WINDOW – A Code to Compute Ion Beam Power Constraints

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I. INTRODUCTION

The propagation of intense ion beams down plasma channels is an essential part of most light ion beam fusion reactor designs. Plasma channels provide renewable connections between the ion diodes and the fusion target and thus allow the diodes to be moved to a safe distance from the target explosion. In addition, the pulse width of the ion beam may be shortened in the channel through time-of-flight bunching, thus easing the pulse width constraints on the diode and pulsed power machinery. This requires a diode voltage that is increasing with time so that the tail of the beam will catch up with the head of the beam while the beam is in the channel.

The success of this scheme of ion beam propagation depends on the ion beam and channel remaining stable during transport. One can identify five ways that the system would cease to work: (1) the beam may generate electrostatic turbulence in the channel; (2) the beam may filament through interaction with the channel; or (3) it may cause filamentation in the channel return current; (4) the beam may heat the channel to such a degree that the channel experiences significant radial expansion during the beam transit; and (5) there may be too much beam energy lost to the channel. These effects have been translated into limits on the beam power per channel(1-4) and determine an operational window for the beam-channel system.

The WINDOW computer code(1) was developed as a design tool to find the power limits on the beam. The length of the channel, the power compression due to bunching, the energy spread in the beam, and the fractional energy loss by the beam while it is in the channel were fixed in the code so the operator did not have control over them. In the new version of WINDOW presented here, the user is given the choice of channel length, power compression factor and fractional energy loss. The beam energy spread is calculated by the code. Also, the optimum density for minimum energy loss is calculated while it was not in the earlier version.

This report begins with a summary of the formalism modeling the behavior of the beam, with an emphasis on the improvements made. Then the WINDOW code is described. Finally sample results are presented.

II. FORMALISM

A. Summary of Existing Theory

The earlier work done at the University of Wisconsin and at Fusion Power Associates[1] followed the formalism laid down in a series of articles written at NRL.[2-4] This work has provided limits on the beam power that may be transported down a plasma channel as functions of the many channel and beam parameters, most importantly the maximum angle of injection into the channel of the ions. These power limits are derived from considerations of the growth of electrostatic microturbulence (ES), beam current filamentation (BFIL), channel current filamentation (CFIL), the avoidance of excessive magnetohydrodynamic motion of the channel during beam propagation (MHD) and beam energy loss in the channel (ELOSS).

For the present, the limits on beam power due to ES, BFIL, CFIL, and MHD are taken from the earlier work. Thus,

$$P_{ES} = 1.6 \times 10^{-3}[x^6(r_b)^8(\lambda_{ei})^4R^{12}/\{(Z_b)^{26}(\tau_b)^3F^{12}\}]^{1/7} \frac{\overline{E}^3}{(A_b)^{1/2}} , \quad (1)$$

$$(R/F)_{BFIL} = 2.5 \times 10^{-2} [(\lambda_{ei})^4(\overline{E})^2/\{(A_b)^6(Z_b)^8\}]^{1/8} , \quad (2)$$

$$P_{CFIL} = \{1.76 \times 10^5(x)^2(R)^4/F^4\}\{(\overline{E})^{14}(\tau_b)^2(A_b)^4/(Z_b)^{20}\}^{1/6} , \quad (3)$$

and $$P_{MHD} = 1.5 \times 10^{-21}(x)^2(r_b)^4(\bar{E})^2/\{(\tau_b)^3(Z_b)^2(A_b)\} \tag{4}$$

are also used in the present formalism. In these equations, the power per channel P is in TW, x is the ratio of the channel gas density to the optimum density described in Section II.B and r_b is the channel radius in cm. R is the diameter of the diode in cm, F is the focal length of the diode in cm and λ_{ei} is the Coulomb logarithm,

$$\lambda_{ei} = 24 - \ln[(10^2\ Rr_b/FA_b)(\bar{E}^9x^2/\{(Z_b)^{10}P^3(\tau_b)^3\})^{1/4}] \ . \tag{5}$$

Z_b is the charge state of the beams ions in units of electronic charge, A_b is the ion mass in amu and τ_b is the pulse width of the beam before bunching in seconds. $\bar{E}$ is the average ion energy in the beam in MeV.

B. Bunching Requirements and Channel Length

The bunching of the beam to compress the power through pulse width shorting is dependent on waveform of the diode voltage and the length of the channel. In the work presented here, the bunching factor and the energy spread of the beam are used explicitly in an expression for the channel length. In the earlier work values for these were assumed and the choice of channel length had been removed. In the WINDOW computer code that is described in Section III, the channel length and the bunching factor must be provided by the user and the energy spread is calculated by the code.

The diode voltage may be ramped to bunch the beam while it is in the channel. The ideal diode voltage waveform is

$$\phi(t) = \phi_o(1 - t/\tau_t)^{-2} \ . \tag{6}$$

This leads to a velocity of ions leaving the diode which is also ramped

$$v_i(t) = v_i(t=0)(1 - t/\tau_t)^{-1} \ . \tag{7}$$

The length which is required for the pulse to converge by a factor of α is[4]

$$L = 1.3 \times 10^9 \ (1 - \frac{1}{\alpha})(\tau_t - \tau_b)(\frac{E(\tau_b)}{A_b})^{1/2} \text{ cm} \ . \tag{8}$$

$E(\tau_b)$ is the final ion energy at the end of the pulse in MeV and τ_b is the initial duration of the pulse.

From Eq. (7), one can find τ_t in terms of the voltage spread between the end and beginning of the pulse

$$\frac{\phi(\tau_b)}{\phi(0)} = \frac{1}{(1 - \frac{\tau_b}{\tau_t})^2} \ , \tag{9}$$

which converts to

$$\tau_t = \tau_b\left(\frac{1}{1 - \frac{1}{(1 + \delta)^{1/2}}}\right) \tag{10}$$

if one expresses the energy spread of the beam as

$$\phi(\tau_b) \equiv \phi(0)(1 + \delta) \ . \tag{11}$$

Thus the channel length L can be written in terms of α, δ and τ_b with Eqs. (8) and (10),

$$L = 1.3 \times 10^9 \left(1 - \frac{1}{\alpha}\right) \tau_b \frac{\bar{E}^{1/2}}{A_b^{1/2}} \left(\frac{(1 + \frac{\delta}{2})^{1/2}}{(1 + \delta)^{1/2} - 1}\right) \quad \text{cm} . \qquad (12)$$

C. Beam Energy Loss in Channel

While the ion beam is propagating through the plasma channel, it will lose some of its energy. The energy loss consists of a collisional slowing down component and a loss due to work done by the beam ions against an axial electric field.(3,4) This electric field has an ohmic term and an inductive term proportional to the inverse of the channel gas density. The inductive part of the axial electric field is due to the radial hydrodynamic motion of the channel and the azimuthal magnetic field of the channel. We will neglect the ohmic contribution to the axial electric field. The collisional energy loss is proportional to the channel gas density. Because of the inverted dependence on the plasma density of the collisional and inductive terms, there is an optimum density where the energy loss is a minimum.(4) For a deuterium gas, the optimum channel plasma density is

$$\rho_{opt} = 0.167 \; E^{1/2} P^{1/2} (\tau_b)^{1/2} (\theta_m)^2 / \{(r_b)^2 Z_b\} \qquad (13)$$

where θ_m is the maximum angle at which ions are injected into the channel and is equal to R/F for most cases of interest. This equation depends on an expression for the radial expansion of the plasma channel which may be valid only for a deuterium channel.(2) This equation and those that follow dealing with the energy loss are only valid for deuterium.

With the help of expressions found in Refs. 3 and 4, the fractional energy loss for the beam ions can be written as

$$f_E = \frac{1.7 \times 10^2 (x + 1/x)\ A_b \theta_m^2 Z_b^{1/2} I_b^{1/2} \tau_b^{1/2} L}{\bar{E} r_b^2} \tag{14}$$

where I_b is the beam ion current. There is a strong dependence on f_E of θ_m because the main energy loss mechanism occurs through a $v_r \times B_\theta$ force on the beam ions. B_θ is the confining field, whose required magnitude is $\propto \theta_m^2$. One can invert this expression and find the power limit on the beam due to energy loss,

$$P_{ELOSS} = \frac{I_b \bar{E}}{Z_b} = \frac{3.5 \times 10^{-5}\ f_E^2 \bar{E}^3 r_b^4}{(x + 1/x)^2\ A_b^2 Z_b^2 \tau_b L^2 \theta_m^4} \ . \tag{15}$$

III. WINDOW COMPUTER CODE

A. General Code Description

The WINDOW computer code uses the theory outlined in Section II to study the constraints on the propagation of an ion beam down a plasma channel. A listing of the code is given in Appendix A. The code calculates the limits on the beam power per channel for a range of R/F. The optimum mass density of the gas in the channel is also found at the maximum allowable beam power as a function of R/F. Results are both printed out and are stored in files for plotting. WINDOW is written in FORTRAN 77, is 199 lines long including comments, and requires 51 kilobytes of memory on an IBM personal computer for just the executable code. The input and the output are described in the next two subsections.

B. Input

The input for WINDOW is through a formatted read of a file labeled INPUT.DAT. A sample of this input file is given in Appendix B. Obviously, the filename would have to be adapted if one wished to use WINDOW on a computer other than an IBM PC. The input consists of 14 lines with one E10.4 formatted record per line. The first line is a flag called IPLOT that is not used in the present version of WINDOW. Next comes the number of values for R/F, then the minimum and maximum values for R/F in radians and a maximum value for the power per channel in TW. This is followed by x, r_b in cm, A_b in amu, τ_b in seconds, $\overline{E}$ in MeV and α. The last two lines are the channel length in cm and the fractional energy loss in the channel. To facilitate reading the input, we recommend that a portion of each line past column 11 be used to describe the input as is shown in Appendix B. This will not affect the code in any way but it helps one keep the input in the proper order.

C. Output

WINDOW provides both printed output and output filed for plotting. The output files are summarized in Table I. An example of the printed output is given in Appendix C for the input shown in Appendix B. The output for printing is stored in the file PRNT.OUT. The output begins with a list of the input parameters. Then comes the calculated value of the energy spread parameter δ. WINDOW uses an iteration method for finding δ which involves recalculation of the channel length until it equals the inputted channel length, so, as a check, the calculated and inputted values for the channel length are then given. This is followed by a table of the calculated power limits due to ES, CFIL, MHD, and ELOSS in TW as well as ρ_{opt} in g/cm^3 for all of the values of R/F in radians. The final entry is a table of the power limit due to BFIL in TW versus R/F. The same things that are given in these two tables are stored in the files PLT1.DAT, PLT2.DAT, PLT3.DAT, PLT4.DAT, PLT5.DAT, and PLT6.DAT, as described in Table I. These plot files are written as one record

Table I. Output Files

Filename	Description
PRNT.OUT	Output to be printed
PLT1.DAT	ES power limit in TW versus R/F in radians
PLT2.DAT	CFIL power limit in TW versus R/F in radians
PLT3.DAT	MHD power limit in TW versus R/F in radians
PLT4.DAT	ELOSS power limit in TW versus R/F in radians
PLT5.DAT	BFIL power limit in TW versus R/F in radians
PLT6.DAT	ρ_{opt} in g/cm^3 versus R/F in radians

per line where the record contains R/F followed by the quantity to be plotted in a (G10.4,1X,G10.4) format.

IV. EXAMPLE CALCULATIONS

We have performed several sample calculations with WINDOW, which we will now present. These include determination of beam transport windows for a base case light ion fusion reactor design and for such a reactor design with a large radius channel. We will also present for the base case design calculations of the optimum mass density, a study of the dependence of the ELOSS power limit on channel length, and a study of the interrelation of the beam ion energy spread, the channel length and the bunching factor α. The parameters for the base case reactor design are shown in Table II.

A. "Reactor" Base Case

The WINDOW computer code has been used to find that region of beam power per channel versus R/F space where ion beam propagation is possible from the

Table II. Base Case Reactor Design Channel Parameters

Channel Length (cm)	1000
Channel Radius (cm)	0.5
Channel Gas	Deuterium
Ratio of Channel Density to Optimum Density	1.0
Average Beam Ion Energy (MeV)	30
Fractional Energy Spread	0.2
Power Compression Factor	3.0
Ion Type	Li^{+3}
Beam Pulse Width at Diode (ns)	50
Fractional Beam Ion Energy Loss During Transport	0.25

points of view of ES, CFIL, BFIL, MHD, and ELOSS. The power limits due to these five constraints are shown plotted against R/F in Fig. 1 for the base case light ion fusion reactor design. The cross-hatched region is where all of these constraints are met. One can see that the optimum value for R/F is 0.06 radians and the maximum power per channel is roughly 2.5 TW. This means that about 44 such channels would be required to transport the needed 250 TW to the target when one considers the power compression due to bunching and the energy lost during transport down the channel.

B. Large Radius Case

One way to increase the amount of transportable power per channel is to choose a larger channel radius. The transport window is shown in Fig. 2 for the case where all of the parameters are the same as in Table II except that

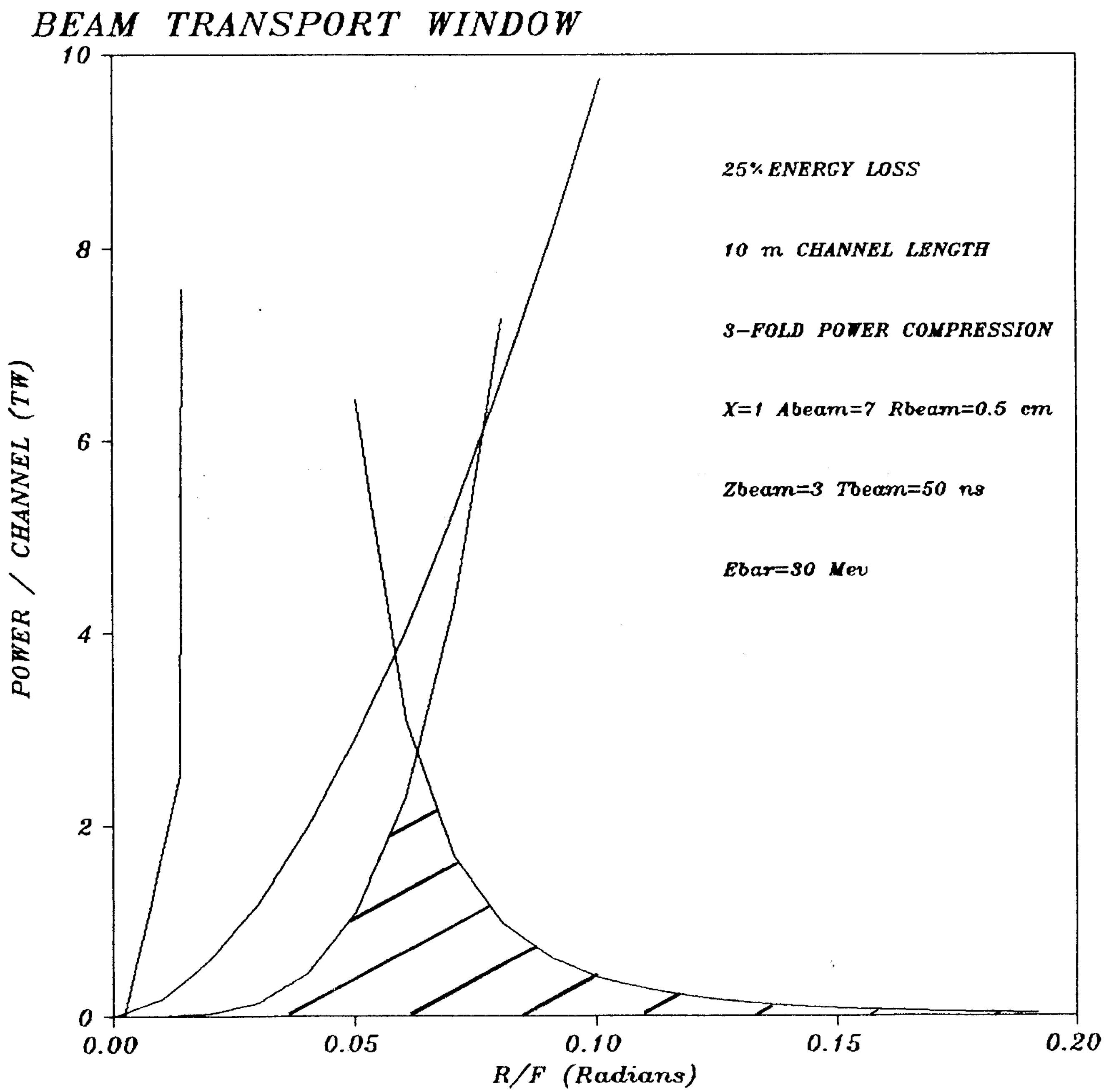

Fig. 1. Beam transport window for light ion fusion reactor base case design.

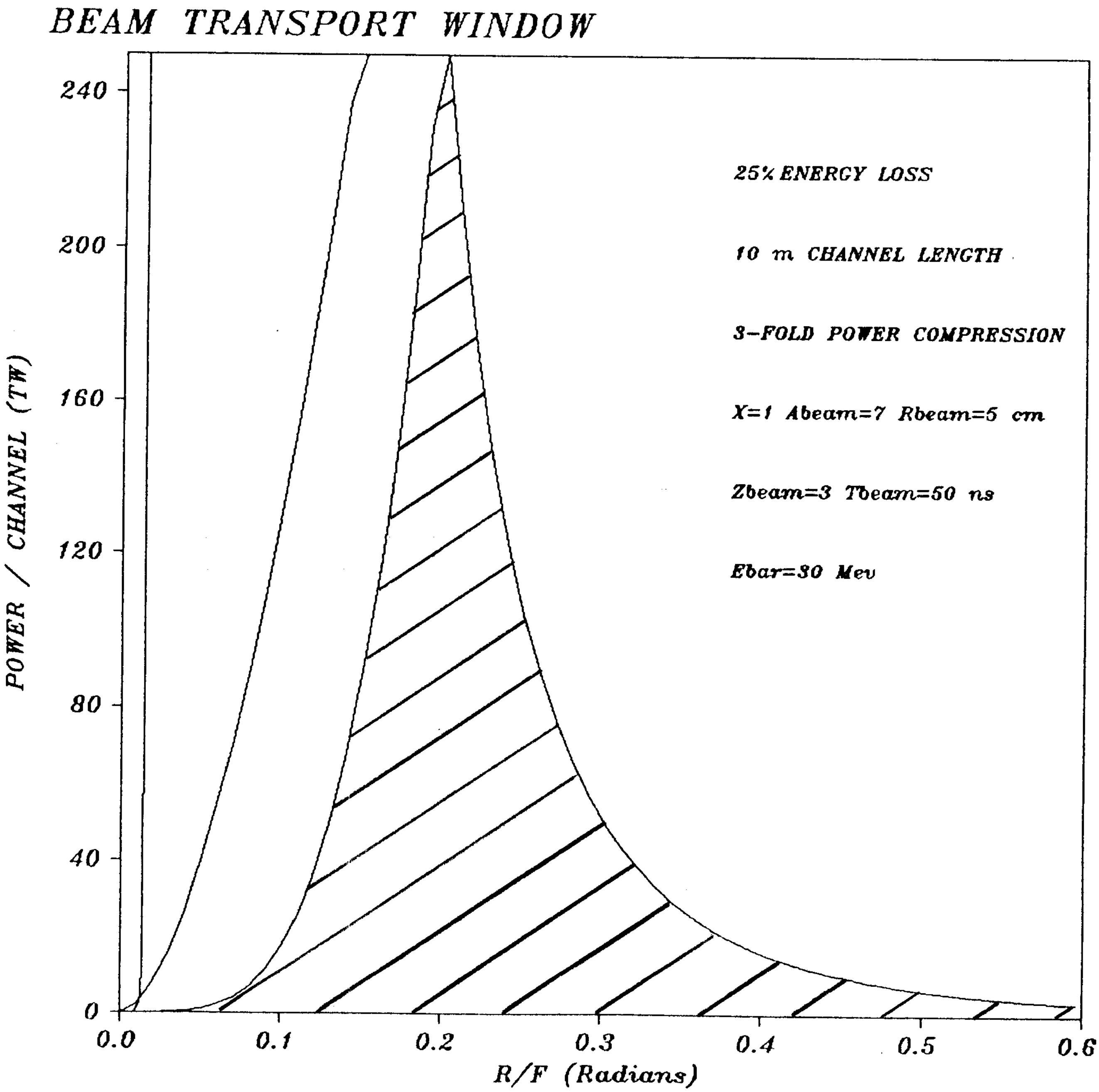

Fig. 2. Beam transport window for light ion fusion reactor design with 5 cm radius channel.

the channel radius is 5 cm, where once again the cross-hatched region is where transport is permitted. For this case a single channel can carry the 250 TW but the diode R/F must be 0.2 radians.

C. Optimum Mass Density

As R/F changes, so does the optimum channel mass density. As Eq. (13) shows, if all else is equal, the density is proportional to (R/F).[4] This behavior is clearly shown in the low R/F part of Fig. 3. This is a plot of the optimum density for the base case light ion fusion reactor. The optimum density is independent of R/F at large values of R/F because here the power limit that is important is the one due to energy loss. From Eq. (15), one can see that P_{ELOSS} varies as $(R/F)^{-4}$, which cancels out the R/F dependence in Eq. (13). A curve such as this is needed when one designs a beam propagation system for a light ion fusion reactor.

D. Energy Loss Power Limit

Since the energy loss power limit P_{ELOSS} defines the high R/F side of the transport window,[4] it is interesting to study its dependence on the channel length. Figure 4 shows P_{ELOSS} plotted against R/F for channel lengths of 4, 6, 8, 10, and 12 m for the base case parameters. One should not take the convergence of the 6, 8, and 10 m lines at R/F = 0.02 radians seriously because it is due to the setting of the maximum power to 250 TW. Equation (15) has an L^{-2} dependence, a behavior that is shown in Fig. 4. The maximum transportable power in Fig. 1 was 2.5 TW at 0.06 radians, where the channel length was 10 m. In Fig. 4, at 0.06 radians one finds that for a channel length of 4 m, P_{ELOSS} is about 15 TW. So, by going to a shorter channel and a slightly higher R/F, one can substantially increase the power per channel. However, the channel length is often dictated by geometrical constraints which cannot be relaxed

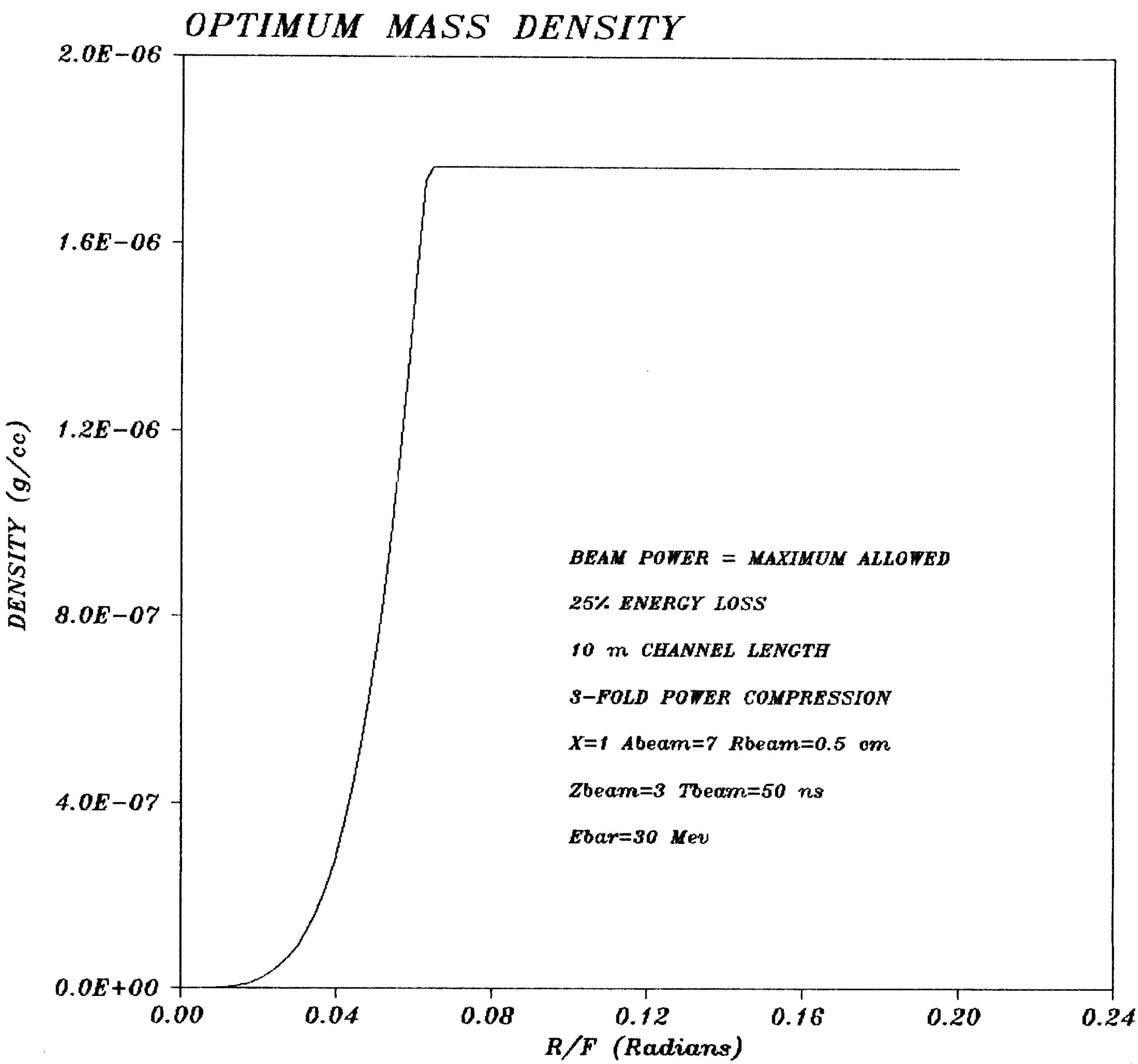

Fig. 3. Optimum channel mass density versus R/F for base case light ion fusion reactor design.

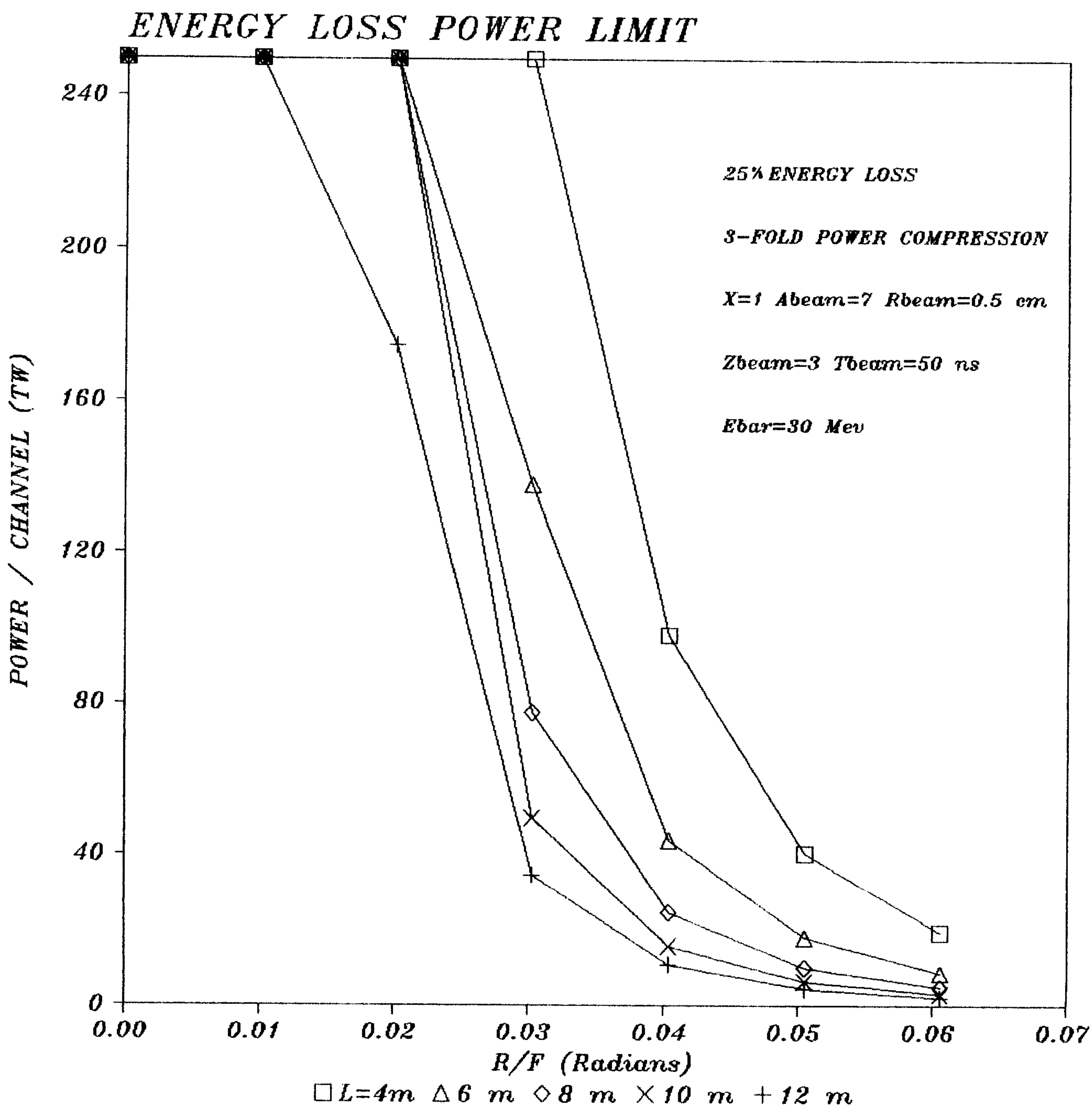

Fig. 4. Energy loss power limit versus R/F for various channel lengths for base case light ion fusion reactor design.

but one can have the same effect by increasing the allowable fractional energy loss.

E. Bunching Requirements

The WINDOW code has been used to study the interrelations between the parameters governing bunching of the beam. The fractional energy spread is shown in Fig. 5, plotted against the channel length for a few values of the power compression factor, α. The base case parameters of a 10 m channel length and a 3-fold power compression yield a fractional energy spread of 0.2. There is a minimum achievable fractional energy spread for today's diodes somewhere around 0.1 that puts an upper limit on the channel length. One can see that at moderate and large channel lengths the fractional energy spread, δ, has only a rather weak dependence on the power compression factor, α.

V. CONCLUSIONS

The WINDOW computer code has been updated to allow the user to explicitly choose values for the channel length, the power compression during bunching and the fractional energy loss by the ion beam while it is moving down the channel. The code still is only valid for deuterium channel gases. The code provides separate files for plotting all of the power limits as well as the optimum channel mass density and now has a detailed printed output.

WINDOW has been used to study the bunching of beams while in the channel and the sensitivity of the beam transport window to the channel length and radius. The dependence of the optimum channel gas mass density on the maximum beam injection angle has also been studied. In these and other ways WINDOW will be a useful tool in light ion beam fusion reactor design studies.

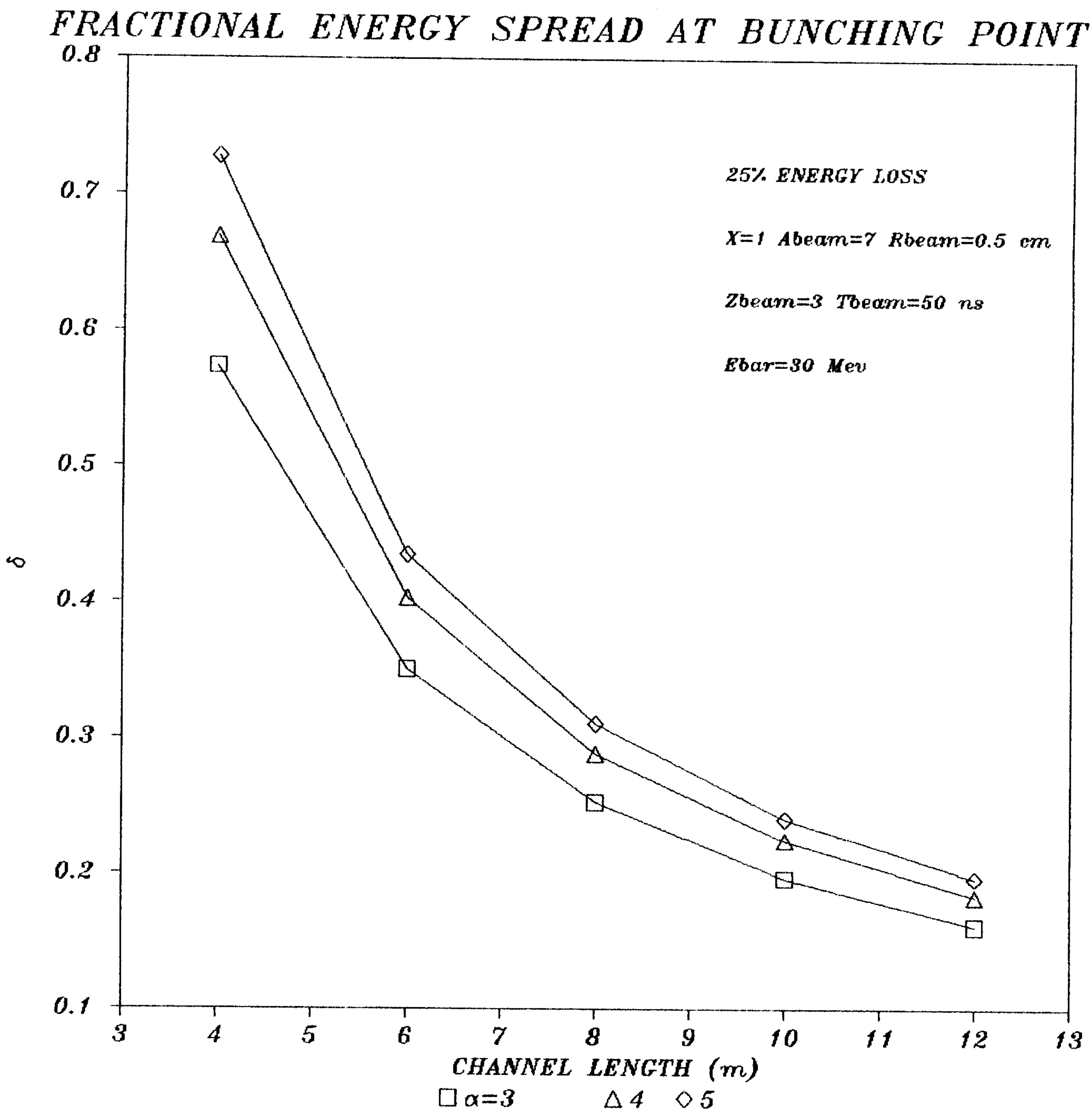

Fig. 5. Fractional beam ion energy spread versus channel length and power compression factor for base case light ion fusion reactor design.

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2. D.G. Colombant, S.A. Goldstein and D. Mosher, "Hydrodynamic Response of Plasma Channels to Propagating Ion Beams," Phys. Rev. Lett. 45, 1253 (1980).

3. D. Mosher, D.G. Colombant and S.A. Goldstein, "Beam Requirements for Light-Ion-Driven Inertial-Confinement Fusion," Comments Plasma Physics 6, 101 (1981).

4. P.F. Ottinger, S.A. Goldstein and D. Mosher, "Constraints on Transportable Ion Beam Power," Naval Research Laboratory Memorandum Report 4948 (November 1982).

APPENDIX A:

WINDOW SOURCE LISTING

```
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 LAMDA1, LAMDA2
      REAL*4 SA,SZ,SR,SX,SE,ST,RLABEL,PELOSS,PMHD,PCFIL,DPBF
      REAL*4 PBFIL, PES, ROVERF, RF1, PVAR, SC1, SC2, SCDATA, SCRF
      REAL*4 PLAB, RFLAB, PLAB1, RFLAB1, DUMIN, LCHAN, CPMIN
      REAL*4 DELTA, DELTAH, DELTAL, L1
      DIMENSION PELOSS(200), PMHD(200),PCFIL(200), PBFIL(200),
     1     PES(200), ROVERF(200), PVAR(200,5), SCDATA(10),
     2     SCRF(10), SC1(10), SC2(10), RLABEL(20), RF1(200), ILAB(10)
      DIMENSION DUMIN(20), RHOOPT(200)
      EQUIVALENCE (PVAR(1,1), PELOSS(1)), (PVAR(1,2), PMHD(1)),
     1     (PVAR(1,3), PCFIL(1)), (PVAR(1,4), PES(1)),
     2     (PVAR(1,5), PBFIL(1))
      COMMON/COMV/PVAR
     +     NUM,           PMAX,           IPLOT
C
C   READ IN PARAMETERS
      OPEN(5,FILE='INPUT.DAT',FORM='FORMATTED')
      READ(5,1000) (DUMIN(I),I=1,14)
 1000 FORMAT(E10.0)
      IPLOT = DUMIN(1) + 0.0001
      NUM = DUMIN(2) + 0.0001
      RFMIN = DUMIN(3)
      RFMAX = DUMIN(4)
      PMAX = DUMIN(5)
      X = DUMIN(6)
      RBEAM = DUMIN(7)
      ABEAM = DUMIN(8)
      ZBEAM = DUMIN(9)
      TBEAM = DUMIN(10)
      EBAR = DUMIN(11)
      ALPHA = DUMIN(12)
      LCHAN = DUMIN(13)
      FELOSS = DUMIN(14)
C    PRINT OUT INPUT
      OPEN(15,FILE='PRNT.OUT',FORM='FORMATTED',STATUS='NEW')
      WRITE(15,1999) NUM,RFMIN,RFMAX,PMAX
      WRITE(15,2000) X,RBEAM,ABEAM,ZBEAM,TBEAM,EBAR
      WRITE(15,2001) ALPHA, LCHAN,FELOSS
 1999 FORMAT(' NUM=',I4,'   RFMIN=',G10.4,'    RFMAX=',G10.4/
     1     '   PMAX=',G10.4,' TW')
 2000 FORMAT(' X=',G10.4/'RBEAM=',G10.4,' CM'/'ABEAM=',G10.4/
     1     'ZBEAM=',G10.4,' E'/
     2     'TBEAM=',G10.4,' SEC'/'EBAR=',G10.4,' MEV')
 2001 FORMAT(' ALPHA=',G10.4/' LCHAN=',G10.4,' CM'/' FELOSS=',G10.4)
C
C   INITIALIZE
      EZERO = EBAR / (DSQRT(ABEAM) * ZBEAM)
      LAMDA2 = 10.D0
      NPTS = 800
C
C  FIND ENERGY SPEAD PARAMETER 'DELTA' USING AN INTERVAL HALVING METHOD
      ITERA = 0
      DELTAH = 10.E0
      DELTAL = 1.E-6
```

```
   11 ITERA = ITERA + 1
      IF( ITERA .GT. 100 ) GOTO 4
      DELTA = .5 * ( DELTAL + DELTAH )
      L1 = 1.3E9 * ( 1. - (1./ALPHA) ) * TBEAM *
     1     SQRT( EBAR / ABEAM ) *
     2     ( SQRT( 1. + ( .5 * DELTA) ) / ( SQRT( 1 + DELTA ) - 1. ) )
      IF( ABS((L1-LCHAN)/(L1+LCHAN)) .LT. 1.E-3 ) GOTO 4
      IF( LCHAN .GT. L1 ) GOTO 12
C
C  IF THE CALCULATED CHANNEL LENGTH IS TOO LARGE, DELTA IS TOO SMALL.
      DELTAL = DELTA
      GOTO 11
C
C  IF THE CALCULATED CHANNEL LENGTH IS TOO SMALL, DELTA IS TOO LARGE.
   12 DELTAH = DELTA
      GOTO 11
    4 CONTINUE
      WRITE(15,350) DELTA, L1, LCHAN
  350 FORMAT(' DELTA (FRACTIONAL ENERGY SPREAD AT FOCUS) = ',G10.4/
     1       ' CALCULATED CHANNEL LENGTH = ',G10.4,' CM'/
     2       ' INPUTTED CHANNEL LENGTH   = ',G10.4,' CM')
      DO 5 I = 1, NUM
      PELOSS(I) = 0.E0
      PMHD(I) = 0.E0
      PCFIL(I) = 0.E0
      PBFIL(I) = 0.E0
    5 PES(I) = 0.E0
C
C  CREATE MESH
      DRF = ( RFMAX - RFMIN ) / ( NUM - 1 )
      DO 10 IRF = 1, NUM
   10 ROVERF(IRF) = RFMIN + DRF * ( IRF - 1 )
C
C  FIND CRITICAL POWERS
      DO 20 IRF = 1, NUM
C
C  ES
      ITER = 0
    1 LAMDA1 = LAMDA2
      ITER = ITER + 1
      IF( ITER .GE. 20 ) GOTO 100
      PES(IRF) = 1.6D-3 * (((X**6) * (RBEAM**8) * (ABEAM**6.5)
     1     * (LAMDA1**4) * (ROVERF(IRF)**12) / ( (ZBEAM**5) *
     2     (TBEAM**3) ) ) ** 0.14286 ) * (EZERO**3)
      IF( PES(IRF) .LT. 1.D-6 ) PES(IRF) = 1.D-6
      LAMDA2 = 24.D0 - DLOG(( 1.D2 * RBEAM * (( EZERO**9) *
     1     DSQRT(ABEAM) * X * X / ( ZBEAM * ( (PES(IRF) * TBEAM) ** 3 )
     2     )) ** 0.25 ) * ROVERF(IRF) )
      IF( DABS((LAMDA1 - LAMDA2)/(LAMDA1 + LAMDA2)).GT.1.D-2) GOTO 1
      IF( PES(IRF) .GT. PMAX ) PES(IRF) = PMAX
C
C  CHANNEL FILAMENTATION
      PCFIL(IRF) = 1.76D5 * X * X * (ROVERF(IRF)**4)
     1     * ( ( (EZERO**14) * (TBEAM**2) * (ABEAM**11) ) ** .166667 )
     2     / ZBEAM
```

```
      IF(PCFIL(IRF) .GT. PMAX) PCFIL(IRF) = PMAX
C
C  MHD CHANNEL EXPANSION
      PMHD(IRF) = 1.5D-21 * X * X * (RBEAM**4) * EZERO * EZERO /
     1     (TBEAM**3)
      IF(PMHD(IRF) .GT. PMAX) PMHD(IRF) = PMAX
      PELOSS(IRF) = FELOSS * FELOSS * EBAR * EBAR * EBAR *
     1     ( RBEAM  ** 4) * 3.5E-5
     2     /( ZBEAM * ZBEAM * ABEAM * ABEAM
     3     * ( ROVERF(IRF) ** 4 ) * TBEAM * LCHAN * LCHAN
     4     * ( ( X + 1./X ) ** 2 ) )
      IF(PELOSS(IRF) .GT. PMAX) PELOSS(IRF) = PMAX
C
C  FIND THE OPTIMUM MASS DENSITY 'RHOOPT'
C
C          FIRST FIND THE MINIMUM POWER LIMIT (EXCLUDING BEAM FILIMENT)
      CPMIN = PES(IRF)
      IF( CPMIN .GT. PCFIL(IRF) ) CPMIN = PCFIL(IRF)
      IF( CPMIN .GT. PMHD(IRF) ) CPMIN = PMHD(IRF)
      IF( CPMIN .GT. PELOSS(IRF) ) CPMIN = PELOSS(IRF)
C
C   NOW FIND RHOOPT
      RHOOPT(IRF) = 0.167 * SQRT( EBAR  * CPMIN * TBEAM )
     1     * ROVERF(IRF) * ROVERF(IRF) / ( RBEAM * RBEAM * ZBEAM )
C
 20   CONTINUE
C
C
C  BFIL
      DPBF = PMAX / ( NUM - 1 )
      DO 40 IRF = 1, NUM
      PBFIL(IRF) = DPBF * (IRF - 1) + 1.E-6
      ITER = 0
 2    ITER = ITER + 1
      LAMDA1 = LAMDA2
      IF( ITER .LE. 50 ) GOTO 209
 209  RF1(IRF) = 2.5D-2 * ( ( (LAMDA1**4) * (EZERO**2) /
     1     ( (ABEAM**5) * (ZBEAM**6) ) ) ** .125 )
      LAMDA2 = 24.D0 - DLOG( ( 1.D2 * RBEAM * ( (EZERO**9) *
     1     DSQRT(ABEAM) * X * X / ( ZBEAM * ( (PBFIL(IRF)*TBEAM) ** 3 )
     2     )) ** 0.25 ) * RF1(IRF) )
      IF( DABS((LAMDA1 - LAMDA2)/(LAMDA1 + LAMDA2)).GT.1.D-2) GOTO 2
      IF( RF1(IRF) .GT. RFMAX ) RF1(IRF) = RFMAX
 40   CONTINUE
C
C
C    CREATE FILES FOR ADDITIONAL PLOTTING
C
      OPEN(20,FILE='PLT1.DAT',STATUS='NEW')
      OPEN(21,FILE='PLT2.DAT',STATUS='NEW')
      OPEN(22,FILE='PLT3.DAT',STATUS='NEW')
      OPEN(23,FILE='PLT4.DAT',STATUS='NEW')
      OPEN(24,FILE='PLT5.DAT',STATUS='NEW')
      OPEN(25,FILE='PLT6.DAT',STATUS='NEW')
      DO 102 I = 1, NUM
```

```
          WRITE(20,600) ROVERF(I), PES(I)
          WRITE(21,600) ROVERF(I),PCFIL(I)
          WRITE(22,600) ROVERF(I), PMHD(I)
          WRITE(23,600) ROVERF(I), PELOSS(I)
          WRITE(24,600) RF1(I), PBFIL(I)
          WRITE(25,600) ROVERF(I), RHOOPT(I)
  102     CONTINUE
C
C   PRINT OUT RESULTS
          WRITE(15,700)
          DO 104 I = 1, NUM
          WRITE(15,701) ROVERF(I), PES(I), PCFIL(I), PMHD(I),
     1      PELOSS(I), RHOOPT(I)
  104     CONTINUE
          WRITE(15,702)
          DO 105 I = 1, NUM
          WRITE(15,703) RF1(I), PBFIL(I)
  105     CONTINUE
          STOP
C
  100     CONTINUE
C
          STOP
C
  600     FORMAT(G10.4,1X,G10.4)
  700     FORMAT(' '/'  R/F(RAD)   PES(TW)   ',
     1      '  PCFIL(TW)  PMHD(TW) ',
     2      ' PELOSS(TW) RHOOPT(G/CC)'
     3      /' ')
  701     FORMAT(6(1X,G10.4))
  702     FORMAT(' '/'  R/F(RAD)   PBFIL(TW)'/' ')
  703     FORMAT(2(1X,G10.4))
C
          END
```

APPENDIX B

SAMPLE WINDOW INPUT FILE

FILE LISTING FOR input.dat 12-04-1984 09:23:38

```
1    0.0000E+00   IPLOT-PLOT FLAG
2    1.0000E+02   NUM-NUMBER OF PLOT POINTS
3    0.1000E-05   RFMIN-MINIMUM R/F (DIVERGENCE)
4    1.0000E+00   RFMAX-MAXIMUM R/F (DIVERGENCE)
5    0.2500E+03   PMAX-MAXIMUM POWER PER CHANNEL (TW)
6    0.1000E+01   X-RATIO OF DENSITY TO OPTIMUM DENSITY
7    0.5000E+00   RBEAM-RADIUS OF CHANNEL (CM)
8    0.7000E+01   ABEAM-ATOMIC MASS OF BEAM IONS (AMU)
9    0.3000E+01   ZBEAM-CHARGE OF BEAM IONS (E)
10   5.0000E-08   TBEAM-DURATION OF BEAM PULSE (SEC)
11   0.3000E+02   EBAR-AVERAGE ENERGY OF BEAM IONS (MEV)
12   3.00000+00   ALPHA-POWER BUNCHING FACTOR
13   1.00000+03   LCHAN-CHANNNEL LENGTH (CM)
14   0.25000+00   FELOSS-FRACTIONA. ENERGY LOSS IN CHANNEL
```

APPENDIX C

SAMPLE WINDOW PRINTED OUTPUT FILE

FILE LISTING FOR B:PRNT.OUT 11-28-1984 17:15:10

NUM= 100 RFMIN= .1000E-05 RFMAX= 1.000
PMAX= 250.0 TW
X= 1.000
RBEAM= .5000 CM
ABEAM= 7.000
ZBEAM= 3.000 E
TBEAM= .5000E-07 SEC
EBAR= 30.00 MEV
ALPHA= 3.000
LCHAN= 1000. CM
FELOSS= .2500
DELTA (FRACTIONAL ENERGY SPREAD AT FOCUS) = .1965
CALCULATED CHANNEL LENGTH = 1002. CM
INPUTTED CHANNEL LENGTH = 1000. CM

R/F(RAD)	PES(TW)	PCFIL(TW)	PMHD(TW)	PELOSS(TW)	RHOOPT(G/CC)
.1000E-05	.1000E-05	.1704E-18	10.71	250.0	.1126E-24
.1010E-01	.1818	.1775E-02	10.71	250.0	.1172E-08
.2020E-01	.5988	.2839E-01	10.71	250.0	.1875E-07
.3030E-01	1.201	.1437	10.71	49.63	.9493E-07
.4041E-01	1.982	.4541	10.71	15.70	.3000E-06
.5051E-01	2.924	1.109	10.71	6.432	.7325E-06
.6061E-01	4.016	2.299	10.71	3.102	.1519E-05
.7071E-01	5.251	4.259	10.71	1.674	.1764E-05
.8081E-01	6.622	7.266	10.71	.9815	.1764E-05
.9091E-01	8.126	11.64	10.71	.6127	.1764E-05
.1010	9.759	17.74	10.71	.4020	.1764E-05
.1111	11.52	25.97	10.71	.2746	.1764E-05
.1212	13.39	36.78	10.71	.1939	.1764E-05
.1313	15.39	50.66	10.71	.1408	.1764E-05
.1414	17.50	68.14	10.71	.1047	.1764E-05
.1515	19.73	89.80	10.71	.7941E-01	.1764E-05
.1616	22.07	116.3	10.71	.6134E-01	.1764E-05
.1717	24.52	148.2	10.71	.4813E-01	.1764E-05
.1818	27.07	186.2	10.71	.3830E-01	.1764E-05
.1919	29.74	231.2	10.71	.3085E-01	.1764E-05
.2020	32.51	250.0	10.71	.2513E-01	.1764E-05
.2121	35.38	250.0	10.71	.2067E-01	.1764E-05
.2222	38.35	250.0	10.71	.1716E-01	.1764E-05
.2323	41.43	250.0	10.71	.1437E-01	.1764E-05
.2424	44.60	250.0	10.71	.1212E-01	.1764E-05
.2525	47.87	250.0	10.71	.1029E-01	.1764E-05
.2626	51.24	250.0	10.71	.8798E-02	.1764E-05
.2727	54.71	250.0	10.71	.7565E-02	.1764E-05
.2828	58.27	250.0	10.71	.6541E-02	.1764E-05
.2929	61.93	250.0	10.71	.5684E-02	.1764E-05
.3030	65.68	250.0	10.71	.4963E-02	.1764E-05
.3131	69.52	250.0	10.71	.4353E-02	.1764E-05
.3232	73.46	250.0	10.71	.3834E-02	.1764E-05
.3333	77.49	250.0	10.71	.3390E-02	.1764E-05
.3434	81.60	250.0	10.71	.3008E-02	.1764E-05
.3535	85.81	250.0	10.71	.2679E-02	.1764E-05
.3636	90.11	250.0	10.71	.2394E-02	.1764E-05
.3737	94.49	250.0	10.71	.2145E-02	.1764E-05

56	.3838	98.96	250.0	10.71	.1928E-02	.1764E-05
57	.3939	103.5	250.0	10.71	.1738E-02	.1764E-05
58	.4040	108.2	250.0	10.71	.1570E-02	.1764E-05
59	.4141	112.9	250.0	10.71	.1423E-02	.1764E-05
60	.4242	117.7	250.0	10.71	.1292E-02	.1764E-05
61	.4343	122.6	250.0	10.71	.1176E-02	.1764E-05
62	.4444	127.6	250.0	10.71	.1073E-02	.1764E-05
63	.4545	132.7	250.0	10.71	.9804E-03	.1764E-05
64	.4646	137.8	250.0	10.71	.8979E-03	.1764E-05
65	.4747	143.1	250.0	10.71	.8239E-03	.1764E-05
66	.4848	148.4	250.0	10.71	.7574E-03	.1764E-05
67	.4949	153.8	250.0	10.71	.6974E-03	.1764E-05
68	.5051	159.3	250.0	10.71	.6433E-03	.1764E-05
69	.5152	164.8	250.0	10.71	.5943E-03	.1764E-05
70	.5253	170.5	250.0	10.71	.5499E-03	.1764E-05
71	.5354	176.2	250.0	10.71	.5095E-03	.1764E-05
72	.5455	182.0	250.0	10.71	.4728E-03	.1764E-05
73	.5556	187.9	250.0	10.71	.4394E-03	.1764E-05
74	.5657	193.9	250.0	10.71	.4088E-03	.1764E-05
75	.5758	199.9	250.0	10.71	.3809E-03	.1764E-05
76	.5859	206.0	250.0	10.71	.3553E-03	.1764E-05
77	.5960	212.2	250.0	10.71	.3318E-03	.1764E-05
78	.6061	218.5	250.0	10.71	.3102E-03	.1764E-05
79	.6162	224.8	250.0	10.71	.2904E-03	.1764E-05
80	.6263	231.3	250.0	10.71	.2721E-03	.1764E-05
81	.6364	237.8	250.0	10.71	.2552E-03	.1764E-05
82	.6465	244.3	250.0	10.71	.2396E-03	.1764E-05
83	.6566	250.0	250.0	10.71	.2252E-03	.1764E-05
84	.6667	250.0	250.0	10.71	.2119E-03	.1764E-05
85	.6768	250.0	250.0	10.71	.1995E-03	.1764E-05
86	.6869	250.0	250.0	10.71	.1880E-03	.1764E-05
87	.6970	250.0	250.0	10.71	.1774E-03	.1764E-05
88	.7071	250.0	250.0	10.71	.1674E-03	.1764E-05
89	.7172	250.0	250.0	10.71	.1582E-03	.1764E-05
90	.7273	250.0	250.0	10.71	.1496E-03	.1764E-05
91	.7374	250.0	250.0	10.71	.1416E-03	.1764E-05
92	.7475	250.0	250.0	10.71	.1341E-03	.1764E-05
93	.7576	250.0	250.0	10.71	.1271E-03	.1764E-05
94	.7677	250.0	250.0	10.71	.1205E-03	.1764E-05
95	.7778	250.0	250.0	10.71	.1144E-03	.1764E-05
96	.7879	250.0	250.0	10.71	.1086E-03	.1764E-05
97	.7980	250.0	250.0	10.71	.1032E-03	.1764E-05
98	.8081	250.0	250.0	10.71	.9815E-04	.1764E-05
99	.8182	250.0	250.0	10.71	.9339E-04	.1764E-05
100	.8283	250.0	250.0	10.71	.8892E-04	.1764E-05
101	.8384	250.0	250.0	10.71	.8471E-04	.1764E-05
102	.8485	250.0	250.0	10.71	.8075E-04	.1764E-05
103	.8586	250.0	250.0	10.71	.7702E-04	.1764E-05
104	.8687	250.0	250.0	10.71	.7350E-04	.1764E-05
105	.8788	250.0	250.0	10.71	.7018E-04	.1764E-05
106	.8889	250.0	250.0	10.71	.6704E-04	.1764E-05
107	.8990	250.0	250.0	10.71	.6408E-04	.1764E-05
108	.9091	250.0	250.0	10.71	.6128E-04	.1764E-05
109	.9192	250.0	250.0	10.71	.5863E-04	.1764E-05
110	.9293	250.0	250.0	10.71	.5612E-04	.1764E-05

.9394	250.0	250.0	10.71	.5374E-04	.1764E-05
.9495	250.0	250.0	10.71	.5149E-04	.1764E-05
.9596	250.0	250.0	10.71	.4936E-04	.1764E-05
.9697	250.0	250.0	10.71	.4733E-04	.1764E-05
.9798	250.0	250.0	10.71	.4541E-04	.1764E-05
.9899	250.0	250.0	10.71	.4359E-04	.1764E-05
1.000	250.0	250.0	10.71	.4185E-04	.1764E-05

R/F(RAD)	PBFIL(TW)
.2256E-02	.1000E-05
.1389E-01	2.525
.1433E-01	5.051
.1453E-01	7.576
.1467E-01	10.10
.1467E-01	12.63
.1478E-01	15.15
.1487E-01	17.68
.1495E-01	20.20
.1501E-01	22.73
.1507E-01	25.25
.1512E-01	27.78
.1517E-01	30.30
.1521E-01	32.83
.1525E-01	35.35
.1529E-01	37.88
.1532E-01	40.40
.1535E-01	42.93
.1538E-01	45.45
.1541E-01	47.98
.1543E-01	50.51
.1546E-01	53.03
.1548E-01	55.56
.1550E-01	58.08
.1552E-01	60.61
.1554E-01	63.13
.1556E-01	65.66
.1558E-01	68.18
.1560E-01	70.71
.1562E-01	73.23
.1563E-01	75.76
.1565E-01	78.28
.1566E-01	80.81
.1568E-01	83.33
.1569E-01	85.86
.1571E-01	88.38
.1572E-01	90.91
.1573E-01	93.43
.1575E-01	95.96
.1576E-01	98.48
.1577E-01	101.0
.1578E-01	103.5
.1580E-01	106.1
.1581E-01	108.6
.1582E-01	111.1

166	.1583E-01	113.6
167	.1584E-01	116.2
168	.1585E-01	118.7
169	.1586E-01	121.2
170	.1587E-01	123.7
171	.1588E-01	126.3
172	.1589E-01	128.8
173	.1590E-01	131.3
174	.1591E-01	133.8
175	.1592E-01	136.4
176	.1592E-01	138.9
177	.1593E-01	141.4
178	.1594E-01	143.9
179	.1595E-01	146.5
180	.1596E-01	149.0
181	.1596E-01	151.5
182	.1597E-01	154.0
183	.1598E-01	156.6
184	.1599E-01	159.1
185	.1600E-01	161.6
186	.1600E-01	164.1
187	.1601E-01	166.7
188	.1602E-01	169.2
189	.1602E-01	171.7
190	.1603E-01	174.2
191	.1604E-01	176.8
192	.1604E-01	179.3
193	.1605E-01	181.8
194	.1606E-01	184.3
195	.1606E-01	186.9
196	.1607E-01	189.4
197	.1608E-01	191.9
198	.1608E-01	194.4
199	.1609E-01	197.0
200	.1609E-01	199.5
201	.1610E-01	202.0
202	.1611E-01	204.5
203	.1611E-01	207.1
204	.1612E-01	209.6
205	.1612E-01	212.1
206	.1613E-01	214.6
207	.1613E-01	217.2
208	.1614E-01	219.7
209	.1614E-01	222.2
210	.1615E-01	224.7
211	.1615E-01	227.3
212	.1616E-01	229.8
213	.1616E-01	232.3
214	.1617E-01	234.8
215	.1617E-01	237.4
216	.1618E-01	239.9
217	.1618E-01	242.4
218	.1619E-01	244.9
219	.1619E-01	247.5
220	.1620E-01	250.0