Basic Theory for Three-Dimensional Motion of LIBRA INPORT Tubes

R.L. Engelstad and E.G. Lovell

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FUSION POWER ASSOCIATES

2 Professional Drive, Suite 248
Gaithersburg, Maryland 20879
(301) 258-0545

1500 Engineering Drive
Madison, Wisconsin 53706
(608) 263-2308

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R.L. Engelstad

E.G. Lovell

Fusion Power Associates
6515 Grand Teton Plaza
Room 245
Madison, Wisconsin 53719

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I. INTRODUCTION

The proposed first wall protection scheme for the light ion beam reactor LIBRA (Fig. 1) consists of an annular tube bank encircling the cavity of the reaction chamber. Individual vertical tubes, identified as INPORTs, are made of silicon carbide fiber, braided to produce a porous component. Liquid lithium/lead, used as a coolant and breeder, flows axially within the INPORT and also through the tube wall to develop a thin protective outer film as indicated in Fig. 2. The tubes are elastically supported at both the top and the bottom as shown by the preliminary design of Fig. 3. This would permit relatively convenient assembly and would allow tensile preloading of the INPORTs by means of the compression spring system. In addition, a modification of this support mechanism could be used which allows end rotation, essentially as a ball-and-socket joint.

The mechanical shock from the fireball is transmitted through the cavity gas and produces a repetitive transverse pressure on the first two rows of INPORTs. Key design considerations for the LIBRA cavity depend upon the mechanical response of the tubes under this dynamic loading. Previous analytical calculations have been done for planar motion.[1,2] However, out-of-plane motion may be triggered by imperfections which would naturally occur in the system. In the work which follows, the complete equations of motion are derived using Hamilton's principle and variational calculus procedures.

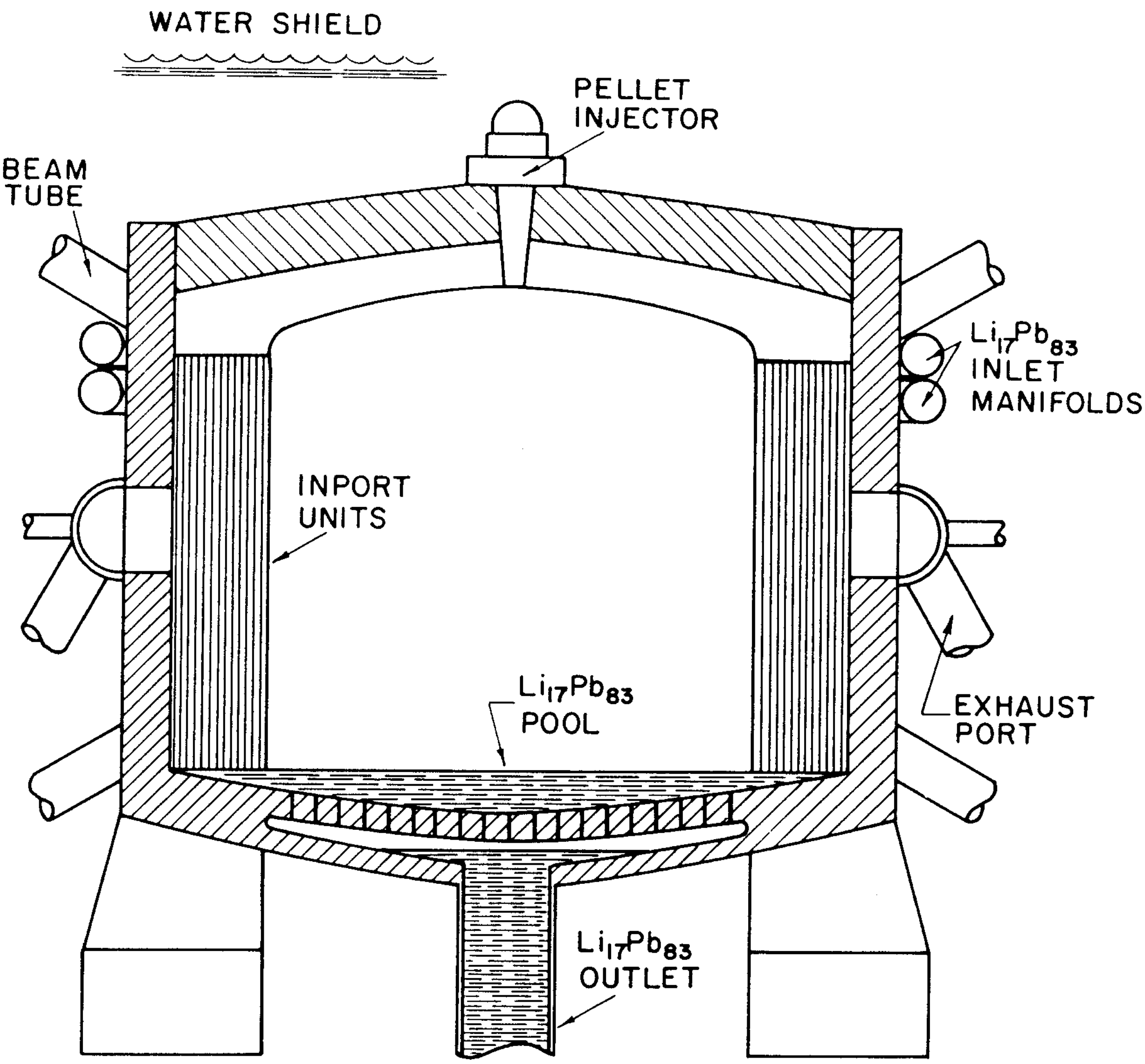

Figure 1. Schematic of LIBRA Reactor Chamber.

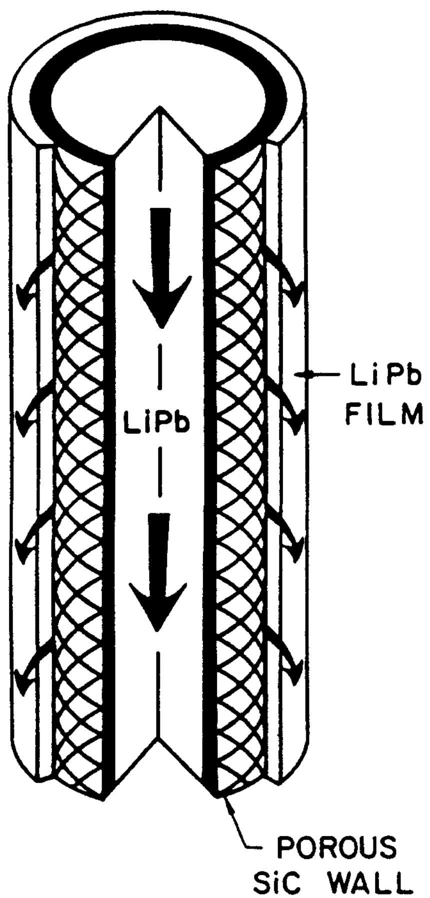

Figure 2. Sectioned INPORT Unit.

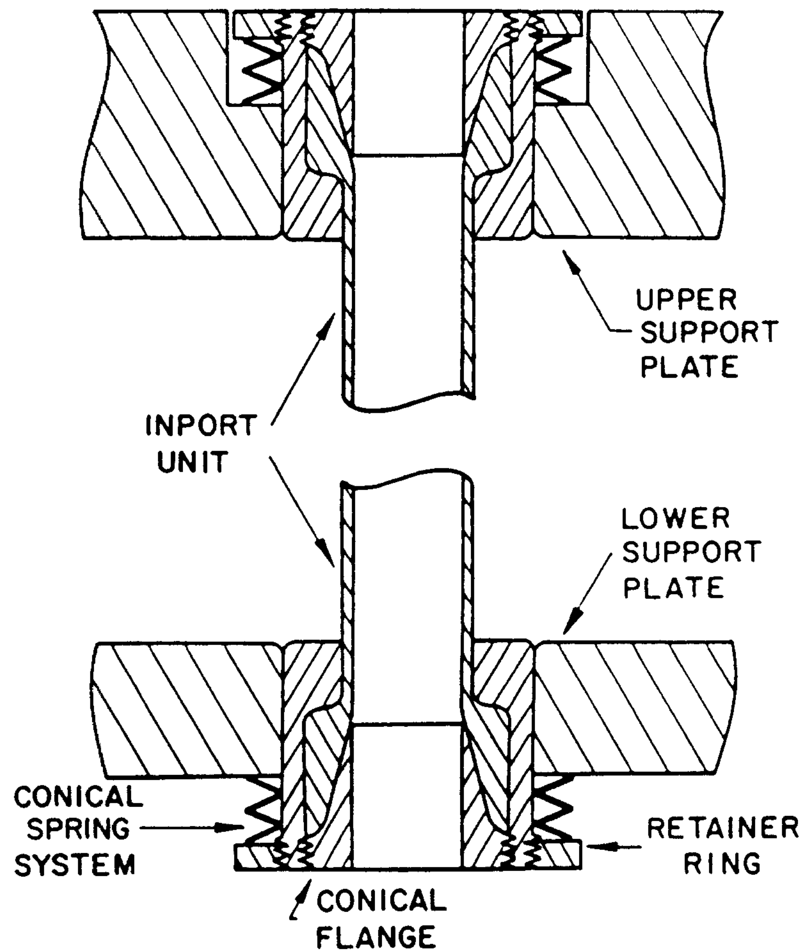

Figure 3. Support Mechanisms for INPORTs.

II. NOMENCLATURE

A_f - cross-sectional flow area

A_t - cross section of tube

c - flow velocity

c_o - mean velocity

E - elastic modulus of the tube

F - forcing function

g - gravitational constant

I - moment of inertia of tube section

K - kinetic energy

ℓ - tube length

m_f - mass/length of fluid

m_t - mass/length of tube

p - internal mean pressure

t - time

T - absolute tension

T_o - static pretension

u - x displacement

U - potential energy

v - y displacement

w - z displacement

W_{nc} - work done by nonconservative forces

x - axial coordinate

y - transverse coordinate

z - transverse coordinate

κ_o - damping coefficient

μ - amplification factor on fluid forcing frequency

ν - Poisson's ratio for tube

Ω - fluid forcing frequency

III. MODEL DESCRIPTION

The system under consideration (Fig. 4) consists of a uniform tube of length ℓ supported at each end. It has a cross-sectional area A_t, mass per unit length m_t and flexural rigidity EI. The internal fluid flows axially with velocity c, cross-sectional flow area A_f and mass per unit length m_f. The mean pressure within the tube is p, measured above atmospheric.

In its undeformed (equilibrium) position the longitudinal axis of the tube coincides with the x axis. With this vertical configuration, gravity effects will be assessed. Free and forced response of the tube is allowed in both the x-y and x-z planes along with longitudinal deformations.

In the problem formulation, various assumptions have been made concerning both the tube and the fluid. They include:

1. The effects of rotary inertia and shear deformation of the tube are neglected.
2. Nominal dimensions of the tube do not change significantly with internal pressure or displacements.
3. External drag forces are neglected.
4. The fluid is viscous and incompressible.
5. Secondary flow effects and radial variations in the flow velocity are neglected.
6. Only the mechanical response of the tube is considered; thermal effects are not assessed.

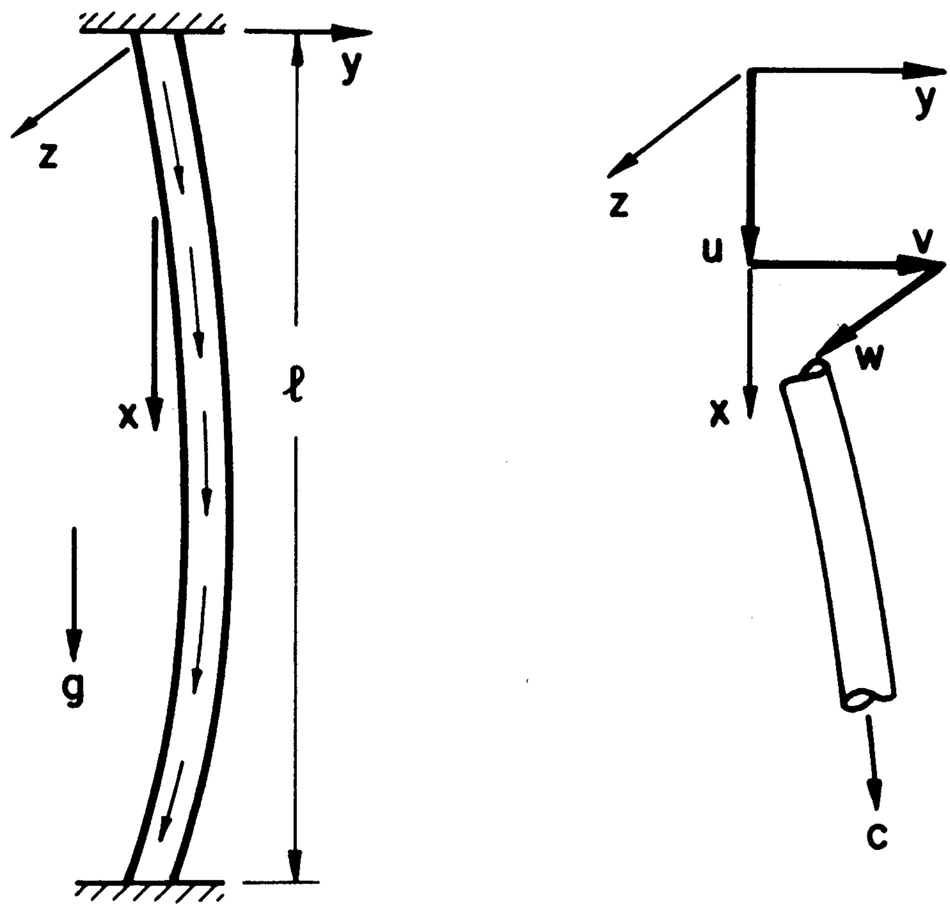

Figure 4. Tube Geometry and Coordinate System.

IV. DERIVATION OF THE EQUATIONS OF MOTION

An energy approach has been used to derive the equations of motion for the system. Hamilton's principle can be expressed in the form

$$\delta \int_{t_1}^{t_2} (K - U)\, dt + \int_{t_1}^{t_2} \delta W_{nc}\, dt = 0 \tag{1}$$

where K and U represent the kinetic and potential energies, and W_{nc} accounts for the work done by nonconservative forces.

The kinetic energy associated with the motion of the tube is given by

$$K_{tube} = \frac{1}{2} m_t \int_0^{\ell} \left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2 \right] dx \tag{2}$$

where u, v and w are the displacement components in the x, y and z directions, respectively.

For the fluid, the magnitude of the flow velocity c may have a harmonic component to include the possibility of pulsating flow, i.e.,

$$c = c_o(1 + \mu \cos \Omega t) \tag{3}$$

where c_o is the mean velocity, μ is the amplification factor and Ω is the forcing frequency. The velocity components of the fluid flow can be described using Fig. 4, which shows the direction of c tangent to the deformed tube centerline. Thus, the kinetic energy due to the flowing fluid can be expressed as

$$K_{fluid} = \frac{1}{2} m_f \int_0^{\ell} \left[\left(\frac{\partial u}{\partial t} + c\right)^2 + \left(\frac{\partial v}{\partial t} + c \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial t} + c \frac{\partial w}{\partial x}\right)^2 \right] dx \; . \tag{4}$$

It should be noted that nonlinear inertia effects from the fluid have not been included.

The potential energy of the system consists of the elastic energy stored in tension and the elastic strain energy due to bending. The contribution from the axial load is

$$U_{tension} = \frac{1}{2A_tE} \int_0^{\ell} T^2 \, dx \tag{5}$$

where T is the absolute tension in the tube. In general, this tension is comprised of a number of components including a static pretensile load T_o and the weight of the tube and fluid which results in a linear axial variation. Also, since the lower end of the tube is not free and the fluid is allowed to discharge into a pool, there will be an additional tensile term equal to $pA_t(2\nu - 1)$ for a thin tube. In order to take into account the possibility of large amplitude motion, nonlinear tension effects can be included by considering higher order terms in the expression for the tube extension. Thus, the tension can be written as

$$T = T_o - pA_t(1 - 2\nu) + (m_f + m_t)g(\ell - x) + EA_t \left(\frac{ds - dx}{dx}\right) \tag{6}$$

where

$$\frac{ds}{dx} = [(1 + \frac{\partial u}{\partial x})^2 + (\frac{\partial v}{\partial x})^2 + (\frac{\partial w}{\partial x})^2]^{1/2} \tag{7}$$

$$\cong 1 + \frac{\partial u}{\partial x} + \frac{1}{2}(\frac{\partial v}{\partial x})^2 + \frac{1}{2}(\frac{\partial w}{\partial x})^2 - \frac{1}{2}\frac{\partial u}{\partial x}(\frac{\partial v}{\partial x})^2 - \frac{1}{2}(\frac{\partial u}{\partial x})(\frac{\partial w}{\partial x})^2 + \frac{1}{2}(\frac{\partial u}{\partial x})^2(\frac{\partial v}{\partial x})^2$$

$$+ \frac{1}{2}(\frac{\partial u}{\partial x})^2(\frac{\partial w}{\partial x})^2 - \frac{1}{8}(\frac{\partial v}{\partial x})^4 - \frac{1}{4}(\frac{\partial v}{\partial x})^2(\frac{\partial w}{\partial x})^2 - \frac{1}{8}(\frac{\partial w}{\partial x})^4 + \ldots .$$

The strain energy due to bending is given by

$$U_{bending} = \frac{1}{2EI}\int_0^{\ell} [(\frac{EI\,\frac{\partial^2 v}{\partial x^2}}{[1 + (\frac{\partial v}{\partial x})^2]^{3/2}})^2 + (\frac{EI\,\frac{\partial^2 w}{\partial x^2}}{[1 + (\frac{\partial w}{\partial x})^2]^{3/2}})^2]\, dx . \tag{8}$$

Using a binomial expansion, Eq. (8) becomes

$$\begin{aligned} U_{bending} = \frac{1}{2}\int_0^{\ell} \{ & EI\,(\frac{\partial^2 v}{\partial x^2})^2[1 - 3(\frac{\partial v}{\partial x})^2 + 6(\frac{\partial v}{\partial x})^4 - \ldots] \\ & + EI\,(\frac{\partial^2 w}{\partial x^2})^2[1 - 3(\frac{\partial w}{\partial x})^2 + 6(\frac{\partial w}{\partial x})^4 - \ldots]\}\, dx . \end{aligned} \tag{9}$$

The work done by nonconservative damping forces can be expressed as

$$\int_{t_1}^{t_2} \delta W_{nc}\, dt = \int_{t_1}^{t_2}\int_0^{\ell} \kappa_o m_t [\frac{\partial u}{\partial t}\,\delta u\, dx + \frac{\partial v}{\partial t}\,\delta v\, dx + \frac{\partial w}{\partial t}\,\delta w\, dx]\, dt \tag{10}$$

where κ_o is considered to be an equivalent damping coefficient that includes both internal structural damping and viscous damping due to the friction of the tube with the surrounding medium. It should be noted that κ_o can be adjusted to comply with the conditions of the problem.

Finally, substituting into Eq. (1) and employing variational calculus, the three-dimensional equations of motion are

$$(m_f + m_t)\frac{\partial^2 u}{\partial t^2} + m_f \frac{\partial c}{\partial t} + \kappa_o m_t \frac{\partial u}{\partial t} - \frac{T^*(m_f + m_t)g}{A_t E} + (m_f + m_t)g\Big[1 - \frac{\partial u}{\partial x}$$
$$- \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial w}{\partial x}\right)^2\Big] - EA_t \frac{\partial^2 u}{\partial x^2} \tag{11}$$
$$- \frac{\partial}{\partial x}\Big\{(EA_t - T^*)\Big[\frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial x}\right)^2 - \left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial w}{\partial x}\right)^2\Big]\Big\} = 0$$

$$(m_f + m_t)\frac{\partial^2 v}{\partial t^2} + 2m_f c \frac{\partial^2 v}{\partial x \partial t} + m_f c^2 \frac{\partial^2 v}{\partial x^2} + m_f \frac{\partial c}{\partial t}\frac{\partial v}{\partial x} + \kappa_o m_t \frac{\partial v}{\partial t} - \frac{\partial}{\partial x}\Big\{T^* \frac{\partial v}{\partial x}$$
$$+ (EA_t - T^*)\Big[\frac{\partial u}{\partial x} - \left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\Big]\frac{\partial v}{\partial x}\Big\} + EI\frac{\partial^4 v}{\partial x^4} \tag{12}$$
$$- 3EI\left(\frac{\partial v}{\partial x}\right)^2 \frac{\partial^4 v}{\partial x^4} - 12EI\left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial^2 v}{\partial x^2}\right)\left(\frac{\partial^3 v}{\partial x^3}\right) - 3EI\left(\frac{\partial^2 v}{\partial x^2}\right)^3 = 0$$

$$(m_f + m_t)\frac{\partial^2 w}{\partial t^2} + 2m_f c \frac{\partial^2 w}{\partial x \partial t} + m_f c^2 \frac{\partial^2 w}{\partial x^2} + m_f \frac{\partial c}{\partial t}\frac{\partial w}{\partial x} + \kappa_o m_t \frac{\partial w}{\partial t} - \frac{\partial}{\partial x}\Big\{T^* \frac{\partial w}{\partial x}$$
$$+ (EA_t - T^*)\Big[\frac{\partial u}{\partial x} - \left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\Big]\frac{\partial w}{\partial x}\Big\} + EI\frac{\partial^4 w}{\partial x^4} \tag{13}$$
$$- 3EI\left(\frac{\partial w}{\partial x}\right)^2 \frac{\partial^4 w}{\partial x^4} - 12EI\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^3 w}{\partial x^3}\right) - 3EI\left(\frac{\partial^2 w}{\partial x^2}\right)^3 = 0$$

where $$T^* = T_o - pA_f(1 - 2\nu) + (m_f + m_t)g(\ell - x)\ . \tag{14}$$

These equations are presented in general form with the order of the nonlinear terms high enough to cover a wider range of potential problems. When different categories of problems are analyzed, simplifications will reduce the complexity of the equations, e.g., negligible flexural stiffness, axial displacements which are much smaller than lateral and transverse components, etc. Such reductions will be developed for particular forced response cases.

V. FORCING FUNCTION

The primary external loading on the INPORTs is the mechanical shock from the fireball that is transmitted through the cavity gas.[3] This occurs with each ignition, typically between 1 and 5 Hz. Figure 5 shows the first two rows of the tube bank with the dynamic radial pressure applied to one side of an INPORT. This loading will have both a time and spatial variation in the axial direction, i.e., $F = F(x,y,t)$.

Although the forcing function, in this case, is considered to be strictly planar, a nonplanar response of the INPORTs is expected. Such "whirling" motion has been observed in both strings and beams subjected to planar excitations.[4,5] For the proposed tube bank of LIBRA, further analysis will be necessary to determine

1. the conditions for which out-of-plane motion will or will not develop;
2. the magnitude of out-of-plane displacement; and
3. possible control or reduction of the displacement amplitudes.

VI. CONCLUSIONS

The nonplanar equations of motion for free and forced vibrations of the INPORTs have been determined. Nonlinear curvature and tension terms are in-

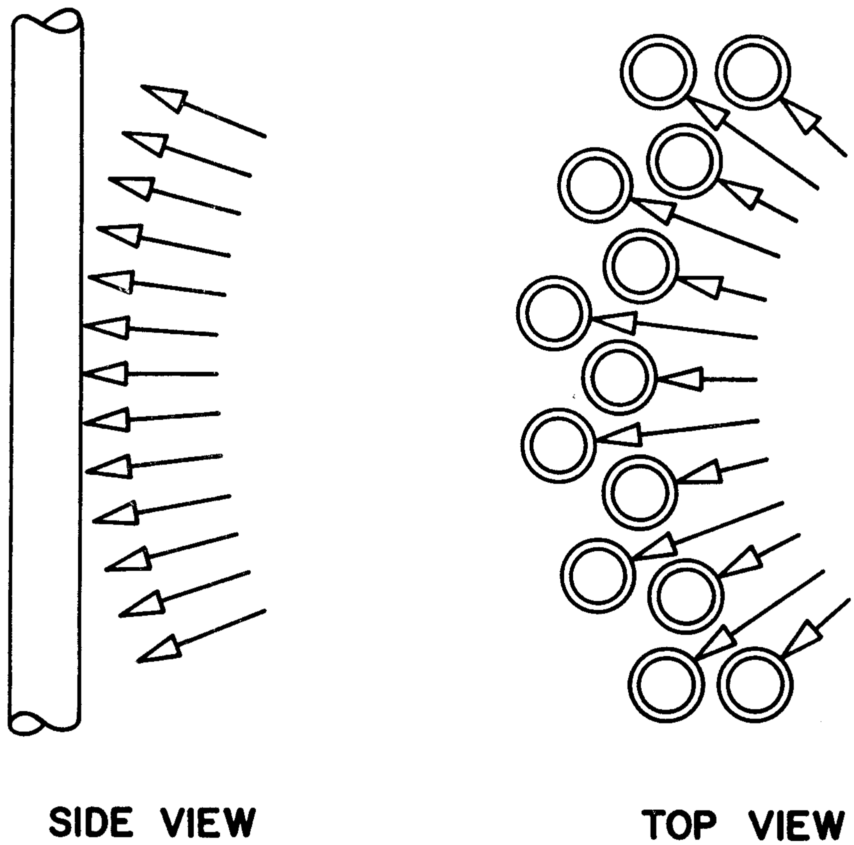

Figure 5. Dynamic Pressure Loading on the INPORTs.

cluded in the derivation to identify the coupling of longitudinal and transverse modes. A planar forcing function is considered that could cause instabilities in the system due to either large amplitude or out-of-plane motion.

General closed form solutions of such highly nonlinear coupled equations are not possible. However, reductions in complexity will be used for particular problems when forced response calculations are made.

VII. REFERENCES

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