A Collection of Diffusion Coefficients for Fusion Studies

J.E. Scharer

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This work is intended to pull together some of the expressions for diffusion coefficients based on various theories which are currently in vogue for toroidal magnetic confinement systems. Perhaps no aspect of plasma physics is less understood nor more important to controlled fusion than the diffusion process. Experimental confirmation of most diffusion processes is meagre at best. It is hoped that this work will simplify the task of sifting and winnowing through the Russian[1] and international conference[2,3] literature and assist in the calculation of diffusion rates for fusion reactors or as a comparison with experimental data on smaller devices. Expressions are both in exact form and with coefficients numerically calculated suitable for scaling studies. MKS units will be used throughout. Any corrections that are found will be welcomed by the author. It is hoped that this will simplify the task of comparison for future delvers into the plasma state at Wisconsin.

1. Bohm diffusion (D_B) is used most often as a comparison to measure relative scaling of other diffusion processes. The C-stellarator exhibited diffusion which agrees closely with the Bohm formula.

$$D_B \ (m^2/s) = \kappa T \ (\text{joules})/16 \ eB \ (\text{Tesla}) \tag{1}$$

where κ is Boltzmann's constant, T is the temperature in °K, e is the electron charge, and B is the magnetic flux density in Tesla.

$$D_B \ (m^2/s) = 6.25\times10^{-2} T \ (\text{eV})/B \ (\text{Tesla}) \quad . \tag{2}$$

2. Classical free diffusion follows from the random walk argument of particles on the average moving an electron gyroradius step size (ρ_e) each collision period (ν_{ei}).

$$D_{Cl} = \rho_e^2 \ \nu_{ei} \tag{3}$$

It is convenient for scaling purposes to normalize this to the Bohm relation which gives

$$D_{Cl}/D_B = 1.6\times10^1 \quad \nu_{ei}/\omega_c \tag{4}$$

where $\omega_c = eB/m$, the electron gyrofrequency. Numerically, this becomes

$$D_{Cl}/D_B = 2.64 \times10^{-16}\left(n(\mathrm{cm}^{-3})\ \ln\Lambda\right)/B(T)\ \left(T(\mathrm{eV})\right)^{3/2} . \tag{5}$$

3. Neoclassical Diffusion: In this regime the effects of toroidicity and helical field windings or field ripple are included in the analysis. For systems with low aspect ratio ($a/R << 1$), we assume that the magnitude of the magnetic field along a field line can be written as

$$B = B_0\left(1-\varepsilon_t \cos\theta-\varepsilon_h(r)\ \cos\ell(\theta-\alpha\phi)\right) \tag{6}$$

where ϕ measures the distance along the magnetic axis, θ is the poloidal angle around the axis, ℓ is 1/2 the number of helical windings around the poloidal direction, $\varepsilon_h(r) << 1$ is the minor radius dependent perturbation due to the helical windings, and $\varepsilon_t << 1$ is the inverse aspect ratio (a/R).

We will assume that for the case of an axisymmetric tokamak ($\varepsilon_h \to 0$) all are familiar with the Pfirsch-Schlüter, transition, and banana regimes of neoclassical theory and the refinements of Rosenbluth, Hazeltine, and Hinton and concentrate on the regime most applicable to fusion studies, the banana regime (realizing full well that during start-up one might have to pass through the other regimes). Banana diffusion occurs in a axisymmetric toroidal system where the <u>effective</u> collision frequency for scattering trapped particles out of the helical mirrors is small compared to the bounce time for particles trapped in the mirrors. This requires $\nu_{eff} \simeq \nu_{ei}/\varepsilon_t < \omega_b$, where ω_b is the bounce time, $\omega_b \simeq V_t(2\varepsilon_t)^{1/2}/R\ q$, V_t is the electron thermal velocity, R the major radius, and $q = r\ B_T/R\ B_p > 1$ is the safety factor. When this condition is obtained, i.e., when $\nu_{ei} < (\varepsilon_t)^{3/2}\ V_t/R\ q$ one obtains banana diffusion due to the trapped particles which have an enhancement factor $q^2 \varepsilon_t^{-3/2}$ over free classical diffusion.

$$D_{NC} = \nu_{ei}\ \rho_e^2\ q^2/\varepsilon_t^{3/2} \tag{7}$$

thus

$$D_{NC}/D_B = 1.6\times 10^1 \quad \nu_{ei} q^2/\omega_c \quad \varepsilon_t^{3/2} \tag{8}$$

$$D_{NC}/D_B = 2.64 \times 10^{-16} \left(n(\mathrm{cm}^{-3})\ \ell n\Lambda\ q^2\right)/B(T) \left(T(\mathrm{eV})\right)^{3/2} \varepsilon_t^{3/2}\ . \tag{9}$$

4. In non-axisymmetric systems other particle motions are possible. In particular, in stellarators where $\varepsilon_t << \varepsilon_h << 1$ we have the possibility of superbanana diffusion. This occurs for a very special class of particles whose banana orbits drift radially and are localized in the poloidal angle (so-called trapped bananas). The slowly drifting guiding center of the trapped banana generates the superbanana which has a net diffusion rate greater than that for a banana orbit in a comparable axisymmetric system. To obtain this condition, one requires that the <u>effective</u> collision frequency for scattering trapped banana orbits be small compared to the bounce frequency for the superbanana orbit. This requires that

$$\nu_{ei} < \nu_3 = \kappa T\ \varepsilon_h^{1/2}\ \varepsilon_t^{3/2}/eBr^2$$

where r is the minor radius. The superbanana diffusion coefficient becomes

$$D_{SB} = \nu_{ei}\ \varepsilon_t^2\ \kappa T/\nu_3\ \varepsilon_h^{1/2}\ eB \tag{10}$$

and

$$D_{SB}/D_B = 1.6\times 10^1 \quad \nu_{ei}\ r^2\ \varepsilon_t^{1/2}\ eB/\kappa T\ \varepsilon_h \tag{11}$$

which obtains a maximum when $\nu_{ei} = \nu_3$ of

$$D_{SB}/D_B = 1.6\times10^1 \; \varepsilon_t^{\,2}/\varepsilon_h^{\,1/2} \tag{12}$$

which is Bohm-like except for geometrical factors.

5. If a radial electric field exists in the device the $\bar{E}\times\bar{B}$ drift velocity and frequency must be taken into account, modifying the previous analysis. Let $\omega_E = E_r/reB$, when $\omega_E > \omega_h = \varepsilon_H \kappa T/eBr^2$, which is satisfied if $reE_r/\varepsilon_h \kappa T > 1$. All of the bananas are then untrapped and superbananas do not occur. Maximum estimates of the diffusion coefficients in this case become

$$D_E/D_B = 16\; \varepsilon_t^{\,2}\; \varepsilon_h^{\,1/2}\; \kappa T/eE_r r \tag{13}$$

which reduces the diffusion coefficient significantly below that for superbananas.

6. Another approach to the diffusion process is pseudo-classical diffusion based on a field fluctuation spectrum in the presence of density gradients which provides a good fit to confinement times observed on most experimental devices to date. The diffusion coefficient can be expressed as

$$D_{PC} = \left((\langle k_\perp^{\,2}/k_\parallel^{\,2}\rangle\; \delta n^2/2n_o^{\,2}) + 1 \right) D_{Cl} \tag{14}$$

where δn is the fluctuation density due to plasma noise and $k_\perp$ and $k_\parallel$ are the wavenumbers of the fluctuations perpendicular and parallel to the magnetic field. A saturation estimate on the diffusion rate gives the expression for a tokamak of

$$D_{PC} = C^2\; D_{Clp} = C^2\; \nu_{ei}\,(mv_\perp/eB_p)^2 \tag{15}$$

where $C^2 \simeq$ 1-10 and D_{Clp} is the classical diffusion coefficient expressed in terms of the poloidal magnetic field B_p. Thus

$$D_{PC}/D_B = 1.6\times10^1 \;\; C^2\; q^2\; \nu_{ei}/\varepsilon_t^{\,2}\; \omega_c \;\; . \tag{16}$$

7. It is theorized that the nonequilibrium nature of a confined plasma can give rise to instabilities which may cause anomolous transport of the plasma. It may be possible that some microinstabilities cannot be stabilized. Due to their short wavelength, high frequency and relatively small amount of energy driving them, it is expected that they will grow rapidly into the nonlinear regime where their growth will be limited. For a tokamak, the drift waves are caused by radial density gradients and a poloidal electric field E_θ.

$$\omega_{*min} = v_{ti}\,\rho_i/r\,r_p \qquad \text{and} \qquad \omega_{*max} = v_{ti}/r_p$$

are the minimum and maximum frequencies of the drift waves in the poloidal direction and $r_p = \left((1/n)(dn/dr)\right)^{-1}$ is the effective plasma radius. $\omega_{Dmin} = k_{\theta min} v_D$ is the minimum drift frequency due to magnetic field curvature. For tokamaks the frequency ordering is typically such that

$$\omega_{Dmin} < \omega_{*min} << \omega_{bi} << \omega_{*max} << \omega_{be}$$

where the subscript b refers to the bounce frequency of the particles in the mirrors formed by the helical magnetic field lines around the torus. For reactor conditions, the (D) mode or dissipative trapped-ion mode is deemed to be the most important. In this regime the effective collision frequency to remove trapped particles from the mirrors is less than the ion bounce frequency and the drift wave is supported only by trapped particles ($\nu_{eff} = \nu_{ei}/\varepsilon_t << \omega_{bi}$). If $\nu_{ii}/\varepsilon_t\,\omega_{bi} < 0.4(m_e/M_i)^{7/18}$ the mode is unstable and, using the strong turbulence (assuming that wave fields strongly perturb the wave-particle resonance and causes the instability to saturate) upper bound estimate of $D_{AT} = \gamma/k_\perp^2$ where $k_\perp = 1/r$ and γ is a linear growth rate yields

$$D_{AT} \sim (\varepsilon_t)^{5/2}\,(\kappa T_i/eBr_n)^2/\nu_{ei} \tag{17}$$

where $\qquad r_n = \left((1/n)(dn/dr)\right)^{-1}$.

Normalized to the Bohm form this yields

$$D_{AT}/D_B \sim 16\ \varepsilon_t^{5/2}\ (\kappa T/eB)/r_n^{\,2}\ \nu_{ei} \quad . \tag{18}$$

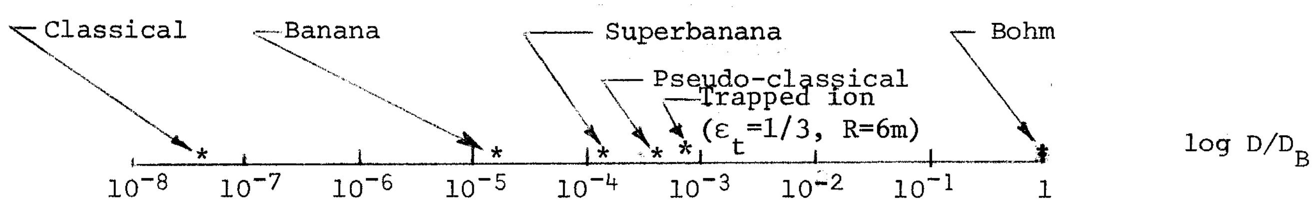
As an exercise, for $n = 10^{14}/\text{cm}^3$, $T = 10^4$ eV, $\ell n\Lambda = 20$, $B = 10$ Tesla, $q^2 = 10$, $r_n^{\,2} = 10\ \text{m}^2$, $\varepsilon_t = 0.1$, $\varepsilon_h = 0$ for all cases except the superbanana regime where $\varepsilon_h = 0.1$, $\varepsilon_t = 0.01$, we obtain

$D_B = 62.5\ \text{m}^2/\text{s}$, $\qquad D_{SB}/D_B = 1.23\times10^{-4}$,

$D_{Cl}/D_B = 5.28\times10^{-8}$, $\qquad D_{PC}/D_B = 5.28\times10^{-4}$, and

$D_{NC}/D_B = 1.67\times10^{-5}$, $\qquad D_{AT}/D_B =$ Dissipative mode stable.

Thus a log scale would look as shown below.



It should be noted that the disspiative trapped-ion mode is stable under the above conditions and one would need to have ε_t=1/3 (R=6m, a=2m) with other parameters unchanged for it to become important.

<u>References</u>

1. Kadomtsev and Pogutse, Nuclear Fusion <u>11</u>, 67-92 (1971).

2. Preprint: Frieman Tokamak Theory paper (1973).

3. S. Yoshikawa and N. C. Cristofilos, IAEA 1971 <u>II</u>, 357-372, Madison meeting.