Neutron Wall Current and Energy Considerations for Alternate Fusion Fuel Cycles and Reactor Approaches

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ALTERNATE FUSION FUEL CYCLES AND REACTOR APPROACHES

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The neutron current to the first wall of various approaches to fusion may be quite different and as a result some reactor choices may be more desirable than others. Let us explore some of these parameters for the three fuel cycles and then look at the four major containment approaches to fusion: low $\beta$ toroidal systems, high $\beta$ toroids, mirrors, and laser fusion.

In looking at the various fuel cycles we will use the standard notation for D, T and He$^3$ as D, T, and 3 respectively. Taking on D-D systems first, we note that for most temperatures the two cross sections $<\sigma v>_{DDp}$ and $<\sigma v>_{DDn}$ are about the same so we take the combined cross section $<\sigma v>_{DD}$ as the sum of the two. Since each D-D reaction produces either a T or a 3 we know the production rate per unit volume-second for either specie is just 1/2 the reaction rate. That is, the production rate of T or 3 is given by

$$R_T = R_3 = \frac{1}{2} \frac{n_D^2}{2} <\sigma v>_{DD}$$

(1)

where we assume $n_D$ is the deuterium density. If we now worry how fast T is burned in a D-D systems we have

$$B_T = n_D n_T <\sigma v>_{DT}$$

(2)

Writing an equation for the amount of T in the system we have from Eqs. (1) and (2)
\[
\frac{dn_T}{dt} = \frac{n_D^2}{4} \langle \sigma v \rangle_{DD} - n_T n_D \langle \sigma v \rangle_{DT} \tag{3}
\]

In the steady state situation then we find an equilibrium value of \( T \) given by

\[
n_T = \frac{n_D}{4} \frac{\langle \sigma v \rangle_{DD}}{\langle \sigma v \rangle_{DT}} \tag{4}
\]

Using this in our D-D system to determine the number and energy of the neutrons produced we write the reaction rate as

\[
R_{DD} = \frac{n_D^2}{2} \langle \sigma v \rangle_{DD} + \frac{n_D^2}{4} \langle \sigma v \rangle_{DD} = \frac{n_D^2}{2} \langle \sigma v \rangle_{DD} \left[ 1 + \frac{1}{2} \right] \tag{5}
\]

where we have used Eq. (4) to give the second term for D-T reactions. The first term of the equality then represents twice the rate of production of 2.45 MeV neutrons from D-D reactions and the second term is the production rate of 14.1 MeV neutrons. The result is that equal amounts of 2.45 and 14.1 MeV neutrons are produced in steady-state D-D reactors independent of the temperature. This does not mean that the production rate of neutrons is the same as other reactor systems as we will see.

Moving to the D-T reactor where \( n_T = n_D = n/2 \) and to make comparisons easier later, we have \( n = n_D \) of the previous analyses. That is, we will keep the plasma density the same for all systems. The reaction rate for this D-T system is then

\[
R_{DT} = \frac{n^2}{4} \langle \sigma v \rangle_{DT} + \frac{n^2}{8} \langle \sigma v \rangle_{DD} \tag{6}
\]
where the second term represents the D-D reactions and again this is twice the production rate of 2.45 MeV neutrons. For comparison purposes Eq. (6) can be written as

\[ R_{DT} = \frac{n^2}{4} <\sigma v>_{DT} \left[ 1 + \frac{<\sigma v>_{DD}}{2<\sigma v>_{DT}} \right]. \]  \hfill (7)

Similarly the D-3 reaction rate is written for \( n_D = n_3 = n/2 \) as

\[ R_{D3} = \frac{n^2}{4} <\sigma v>_{D3} + \frac{n^2}{8} <\sigma v>_{DD} + \frac{n^2}{16} <\sigma v>_{DD}. \]  \hfill (8)

Here we have the first term representing D-3 reactions with no neutron production, the second term representing the D-D reactions which is twice the 2.45 MeV neutron production and the third term representing the 14.1 MeV neutron production from D-T reactions.

Rewriting Eq. (8) to conform to Eqs. (5) and (7),

\[ R_{D3} = \frac{n^2}{4} <\sigma v>_{D3} \left[ 1 + \frac{1}{2} \frac{<\sigma v>_{DD}}{<\sigma v>_{D3}} + \frac{1}{4} \frac{<\sigma v>_{DD}}{<\sigma v>_{D3}} \right]. \]  \hfill (9)

Table 1 shows the ratio of 2.45 MeV neutron production to total neutron production for the three cycles for various temperatures. Notice that for the D-D and the D-3 reactions, the ratio is fixed since the production of T and consequently 14.1 MeV neutrons is limited by \( <\sigma v>_{DD} \) just as is the 2.45 MeV neutron production.
To compare the total neutron production between cycles, one can look at the reaction rates per unit volume which are given by Eqs. (5), (7) and (9). Assuming the plasma density is the same for all cases, we find that the ratios are essentially comparisons of the various cross sections with the added factor of two for D-D. While this comparison is instructive, perhaps it is more instructive to take both the density and the reactor volume to be the same. This results in a variation in the reactor power and we can find the neutron production per watt-second. Using the Q values in MeV of $Q_{DT} = 17.6$, $Q_{Li} = 4.8$, $Q_{Na} = 12.6$, $Q_{D3} = 18.3$ and $Q_{DD} = 3.65$ (averaged between the two branches), we obtain the data in Table II. Further, a total ion density of $1 \times 10^{14}$ ions/cm$^3$ was used and it was assumed that all neutrons produced in the D-T system are absorbed in Li while neutrons produced in the D-D and D-3 systems are absorbed in Na since there is no tritium breeding requirement in these two cases. Figure 1 shows the 2.45, 14.1 MeV and total neutron production for the 3 systems versus temperature.

Moving to calculate the neutron current as a function of plasma parameters and reactor size we can write the current for a D-T reactor as

$$\phi 4\pi^2 r_w R = \frac{n^2}{4} \langle \sigma v \rangle_{DT} 2\pi^2 r_p^2 R.$$  (10)
Here we are assuming a torus but the length cancels out and we get the same result for a linear device where \( r_p \) is the plasma radius and \( r_w \) is the wall radius. Rewriting Eq. (10) where we assume a linear relationship between the plasma radius and wall radius such that \( r_p = y r_w \) and \( y < 1 \),

\[
\phi = \frac{n^2}{8} \langle \sigma v \rangle_{DT} r_p y \quad \text{neutrons/m}^2\text{-sec.} \quad (11)
\]

If we put this into more conventional units where \( n \) is in particles/cm\(^3\), \( \phi \) is in neutrons/cm\(^2\)-sec, \( r \) is in m, and \( \langle \sigma v \rangle \) is in m\(^3\)/sec, we have

\[
\phi = 1.25 \times 10^7 n^2 r_p y \langle \sigma v \rangle_{DT}.
\]

The neutron current is plotted in Fig. 2 as a function of \( n^2 r_p y \) for various values of the temperature. In this, the work of Golovin et al.\(^1\) is taken as a reference point for \( n^2 \langle \sigma v \rangle \) where this represents an average over the radial temperature distribution. In the referenced work the temperature is allowed to vary in a parabolic manner and it is assumed that \( \langle \sigma v \rangle \) varies linearly with temperature. For actual reactors, both \( n \) and \( T \) may vary with both \( r \) and \( z \) and things become more complex. This is especially true when one considers the \( n^2 \) dependence on density. To give an idea of the magnitude of the variation, at low temperatures in going from a parabolic \( n \) and \( T \) to uniform
n and T we find a factor of about 4 difference. At higher temperatures (∼100 keV) this difference becomes less because \( <qv> \) varies more slowly with T.

In low β tokamaks, stellarators or other possible toroidal systems, we are likely to be operating at densities of 1-3\( \times \)10\(^{14} \) cm\(^{-3} \) and a plasma radius of 1-3 m. Using \( y = 0.8 \), which may be too small for \( r_p > 1 \) m, we find the vertical solid lines of Fig. 2 represent a system operating with densities of 1, 2 and 3\( \times \)10\(^{14} \) cm\(^{-3} \) and a plasma radius of 3 m. The circle where the vertical line crosses the 15 keV curve represents a reactor producing 350A MW(th) where A is the aspect ratio. Since the length of the reactor and the total power scales linearly with A, the power can be scaled but the wall current is fixed. Similarly, because the power varies as \( n^2 \), equivalent vertical intercepts with the \( T = 15 \) keV line correspond to 1400A MW and 3150A MW. This suggests that even for the largest toroidal systems that could be envisioned, the wall current is less than 10\(^{14} \) n/cm\(^2\)-sec.

Doing a similar analysis for high β tori we use the parameters suggested by Ribe\(^2 \) and where \( T_i = 10 \) keV, \( n \approx 2.4 \times 10^{16} \), \( r_p = 10 \times 10^{-2} \) with \( y = 0.5 \) and \( T = 0.025 \) sec. Applying this to our curves of Fig. 2 we have \( n^2 r_p y = 2.9 \times 10^{31} \) but with 20 \( T \) sec between pulses or a duty cycle of 0.05. From extrapolations of Fig. 2 we find the peak neutron current does not lie on the curve
but equals $1.0 \times 10^{16}$ and an average value twenty times less or $10^{14}$ neutrons/cm$^2$-sec which is indicated by a closed circle on the plot.

Mirror reactors tend to operate at lower $n$ but higher $T$ than low $\beta$ toroids. Let us take the values of $\langle \sigma v \rangle$ from Post$^3$ for a reactor with a mirror ratio of 3.3 and apply this to the reactor parameters suggested by Sweetman.$^4$ He uses a plasma temperature of 100 keV with $n = 2 \times 10^{14}$ and a wall radius of 2.0 m. This calculates to give $r_p \gamma = 0.98$ m or $n^2 r_p \gamma = 3.9 \times 10^{28}$ and for $\langle \sigma v \rangle = 7.7 \times 10^{-22}$, which is just about ten times that for operation at 15 keV under Golovin's$^1$ distribution, we are at the starred point of Fig. 2 or $3.6 \times 10^{14}$ n/cm$^2$-sec.

If we try to carry out an equivalent calculation for a laser-pellet system we find that the parameters are not so straightforward. For instance, if the energy released per pulse is $10^7$ joules then we need to pulse $10^2$ times per second for each 1000 MW(th) we are required to produce. The $10^9$ watts requires $2.8 \times 10^{20}$ fusions or $2.8 \times 10^{18}$ neutrons per pulse. These neutrons can presumably be spread over any area but let us arbitrarily make the area sufficiently large that any metallic wall facing the blast is not vaporized. That is, the surface is not brought to the melting point.

To determine the surface area required, we calculate the thermal skin depth for a single pulse which we assume to last.
for the time it takes a hot ion (10 keV) to travel the distance to the wall or \( r_w \). Thus

\[
\delta = \left( \frac{K}{\rho c_p v_{th}} \right)^{1/2} \frac{r_w}{r_w}
\]

and the total mass of material to be raised a temperature \( T_m/2 \) is

\[
\rho 4\pi r_w^2 \delta = 4\pi \left( \frac{K r_w}{c_p v_{th}} \right)^{1/2} \frac{r_w^5}{c_p v_{th}}
\]

The energy per pulse is then

\[
Q = 2\pi \left( \frac{\rho K r_w^5}{c_p v_{th}} \right)^{1/2} c_p T_m
\]

or

\[
r_w = \left[ \frac{Q}{2\pi T_m} \right]^{2/5} \frac{v_{th}}{\rho K c_p}
\]

Using \( 10^7 \) joules or \( 2.4 \times 10^6 \) calories, \( K = 0.346 \) cal/sec-cm, \( \rho = 9.09 \) g/cm\(^3\), \( c_p = 0.075 \) cal/g and \( T_m = 1200^\circ C \), we have \( r_w = 68 \) cm. This then gives a wall current of \( 4.8 \times 10^{15} \) n/cm\(^2\)-sec.

Such a current is not too bad when one considers that it increases slowly with a shorter assumed pulse time—say the time for the particle to traverse the pellet radius. Also it decreases rapidly as we go to larger radii. On the other hand, to get the current down to the value of a low \( \beta \) toroidal device
would require a spherical radius of some 470 m which is obviously undesirable.

While this is not meant to be an all inclusive comparison, it does suggest that the neutron damage to low $\beta$ toroids may be less than that from other approaches. By the same token, however, it equivalently suggests and is clear from Table II that the power per unit volume from these systems is significantly less and this seems to suggest that the costs may be higher.

REFERENCES


**TABLE I**

Ratio of 2.45 MeV Neutron Production to Total
Neutron Production for Various Cycles and Temperatures

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<th>D-D Cycle</th>
<th>D-T Cycle</th>
<th>D-3 Cycle</th>
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