MHD Stability Analysis for TASKA

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I. Introduction

Tandem mirrors necessarily employ magnetic field configurations with regions of bad field line curvature which can lead to MHD instabilities. Anchor magnets with good curvature are used to balance the bad curvature of other regions so that the plasma is globally MHD stable. The global MHD stability may be analyzed by taking pressure divided by magnetic field weighted integrals along field lines of the field line curvature. Since the machine outside the anchors has overall bad curvature, the requirement for MHD stability puts an upper limit on the β in the central cell which is particularly sensitive to the anchor cell design.

The tandem mirror engineering test facility study, TASKA,[1] was published in June of 1982 without MHD stability analysis. The central cell β was limited to 50% by assumption based on the experiences of other tandem mirrors. Since the publication of TASKA, the author has performed analysis of the MHD stability for TASKA, which is reported in this paper.

In Section II, the magnetic field geometry for TASKA will be discussed as it pertains to MHD stability. The model for the pressure profile in TASKA will be presented in Section III. The method used to determine the requirements for MHD stability is given in Section IV and results for TASKA are given in Section V. Section VI contains a brief discussion of these results.

II. Magnetic Field Geometry

The geometry of the magnetic fields is, of course, of great importance to the MHD stability of a tandem mirror. Regions of good magnetic field line curvature must be designed into the machine to balance the unavoidable bad curvature in the central cell and transition region. This is done with good curvature yin-yang magnets which form anchor regions.

The magnet design for TASKA is shown in Fig. 1. The system consists of 3 solenoids in the central cell, 2 sets of solenoids which provide the mirror field between the central cell and the thermal barrier, 2 transition coils and 2 recircularizing coils for shaping the plasma, 2 yin-yang systems and 8 coils for field shaping in the thermal barrier region. The magnetic field for this set of conductors, which was designed by W. Maurer, has been calculated by H. Attaya with the EFFI[2] computer code and has been discussed in the TASKA report.[1] The field along a representative field line[†] is shown in Fig. 2. Notice that there is rippling in the central cell. This leads to a rather complex structure in the curvature of the field line.

The curvature of this magnetic field line is shown in Fig. 3. On this graph, positive values mean good curvature and negative values represent bad curvature. There is clearly much bad curvature in the transition region and this is where the greatest contribution to global bad curvature occurs. There is some good curvature also in the transition region but not enough to balance the bad. There is also both good and bad curvature in the central cell and, compared to the transition region, the net bad curvature here is small. There is mainly good curvature in the anchor cell and, though the curvature is small compared to the curvatures in the transition region, the expected high plasma pressure in this region may make this good curvature stabilizing for the whole plasma. Past the anchor cell, the magnitude of the curvature is very large

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[†]The line that goes through the point $x = 5$, $y = 5$ in the center of the machine was chosen. This field line is 45° out of the plane of both yin-yang systems, which is thought to be the most unstable field line.

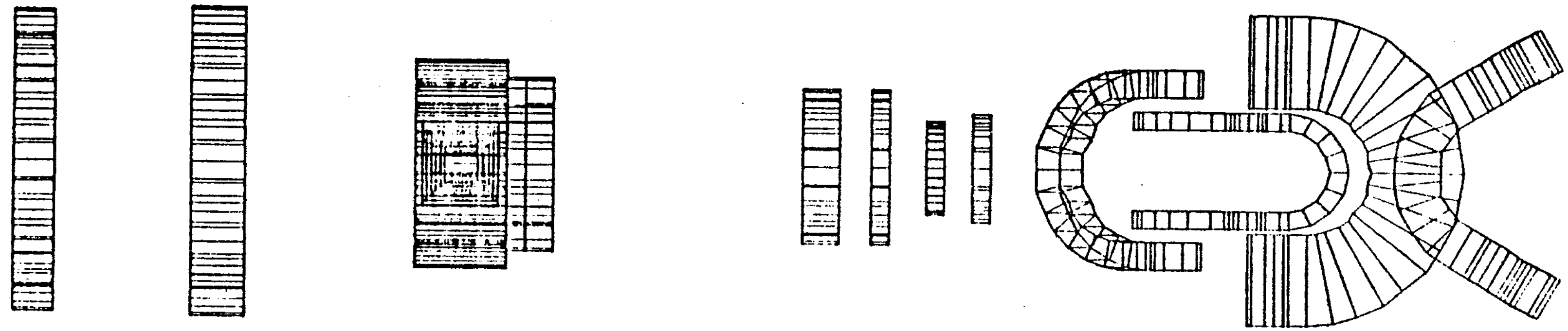

Fig. 1. Magnet Design for TASKA. Only right half of the machine is shown.

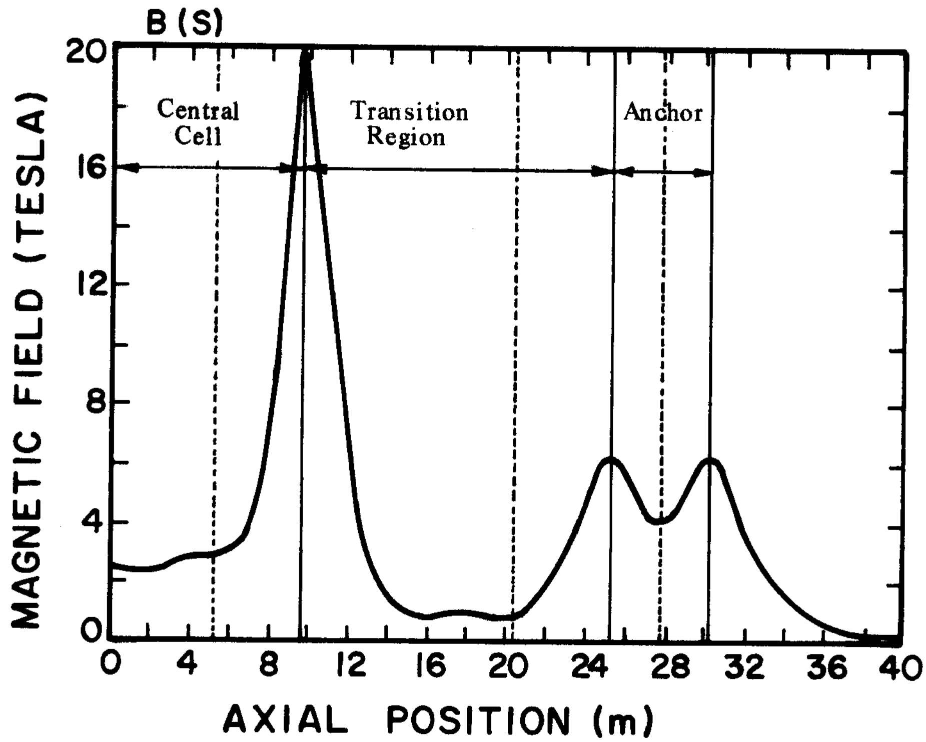

Fig. 2. Magnetic Field in TASKA. The field along a field line that passes through a point 7 cm off of the machine's axis at the center of the machine and lies in a plane tilted 45° from the axis of both sets of yin-yangs is plotted against axial position.

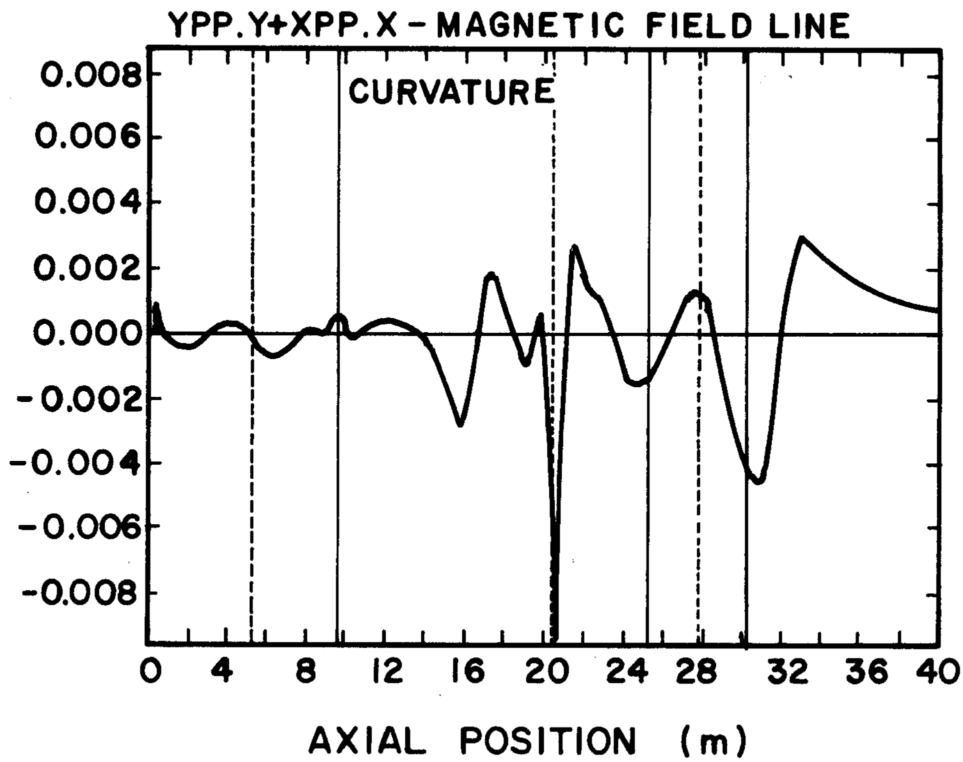

Fig. 3. Curvature of the Magnetic Field Line Described in Fig. 2.

but the plasma pressure is essentially zero in this region so the curvature here has no effect.

III. Pressure Profile in TASKA

Because the magnetic field line curvature oscillates from good to bad very rapidly and because it is the curvature times the plasma pressure that is important to MHD stability, it is important to have a realistic model for the pressure profile in TASKA. The model presented here is based on the conservation of energy for individual ions and includes the effect of the particles working against the barrier potential in TASKA and acceleration of ions down magnetic fields. Electron and ion pressures are calculated separately and are added together in the end.

The electron pressure is simply expressed as a temperature times a density. The electron density is equal to the ion density because a neutral plasma is assumed. The electron temperature is taken as a constant in the central cell and as a different constant outside the central cell. Thus, the electron pressure is,

$$\begin{aligned} P_e &= n_c k_B T_{ec} && \text{; in the central cell} \\ &= n(z) k_B T_{ep} && \text{; outside the central cell ,} \end{aligned} \tag{1}$$

where n_c is the density in the central cell, $n(z)$ is the density outside the central cell as a function of the axial position z, T_{ec} is the electron temperature inside the central cell and T_{ep} is the electron temperature outside the central cell.

The ion pressure consists of components parallel and perpendicular to the magnetic field lines. The ion pressure used in the MHD calculation is taken as the average of the parallel and perpendicular ion pressures. In the central cell, the ion pressure is assumed to be isotropic so that

$$P_{\perp} + P_{\parallel} = 2n_c k_B T_{ic} \qquad \text{in the central cell ,} \tag{2}$$

where $P_{\perp}$ is the perpendicular pressure, $P_{\parallel}$ is the parallel pressure and T_{ic} is the ion temperature in the central cell.

Outside the central cell, the changing plasma potential and magnetic fields affect the parallel and perpendicular pressures differently. Conservation of energy for individual ions requires that

$$\frac{1}{2} m_i v_{\parallel}^2 = \varepsilon - \mu B(z) - e\phi \ , \tag{3}$$

where m_i is the ion mass and $v_{\parallel}$ is the component of ion velocity parallel to the magnetic field lines, ε is the total energy of the average ion in the central cell,

$$\varepsilon = \frac{3}{2} k_B T_{ic} \ . \tag{4}$$

μ is the magnetic moment of the ion,

$$\mu = \frac{m_i v_{\perp}^2}{2\ B(z)} \tag{5}$$

and is taken to be a constant of motion. $v_{\perp}$ is the component of the ion velocity perpendicular to the field lines. $B(z)$ is the magnetic field along the field line as a function of axial position and ϕ is the electrostatic potential of the plasma. The parallel ion pressure may be written with the help of Eqs. (3), (4) and (5) as

$$P_{\parallel} = n(z)m_i v_{\parallel}^2 = 3\ n(z)k_B T_{ic} - 2\ n(z)k_B T_{ic} \frac{B}{B_c} - 2\ n(z)e\phi\ . \tag{6}$$

The perpendicular pressure may be written as

$$P_{\perp} = \frac{n(z)m_i v_{\perp}^2}{2} = n_c k_B T_{ic} \frac{B(z)}{B_c} \frac{n(z)}{n_c}\ , \tag{7}$$

where we have used the fact that μ is a constant independent of z.

Now that the parallel and perpendicular electron and ion pressures have been determined, an expression may be given for the plasma pressure. From Eqs. (1), (2) and (7), the total plasma pressure is

$$\begin{aligned} P(z) &= \frac{1}{2} k_B T_{ic} n(z)\left(3 - \frac{B(z)}{B_c}\right) - n(z)e\phi(z) \\ &+ \begin{pmatrix} n_c k_B T_{ec} \\ n(z)k_B T_{ep} \end{pmatrix} \begin{matrix} \text{in the central cell} \\ \text{outside the central cell} \end{matrix}\ . \end{aligned} \tag{8}$$

The determination of the plasma potential $\phi(z)$ and the plasma density n(z) as functions of z still remains. Both quantities are rather difficult to calculate and they depend on the details of the plasma kinetics and of the machine design. There have been calculations made of the plasma density and the plasma potential which are reported on in the TASKA[1] report. These are shown in Fig. 4. These are for the TASKA base case but to investigate the MHD stability, the plasma conditions must be varied. Phenomenological expressions for n(z) and $\phi(z)$ have been fit to the results of Fig. 4 so that they agree for the base case. The pressure in the anchor needed for stability is found

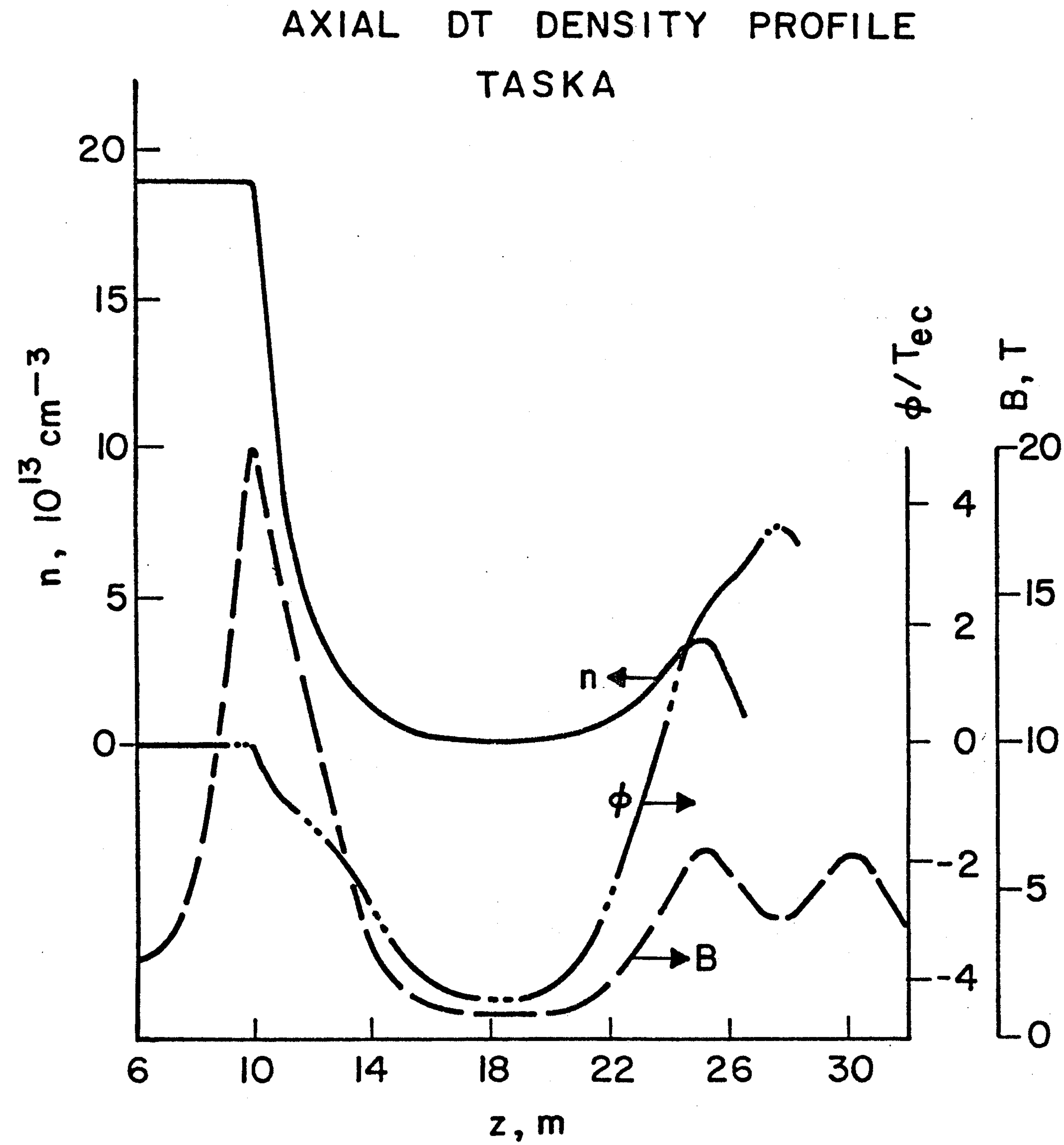

Fig. 4. Plasma Potential, Magnetic Field and Plasma Density Profiles for TASKA.

in the calculations as a function of the density in the transition region. This density and pressure correspond to the marginally stable case and not the base case for TASKA. The density profile for such a marginally stable case is shown in Fig. 5. The plasma potential for this case is shown in Fig. 6 and the total pressure profile is shown in Fig. 7.

IV. Method of Solution

To determine the requirements for MHD stability, the global average of the magnetic field line curvature times the plasma pressure divided by the magnetic field intensity must be found. This global average is expressed as

$$I = \int_0^{s_{max}} \frac{P(s)}{B(s)} (x''x + y''y)\, ds \,, \tag{9}$$

where s is the distance along the field line, which is assumed here to be just the axial position, and s_{max} is the maximum distance, beyond which the plasma pressure vanishes. x and y are the cartesian positions of the field line in the plane normal to the axis of the machine and are functions of s. x'' and y'' are second derivatives with respect to s of x and y. The expression $x''x + y''y$ represents the field line curvature. Since good curvature has a value greater than zero, a positive value for I corresponds to MHD stability.

The stability of TASKA has been investigated with a modified version of the STAB16[3] code, which was developed at Lawrence Livermore National Lab. The original form of this code only may be used when there are 3 peaks in the magnetic field in each half of the machine and for a specific class of pressure profiles. In TASKA, there are 6 local maxima in the magnetic field and a different pressure profile. STAB16 has been modified so that it can handle the

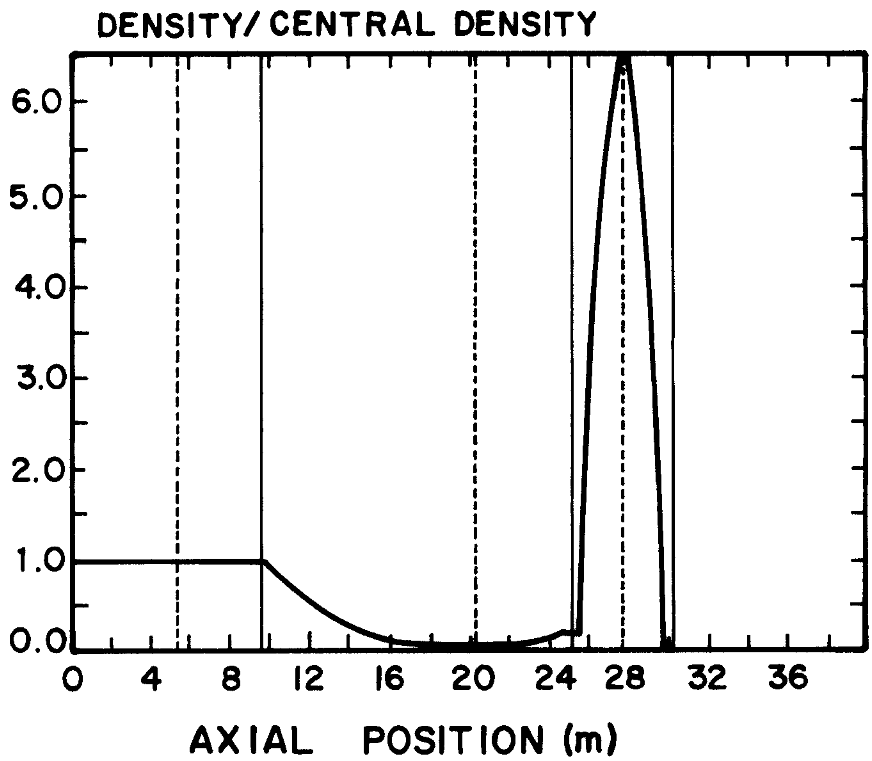

Fig. 5. Plasma Density Profile for Marginally Stable TASKA Case.

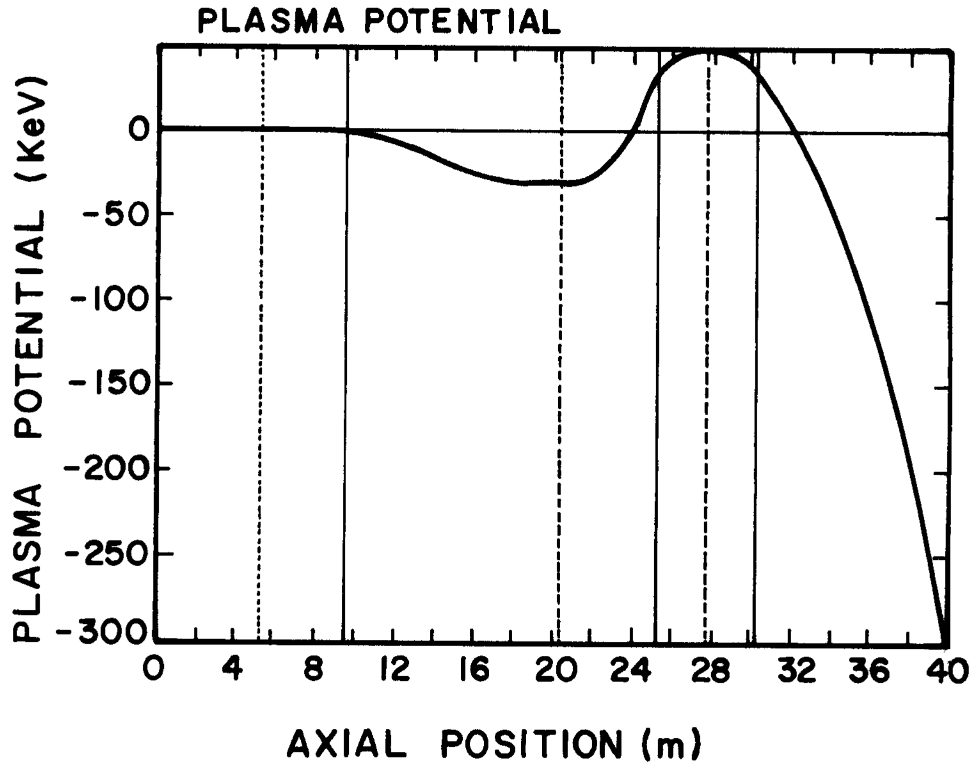

Fig. 6. Plasma Potential Profile for Marginally Stable TASKA Case.

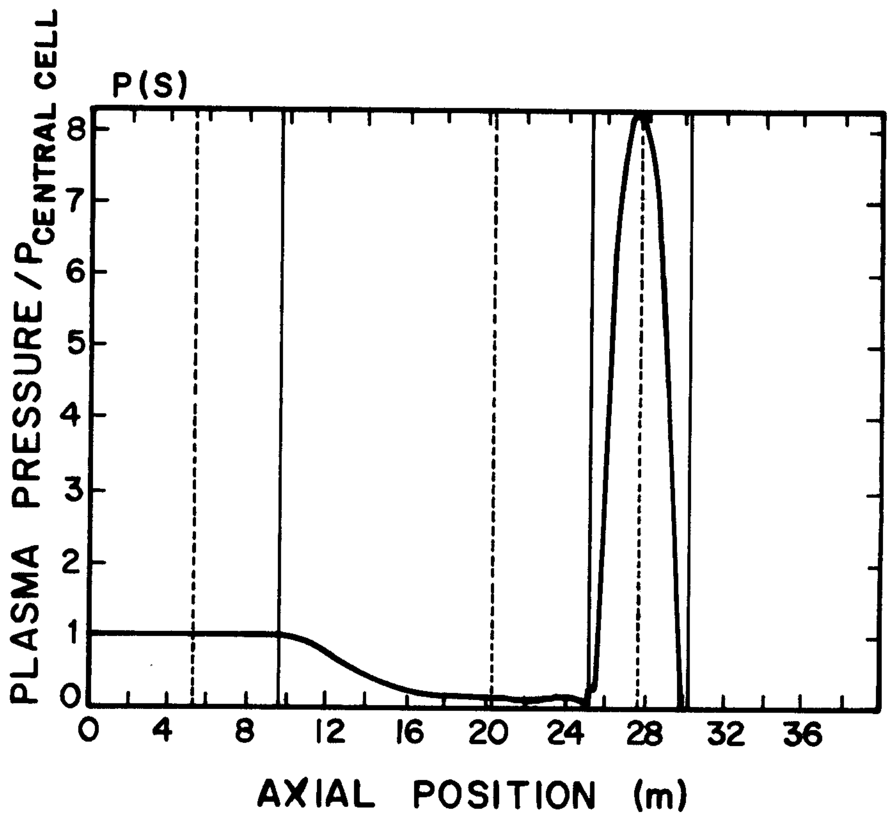

Fig. 7. Total Pressure Profile for Marginally Stable TASKA Case.

additional ripples in the magnetic field and so that it includes the pressure profile described earlier in this article.

STAB16 searches in parameter space for a point where the machine in question is marginally MHD stable. The original and modified versions both find that pressure in the anchor which is needed to make the machine stable for a given pressure profile in the rest of the machine. Other variables affecting the pressure profile are varied and the results are expressed in plots of the marginally stable anchor plasma pressure or β versus these variables. The results presented here will have β_{cc}/β_p plotted against β_T/β_{cc} where β_{cc} is the β in the center of the machine, β_p is the β in the anchor (or plug) and β_T is the minimum β in the transition region. The original version of STAB16 asks the user to provide β_T/β_{cc} while the modified version allows the user to choose the more complicated plasma pressure profile of Section III and to vary it by changing the ratio of the minimum plasma density in the transition region to the plasma density in the center of the machine, with the code then providing the user with β_T/β_{cc}. In both versions, the width of the high pressure region in the anchor may be specified and varied. The codes both have the anchor pressure term expressed as a quadratic with the pressure dropping to the transition region value at the points where the magnetic field reaches some fraction of the local maxima at each side of the anchor. This field is called the cut-off field and the ratio of the cut-off to the local maximum is specified by the user.

V. Results for TASKA

Several versions of STAB16 have been used to investigate the MHD stability of TASKA. Results from two versions will be presented here. By comparing the results from using the two versions, statements can be made concerning the importance to the MHD stability of the plasma pressure in certain parts of the transition region.

The marginally stable value for β_{cc}/β_p is plotted against β_T/β_{cc} in Fig. 8 for an early version of STAB16 which has the general pressure profile shown in the inset of Fig. 8. Values are shown for $B_{cut\text{-}off} = 0.75\ B_{max}$ and $B_{cut\text{-}off} = 0.95\ B_{max}$. $B_{cut\text{-}off}$ is explained in Section IV. A high $B_{cut\text{-}off}$ means that the pressure peak in the anchor is wide while a low $B_{cut\text{-}off}$ corresponds to a narrow pressure peak. From Fig. 3 one can see that the middle of the anchor has good curvature and the edges have bad curvature. Thus, a narrow peak in the pressure in the center of the anchor is better for MHD stability than a wide one. Therefore, a high $B_{cut\text{-}off}$ is worse for MHD stability than a low value. This is why the $B_{cut\text{-}off} = 0.95\ B_{max}$ curve in Fig. 8 requires a lower β_{cc}/β_p than does the $B_{cut\text{-}off} = 0.75\ B_{max}$ curve. All curves for this case allow higher β_{cc}/β_p than TASKA has been designed to.

Results coming from the most recent version of STAB16 are shown in Fig. 9. The pressure profile for this version is that which was described in Section III. An example of such a pressure profile is shown in Fig. 7. The main difference between this profile and the one shown in Fig. 8 is that the pressure does not fall as sharply as one moves into the transition region from the central cell. Figure 3 shows that the curvature in this part of the machine is mostly bad so that the β_{cc}/β_p required for this case to be MHD stable is significantly lower. In Fig. 9, the $B_{cut\text{-}off} = 0.98\ B_{max}$ curve lies below the TASKA design point, which means that if the pressure is as described in Section III and if the pressure peak almost fills the anchor then TASKA is MHD unstable. However, the $B_{cut\text{-}off} = 0.8\ B_{max}$ curve is well above the TASKA point so that by making the pressure peak in the anchor relatively narrow, the

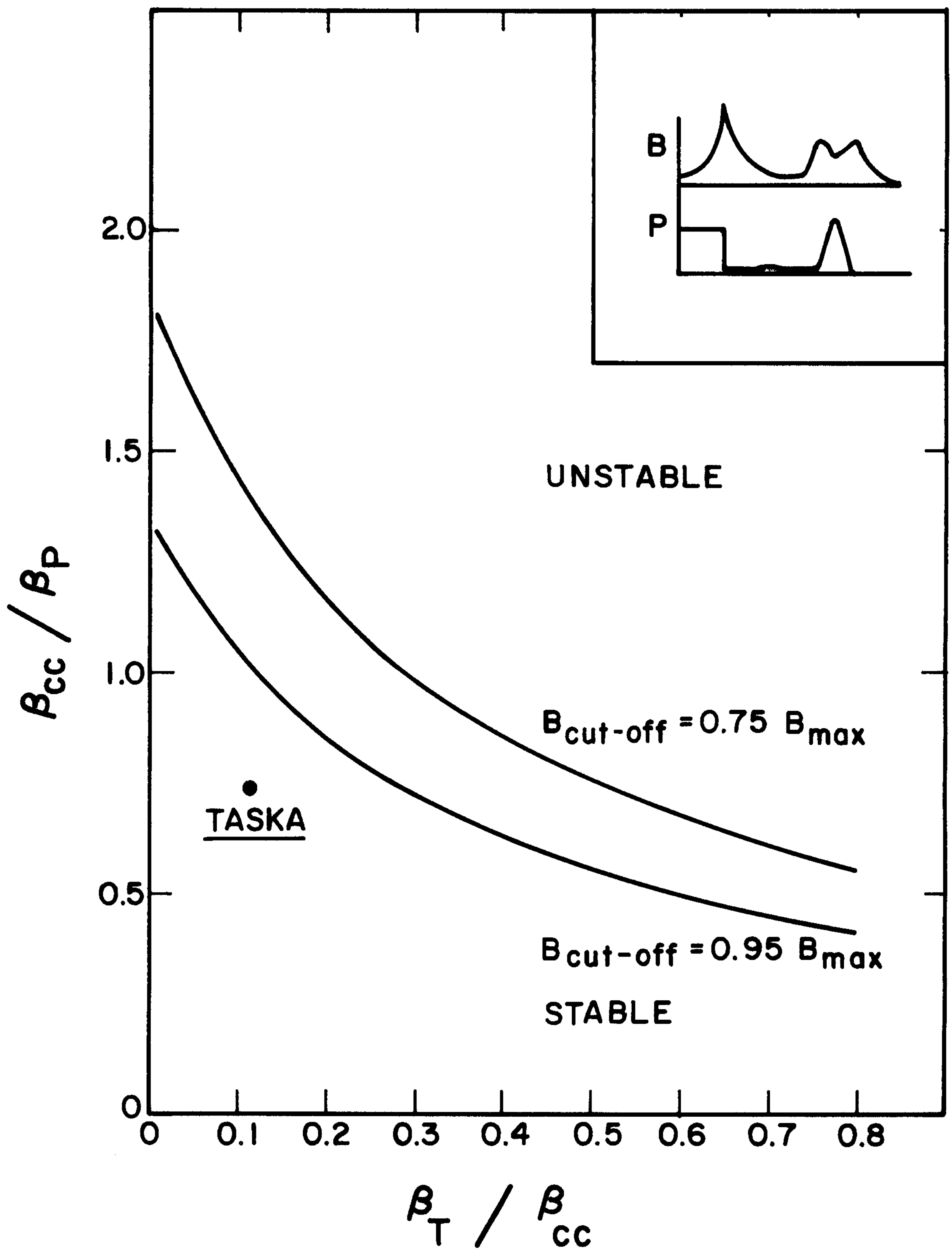

Fig. 8. Stability Diagram for TASKA with Pressure Profile Shown in Inset.

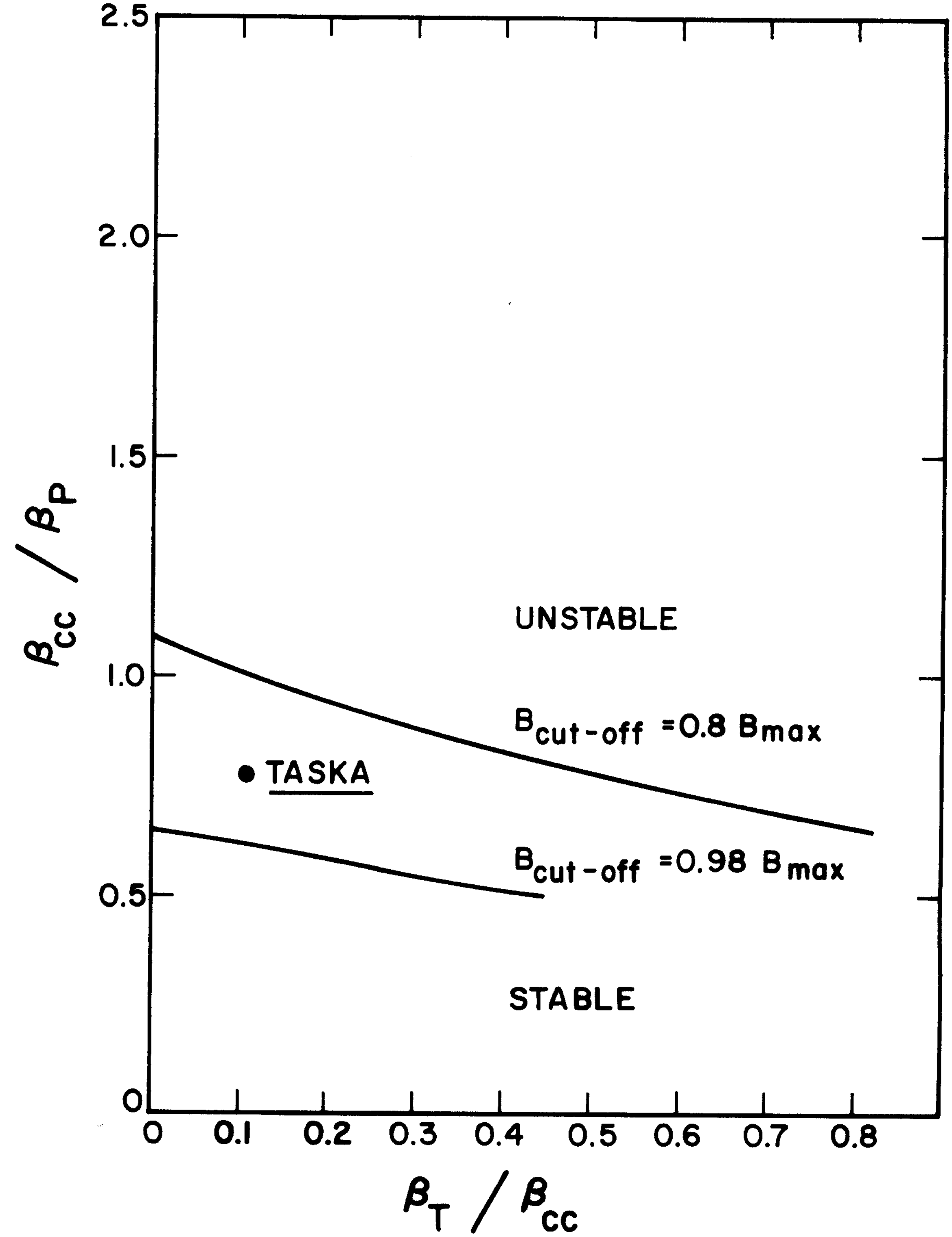

Fig. 9. Stability Diagram for TASKA with Pressure Profile Shown in Fig. 7.

bad curvature in the transition is overcome in TASKA even for this pressure profile.

VI. Discussion

The MHD stability of TASKA has been addressed in this article. Modified versions of the STAB16 code have been used to determine when TASKA is stable to the interchange mode. It has been found that the conditions for stability are sensitive to the plasma pressure profile but that MHD stability should be achievable in TASKA.

There are shortcomings in the STAB16 analysis that should be mentioned. STAB16 only addresses the problems of the interchange mode. Ballooning modes have not been addressed but recent work[4] shows that these may be stabilized. Finite β effects have also been ignored in STAB16 which may have a stabilizing effect.

Acknowledgment

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