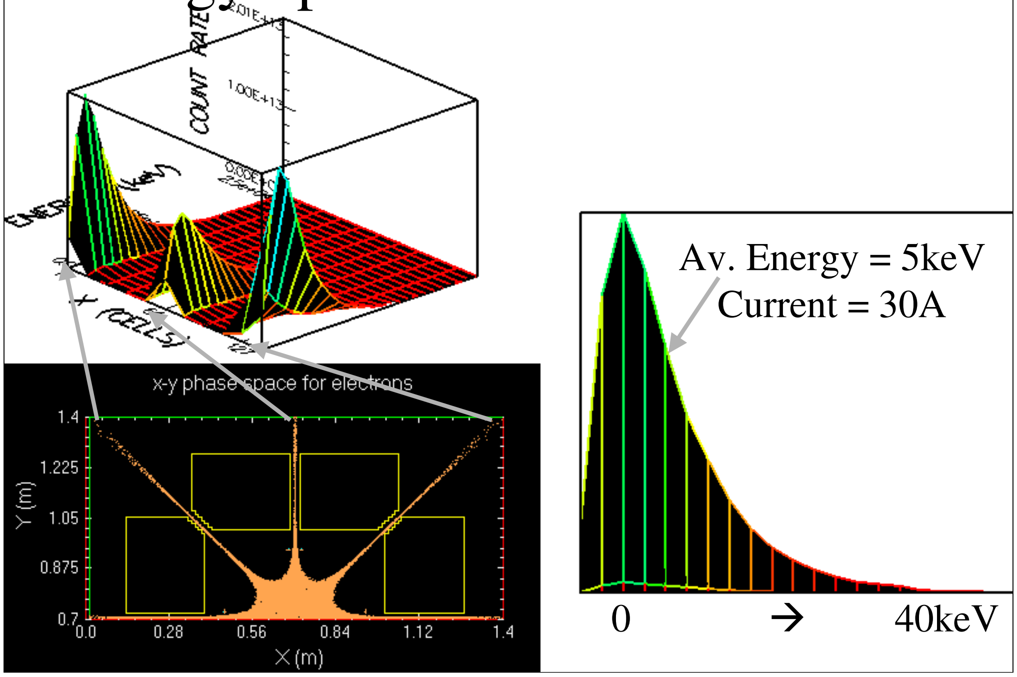
E=15keV gives max yield for DD.

The β=1 condition fixes the product of ion energy(E_i) x ion density(n). Fusion rate is proportional to n^2 which falls with rising energy as E^{-2}. Velocity U rises as $E^{0.5}$.

To make $n^2\langle\sigma U\rangle$ stationary (and therefore Q maximum) requires σ to have slope $E^{0.5}/E^{-2} = E^{1.5}$.

Maximum fusion rate occurs at the ion energy where the slope of the DD yield curve is $E^{1.5}$, indicated by the point where the 1.5-slope line kisses the DD curve.

Energy Spectrum of Lost Electrons



For each of the 4 tank walls, electrons emerge from the core on 3 cusp lines. Face cusp(center) losses are over-estimated in 2d simulation compared to 3d reality.

Face cusp losses were ignored. The average lost electrons' energy was computed from the corner cusp energy spectrum in the right inset.

This energy was multiplied by the loss-current to get lost power, shown in the next slide.

Power Consumed by Lost Electrons

- Lost Power(LP) ≡ Input-Current x Lost-energy
- Simulated LP = (1.5)(30A)(5kV)
- = 225 kW
- An ad hoc factor adjusts the simulation density losses up to the β=1 density losses. Factor = $(n/n_{\beta 1})^{½} = (2.7/1.1)^{½} = 1.6$
- LP = (1.6)(225kW) = 350kW.

For convenience, the lost-energy is expressed in volts (rather than electron-volts) so that the resulting units of lost power come out in conventional watts.

The lost power is adjusted by an ad hoc factor of 1.5 to reduce it by ½ for face cusp losses and raise it by 12/4 for extra edges in 3d not in the 2d simulation.

An additional ad hoc factor of 1.6 adjusts for the accidental shortfall of the simulated density below the desired β=1 steady state density.

Simulated Q and Break-even Radius

- $Q = P_{out}/P_{in}$
- $P_{out} = ½n^2<\sigma U>VE_{DD} = 0.5\ (2.7e19)^2\ (0.75)\ (1.5e\text{-}30)\ (1.2e6)\ (0.32)^3\ (3.7e6)\ (1.6e\text{-}19) = 9.3W$
- P_{in} = 350kW from previous slide.
- Q = 9.3/3.5e5 =2.6e-5
- $R_b = R/Q^{1/5}$ = (0.16m)(8.2) = 1.3m
- **DD fueled Polywell is smaller than ITER.**

In P_{out} n is the density in m^{-3}, σ is the cross section in m^2, U is the velocity in m/s, V is the volume in m^3, and E_{DD} is the DD fusion energy from slide#16.

The $<\sigma U>$ product was averaged over the collision angles of the deuterons, resulting in a factor of 0.75 multiplying the head-on cross section (c.f. next slide).

The value R=0.16m is the simulated radius observed(slide#13 right panel) for the approximately cubic plasma cloud confined in the 1.4m diameter tank.

Cross-section x Velocity Average

$<\sigma v_{rel}> = \pi^{-1}\int d\theta[\sigma_0(E_{cm}/E_0)^{1.5}][2U^2-2U^2\cos(\theta)]^{1/2}$

where: $\sigma_0(E_0)$ is head-on ($\theta=\pi$) cross section, &

$E_{cm} = \frac{1}{2}\mu U^2[1-\cos(\theta)] = 1/4\ MU^2[1-\cos(\theta)]$

Substituting and integrating:

$<\sigma v_{rel}> = 0.75\sigma_0 U$,

In other words, $<\sigma v_{rel}>$ is ¾ the head-on value.

The magnitudes of the individual ions' velocities U are all the same inside the potential well.

However the relative velocity varies from 0 to 2U, depending on the angle(θ) between the two colliding ions' velocity vectors.

The integral shown above averages σU over all possible θ values, taking into account the variation of cross section and relative velocity with θ.

Conclusions

- DD fueled Polywell will be smaller than ITER.
- Wiffleball effect is needed but remains to be seen.
- Neutral gas fuel must be tightly localized to control ions first-pass energy.
- Engineering design of the gas handling system will be challenging.
- Polywell remains a prime contender to solve the world's energy crisis.

ITER is designed to exceed break-even performance, but by how much remains to be seen.

Proving the existence of the Wiffleball effect is an important goal in this ongoing particle-in-cell simulation project.