Double P1 Approximation to Electron Distribution Function for Purposes of Computing Non-Local Electron Transport

Jeffrey Chenhall, Duc Cao, Gregory Moses
Fusion Technology Institute, University of Wisconsin–Madison

This work formulates a Double P1 (DP1) expansion of the Fokker-Planck equation in order to take advantage of the strong correlation between electron energy and direction. Previous formalisms have made use of a P1 expansion of the Fokker-Planck equation to create a diffusion model for electron thermal conduction.

Steady State Transport Equation:
\[ \vec{v} : \vec{\nabla} f - \frac{eE}{m_e} \frac{\partial f}{\partial v} = C(f) \]

1D Slab Steady State TE with Krook collision operator:
\[ n \mu \frac{\partial f}{\partial x} - \frac{eE}{m_e} \frac{\partial f}{\partial v} = C(f) = -n \nu_c (f - f_{\text{SOURCE}}) \]

Expand \( f \) in Legendre Polynomials on the half interval (DP1 expansion):

\[ f(\mu, v, x) = \left\{ \begin{array}{ll} f_0^0(v, x) + (2\mu - 1)f_1^1(v, x) & \text{for isotropic source} \\ f_0^0(v, x) + (2\mu + 1)f_1^1(v, x) & \text{for anisotropic source} \end{array} \right. \]

Half-angular moments can be put in diffusive form:

\[ -\Delta(D_1 \partial_1 \psi_1 + \Sigma \psi_1) = S_0 - 3\Sigma D_1 S_1 + 2\Sigma S_2 - 3\Sigma D_2 S_2 + 3\Sigma \psi_1 \]

Apply a zero current condition to calculate the electric field term:

\[ a(x) = \frac{eE(x)}{m_e} = -\frac{\partial}{\partial x} \int_0^\infty v^2 \psi_1 dv + \int_0^\infty v^2 S_2 dv \]

Where:

\[ \Delta = \frac{\partial}{\partial x} - \frac{a}{v} \frac{\partial}{\partial v} \]

\[ \Sigma(x, v) = \frac{\nu_c}{v} = \left( \frac{v_0}{v} \right)^4 \frac{4n_e e^4 \log \Lambda}{(k_b T_e)^2} \]

\[ S(x, v, \mu) = \Sigma \cdot f_{\text{SOURCE}} \]

\[ \psi_1 = (f_0^0 + f_0^1) + \frac{3}{2}(f_1^1 - f_1^0) \]

\[ \psi_2 = \frac{3}{2}(f_1^1 - f_1^0) \]

\[ D_1 = 2D_2 = \frac{1}{3\Sigma} \]

\[ S_0 = (S_0^0 + S_0^1) \]

\[ S_1 = \frac{1}{2}(S_0^1 - S_0^0) + \frac{1}{4}(S_1^1 + S_1^0) \]

\[ S_2 = \frac{3}{4}(S_1^1 - S_1^0) \]

DP1 Model Gives Improved Results

Test problems to compare DP1 with P1 approximation:

- **Isotropic Problem**
  - Maxwellian source distribution with strong gradients:
    - Left Domain Half: \( T = 0.1 \text{ keV}, n = 1 \times 10^{21} \text{ cm}^{-3} \)
    - Right Domain Half: \( T = 1.0 \text{ keV}, n = 1 \times 10^{22} \text{ cm}^{-3} \)
  - Added anisotropic source term
    \[ S(x, v, \mu) = f_S \cdot \left[ 1 + \frac{1}{3} \left( \mu + 1 \right) \right] \]
    \[ \approx f_S \cdot \left[ 1 + \frac{1}{3} \left( -3 \mu + 3 \cdot 5 \left( \frac{3}{2} \mu^2 - \frac{1}{2} \right) - \frac{1}{4} \cdot 7 \left( \frac{3}{2} \mu^3 - \frac{3}{2} \mu \right) \right) \right] \]

- 101 linearly spaced spatial zones between 0 and 100 \( \mu \)
- 25 linearly spaced velocity groups between 0 and 2 \( \cdot v_{TH} \) @1keV

Discussion

- DP1 and P1 showed good agreement for isotropic problem
- 10% Difference seen in anisotropic problem in half angular moments.
- DP1 model better than P1 for higher anisotropy problems due to its handling of higher order moments

Future Work

- Solve full plasma conduction problem
- Generalize the DP1 model to 2D
- Improve model robustness

\(^{a}\)Schurtz et. al. Phys. Plasmas 7, 4238 (2000)
\(^{c}\)Gelbard et. al. Nucl. Sci. Eng. 5, 36-44 (1959)

\(^{a}\)This work is supported by the University of Rochester Laboratory for Laser Energistics