Double P1 Approximation to Electron Distribution Function for Purposes of Computing Non-Local Electron Transport

Jeffrey Chenhall, Duc Cao, Gregory Moses

Fusion Technology Institute, University of Wisconsin–Madison

This work formulates a Double P1 (DP1) expansion of the Fokker-Planck equation in order to take advantage of the strong correlation between electron energy and direction. Previous formalisms[a,b] have made use of a P1 expansion of the Fokker-Planck equation to create a diffusion model for electron thermal conduction.

Steady State Transport Equation:

$$\vec{v}\cdot\vec{\nabla} f - \frac{e\vec{E}}{m_e}\cdot\frac{\partial f}{\partial \vec{v}} = C(f)$$

1D Slab Steady State TE with Krook collision operator:

$$v\mu\frac{\partial f}{\partial x} - \frac{eE}{m_e}\mu\frac{\partial f}{\partial v} = C(f) = -\nu_e(f - f_{SOURCE})$$

Expand *f* in Legendre Polynomials on the half interval (DP1 expansion):

$$f(\mu,v,x) = \begin{cases} f_0^+(v,x) + (2\mu-1)f_1^+(v,x) \\ f_0^-(v,x) + (2\mu+1)f_1^-(v,x) \end{cases}$$

Half-angular moments can be put in diffusive form[c]:

$$-\mathfrak{D}(D_1\mathfrak{D}\psi_1) + \Sigma\psi_1 = S_0 - 3\mathfrak{D}(D_1S_1) + 2\Sigma\psi_2$$

$$-\mathfrak{D}(D_2\mathfrak{D}\psi_2) + \frac{7}{2}\Sigma\psi_2 = -\frac{3}{4}S_0 + S_2 - 3\mathfrak{D}(D_2S_3) + \frac{3}{4}\Sigma\psi_1$$

Apply a zero current condition to calculate the electric field term:

$$a(x) = \frac{eE(x)}{m_e} = \frac{-\frac{\partial}{\partial x}\int_0^\infty v^5\psi_1 dv + 3\int_0^\infty v^5 S_1 dv}{4\int_0^\infty v^3\psi_1 dv}$$

Where:

$$\mathfrak{D} = \frac{\partial}{\partial x} - \frac{a}{v}\frac{\partial}{\partial v}$$

$$\Sigma(x,v) = \frac{\nu_e}{v} = \left(\frac{v_{th}}{v}\right)^4 \cdot \frac{4\pi n_e e^4 \log\Lambda}{(k_b T_e)^2}$$

$$S(x,v,\mu) = \Sigma \cdot f_{SOURCE}$$

$$\psi_1 = (f_0^+ + f_0^-) + \frac{3}{2}(f_1^+ - f_1^-)$$

$$\psi_2 = \frac{3}{4}(f_1^+ - f_1^-)$$

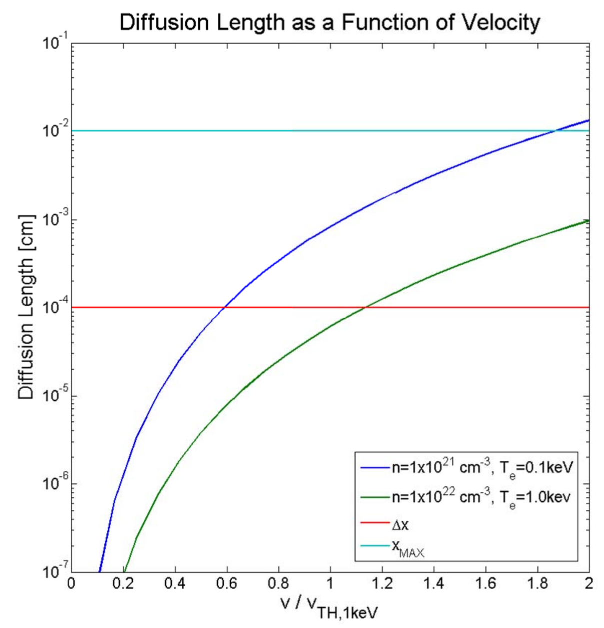
$$D_1 = 2D_2 = \frac{1}{3\Sigma}$$

$$S_0 = (S_0^+ + S_0^-)$$

$$S_1 = \frac{1}{2}(S_0^+ - S_0^-) + \frac{1}{2}(S_1^+ + S_1^-)$$

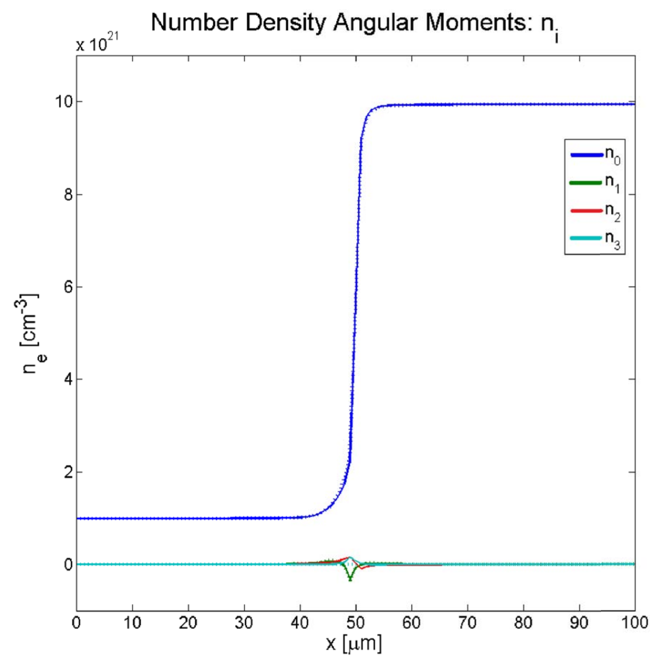
$$S_2 = \frac{3}{4}(S_1^+ - S_1^-)$$

$$S_3 = -\frac{1}{8}(S_0^+ - S_0^-) + \frac{3}{8}(S_1^+ + S_1^-)$$

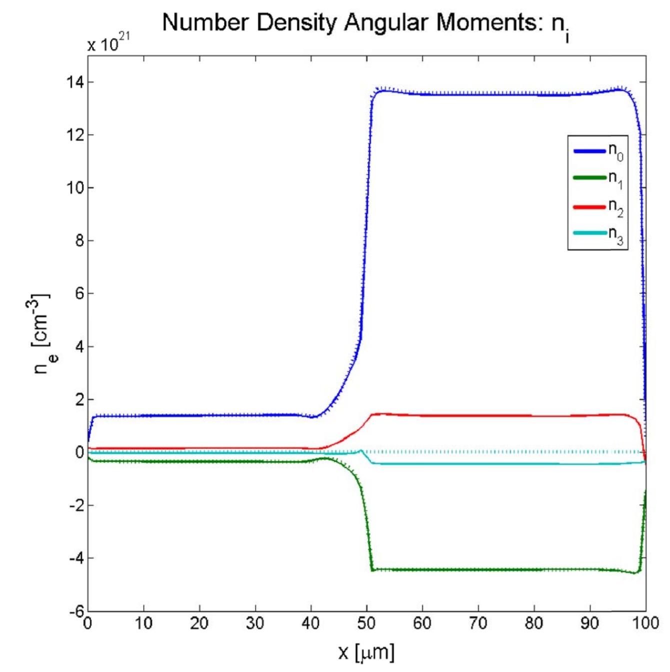


Diffusion length as a function of velocity as compared to problem size (cyan) and grid spacing (red)

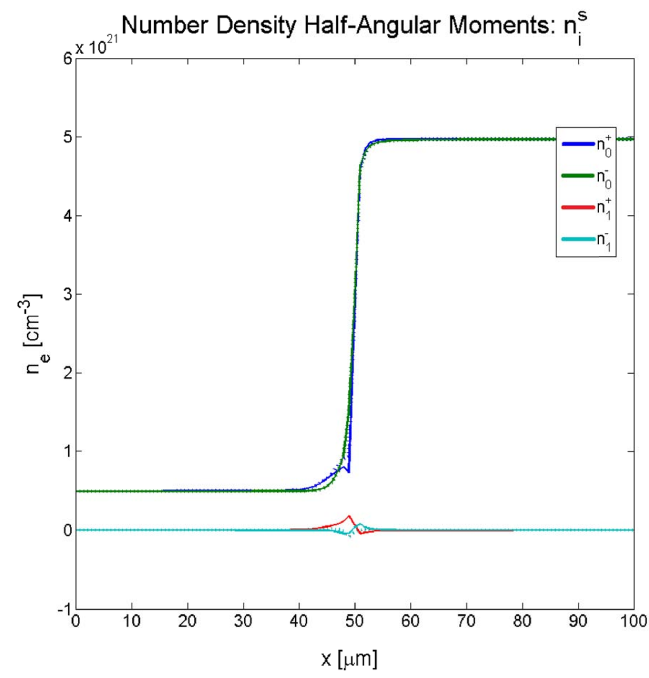
DP1 Model Gives Improved Results



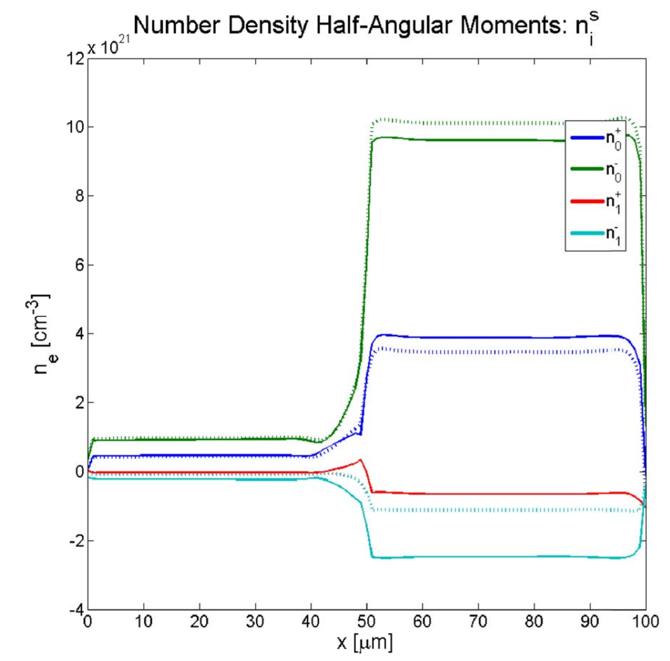
DP1 approximation to first four angular moments for DP1 (solid) and P1 (dashed) for isotropic source



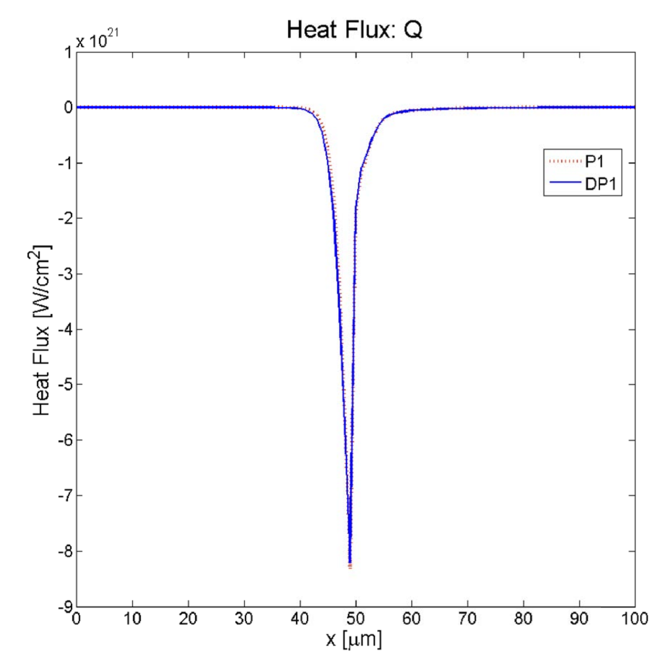
DP1 approximation to first four angular moments for DP1 (solid) and P1 (dashed) for anisotropic source



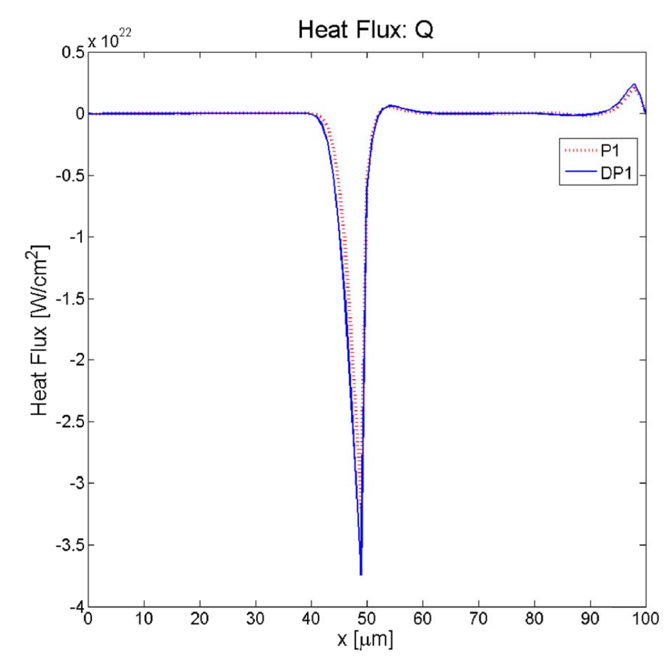
DP1 half-angular moments for DP1 (solid) and P1 (dashed) for isotropic source



DP1 half-angular moments for DP1 (solid) and P1 (dashed) for anisotropic source



Heat flux for P1 and DP1 for isotropic source



Heat flux for P1 and DP1 for anisotropic source

Test problems to compare DP1 with P1 approximation:

- Isotropic Problem
  - Maxwellian source distribution with strong gradients:
    - Left Domain Half: T = 0.1 keV, n=1x10^{21} cm^{-3}
    - Right Domain Half: T = 1.0 keV, n=1x10^{22} cm^{-3}
- Anisotropic Problem
  - Added anisotropic source term
  - $S(x,v,\mu) = f_S \cdot \left[1 + \frac{1}{3}\delta(\mu+1)\right]$

$$\approx f_S \cdot \left[1 + \frac{1}{3}\left(1 - 3\mu + \frac{3}{4}\cdot 5\left(\frac{3}{2}\mu^2 - \frac{1}{2}\right) - \frac{1}{4}\cdot 7\left(\frac{5}{2}\mu^3 - \frac{3}{2}\mu\right)\right)\right]$$

- 101 linearly spaced spatial zones between 0 and 100 μm
- 25 linearly spaced velocity groups between 0 and $2 \cdot v_{TH\ @1keV}$

Discussion

- DP1 and P1 showed good agreement for isotropic problem
- 10% Difference seen in anisotropic problem in half angular moments.
- DP1 model better than P1 for higher anisotropy problems due to its handling of higher order moments

Future Work

- Solve full plasma conduction problem
- Generalize the DP1 model to 2D
- Improve model robustness

- [a]Schurtz et. al. Phys. Plasmas **7**, 4238 (2000)
- [b]Manheimer et. al. Phys. Plasmas **15**, 083103 (2008)
- [c]Gelbard et. al. Nucl. Sci. Eng. **5**, 36-44 (1959)

- This work is supported by the University of Rochester Laboratory for Laser Energetics