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ABSTRACT

Heating of condensed deterium and He^3 by the high energy protons produced by fusion reactions in a D-He^3 plasma is calculated in two limits: a high energy limit, in which energy is deposited uniformly in the pellet, and the low-energy limit, in which the energy is deposited in a thin surface layer. The resultant heating rates are so high that pellets with realistic velocities will penetrate only the outer few centimeters of the plasma.

INTRODUCTION

The heating and ablation of material from pellets injected into a hot plasma is a complicated process involving electron heating of the surface, growth of a cloud of neutral gas around the pellet, etc. In a fusion plasma, the high-energy charged particles from fusion reactions provide an additional heating mechanism. In the case of a D-T plasma, the dominant high-energy charged species is alpha particles. In a D-He^3 fusion plasma, high energy protons are present. The dominant mechanism for energy loss of high-energy charged particles in the range of interest in neutral matter is multiple scattering from atomic electrons. In the following, the units are MKS unless otherwise noted.

ENERGY LOSS FORMULA

For the energies of interest, to a good approximation, the rate of energy loss of a charged massive particle (e.g. proton, deuterium, or alpha particle) in neutral matter is given by the classical formula (MKS units)

$$\frac{dE}{dx} = - \frac{z^2 e^4 NZ}{4\pi\epsilon_0^2 mv^2} \ln\left(\frac{2mv^2}{I}\right) . \tag{1}$$

In the above formula, I is an empirically determined constant with dimensions of energy and is related to the average ionization energy of the electrons in the target. In view of the logarithmic dependence of the dE/dx expression on this quantity, the approximate formula for I,

$$I = 1.84 \times 10^{-18} Z \tag{2}$$

is sufficiently accurate for most purposes. In terms of incident particle energy E rather than velocity, Eq. (1) becomes

$$- \frac{dE}{dx} = f(E) = \frac{z^2 e^4 NZM}{8\pi\epsilon_0^2 mE} \ln\left(\frac{4mE}{MI}\right) . \tag{3}$$

It should be noted that Eqs. (1) and (3) do not apply at the low energies (on the order of kilovolts) in the vicinity of the Bragg peak, for which dE/dx is considerably greater; however, only a negligible fraction of the total energy is in this region.

The two target materials of interest are D_2 and He^3. Using molar volumes of 23.2 and 50.9 cm^3, respectively for these two materials, NZ values of 5.19×10^{28} and 2.36×10^{28} are calculated for D_2 and He^3, respectively. The resulting energy loss values for protons of various energies calculated from Eq. (3) are given in Table I.

Table I. Proton Energy Loss in Solid D_2 and Liquid He^3

Proton Energy (MeV)	dE/dx in D_2 (MeV/mm)	dE/dx in He^3 (MeV/mm)
15	0.49	0.20
10	0.74	0.30
5	1.47	0.59
4	1.84	0.74
3	2.45	0.99
2	3.68	1.48
1	7.36	2.96

From the table it is seen that protons with energies of about 3 MeV or less are completely captured in 1 mm of D_2, and those of energies of about 2 MeV or less are completely captured in 1 mm of He^3. For protons with an energy distribution extending up to 15 MeV, then, the heating in the pellet is strongest in the outer layers of the pellet, but significant bulk heating will occur inside the pellet for pellet diameters of a few millimeters or less.

PROTON ENERGY DISTRIBUTION

Protons generated with $E_0 = 14.7$ MeV energy initially slow down by collisions in the plasma. Initially, the slowdown is dominated by collisions with plasma electrons, but at a critical energy E_c, ion collision energy losses equal electron collision energy losses. According to Ref. 1, the proton energy spectrum has the form

$$g(E) = \frac{s_0\tau_s}{2E(1+(E_c/E)^{3/2})} \qquad E < E_0$$

$$= 0 \qquad E > E_0 \tag{4}$$

Also, according to Ref. 1, E_c is given by the expression

$$E_c = [\frac{3\sqrt{\pi}}{4} \frac{M^{3/2}}{n_e\sqrt{m_e}} \sum_j (\frac{n_j z_j^2}{m_j} \ln\Lambda_j) \frac{1}{\ln\Lambda_e}]^{2/3} kT_e \tag{5}$$

where the sum indicated is over the ion species in the plasma. Taking $\ln\Lambda_e$ to be 17, a 1:1 D-He^3 ratio, and an ion density of $10^{20}/m^3$, one obtains E_c = 0.63 Mev. The slowing down time, according to Ref. 1, is given by

$$\tau_s = \frac{3}{4} \frac{(kT_e)3/2}{\sqrt{2\pi m_e}} (\frac{4\pi E_0}{e^2})^2 \frac{M}{n_e Z \ln\Lambda_e} . \tag{6}$$

Taking kT_e to be 50 kV, one obtains τ_s = 2.8 s. However, for the present numerical estimates this number was not directly used, but rather the product $s_0\tau_s$ was obtained from the model calculations of Ref. 1 using the approximate expression

$$\tau_0 s_0 = \frac{3P_f}{E_0} \tag{7}$$

where f is the fast ion (mostly proton) pressure. According to the model calculations of Ref. 1 (Fig. 3 therein) $P_f \sim 0.13\ P_{thermal}$, so $\tau_0 s_0$ = 2.4×10^{17} (MKS).

CALCULATION OF PELLET HEATING

From the foregoing discussion, it is clear that heating of pellets by fusion-product protons is not simply constant bulk heating or surface heating but bulk heating with a peaking in the outer diameter of the pellet. Given the dE/dx curve f(E) down to zero energy (including the Bragg peak), and ignoring energy straggling and angular deviations, the radial dependence of power density deposited in a sphere of radius a with constant properties, bombarded by charged particles with an energy distribution g(E) is given by the following expression:

$$p(r) = \frac{1}{2} \int_0^{\pi} \int_0^{E_0} \left(\frac{2E}{M}\right)^{1/2} f[E'(\theta,r)] g(E) dE \sin\theta d\theta \qquad (8)$$

where E' is the energy left after the particle has penetrated to radius r (see Fig. 1). The residual energy E' is given by the expressions

$$E' = E - \int_0^{l(r,\theta)} f[E''(x)]dx \qquad l(r,\theta) < R(E)$$

$$= 0 \qquad l(r,\theta) \geq R(E) \qquad (9)$$

where R is the range, or distance of travel for which the particle loses all of its energy. From Fig. 1, it can be seen that the length l is given by

$$l(r,\theta) = (a^2 - r^2\sin^2\theta)^{1/2} - r\cos\theta \ . \qquad (10)$$

It should be noted that in Eq. (10), the positive square root applies for all values of θ between 0 and π. Since dE/dx is not known explicitly as a function of x, but as a function of E, the residual energy is a solution of a first-order differential equation. Upon integration, one has

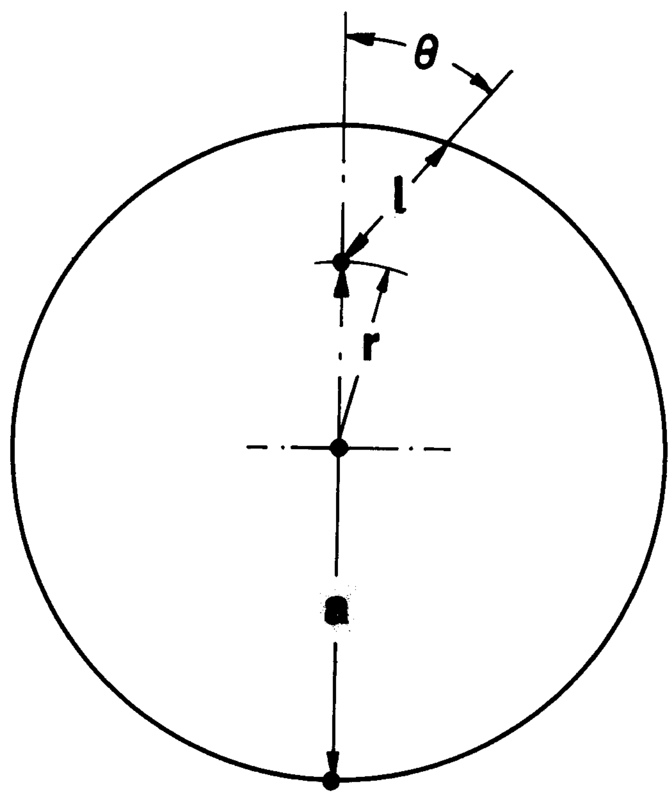

Fig. 1. Geometrical parameters in calculation of pellet heating.

$$\int_{E'}^{E} \frac{dy}{f(y)} = F_E(E') = x \qquad (11)$$

and for the range R

$$\int_{0}^{E} \frac{dy}{f(y)} = R(E) \ . \qquad (12)$$

Eq. (11) can be formally inverted to yield E' as a function of x:

$$E' = F_E^{-1}(x) \ . \qquad (13)$$

In practice x would be determined numerically, and for greatest accuracy, from a model that includes the Bragg peak. With this information, E'(1) can be found and substituted in Eq. (8) for numerical evaluation of the integral.

For the portion of the incident protons with energies above about E_{min} = 4 MeV and pellets of about 1 mm radius, f(E') in Eq. 8 can be approximated by f(E) (i.e. the energy loss can be neglected in calculating dE/dx) and Eq. (8) becomes

$$p = \int_{E_{min}}^{E_0} \left(\frac{2E}{m}\right)^{1/2} f(E)g(E)dE \qquad (14)$$

or

$$p = \frac{s_0 \tau_s M_p^{1/2} e^4 NZ}{8\sqrt{2}\, m\pi E_0^2} \int_{E_{min}}^{E_0} \frac{\ln\left(\frac{4mE}{M_p I}\right)}{E^{3/2} + E_c^{3/2}} dE \ . \qquad (15)$$

Since the logarithm in Eq. (15) varies slowly, it may be taken out of the integral, and one gets

$$p = \frac{s_0 \tau_s M_p^{1/2} e^4 NZ}{8\sqrt{2}\, m\pi E_0^2} \ln\left(\frac{4mE}{M_p I}\right) \int_{E_{min}}^{E_0} \frac{dE}{E^{3/2} + E_c^{3/2}} . \qquad (16)$$

The integral in Eq. (16) is readily evaluated by means of the change of variables $u^2 = E$ and use of the CRC mathematics tables to yield

$$I(E_0) = \int_{E_{min}}^{E_0} \frac{dE}{E^{3/2} + E_c^{3/2}} = \frac{2}{3E_c^{1/2}} \left[\frac{1}{2} \ln \frac{E_c - E_c E_0^{1/2} + E_0}{(E_c^{1/2} + E_0^{1/2})^2}\right.$$

$$- \frac{1}{2} \ln \frac{E_c - (E_c E_{min})^{1/2} + E_{min}}{E_c^{1/2} + E_{min}^{1/2}} + \sqrt{3} \tan^{-1} \frac{2E_0^{1/2} - E_c^{1/2}}{\sqrt{3}\, E_c^{1/2}}$$

$$\left. - \sqrt{3} \tan^{-1} \frac{2E_{min}^{1/2} - E_c^{1/2}}{\sqrt{3}\, E_{min}^{1/2}}\right] . \qquad (17)$$

For E_c = 0.63 MeV, E_0 = 14.7 MeV, and E_{min} = 4 MeV, one obtains

$$I(E_0) = 2.1 \times 10^6 \qquad (\text{MKS})$$

The resultant power densities for deuterium and He^3, respectively, are 1.7×10^{15} and 6.7×10^{14} W/m^3. At this power level, a D_2 pellet would vaporize completely in less than a microsecond. It is not clear if a plasma can really deliver this kind of instantaneous power density, but in any case, the results show that power levels will simply be much too high for appreciable pellet penetration.

Another limit of Eq. (8) is the surface heating limit, applicable to lower energy particles. In this case, one is interested only in the energy flux crossing the pellet surface and the dE/dx behavior is ignored. The surface energy flux is given by

$$q = s_0 \tau_s \left(\frac{2}{M_p}\right)^{1/2} \int_0^{E_{max}} \frac{E^2}{(E^{3/2} + E_c^{3/2})} dE \ . \qquad (18)$$

The integral in Eq. 18 can be evaluated to yield

$$q = s_0 \tau_s \left(\frac{2}{M_p}\right)^{1/2} \left[\frac{E_{max}^{3/2}}{3} + E_c^{3/2} \ln \frac{E_c^{3/2} + E_{max}^{3/2}}{E_c^{3/2}}\right] \ . \qquad (19)$$

For particles with $E < 1$ MeV, this yields

$$q = 1.8 \times 10^{11} \text{ W/m}^2$$

Again, power levels are too high by this calculation to allow significant penetration, since the ablation rate for D_2 becomes 3×10^5 cm/s.

CONCLUSIONS

The heating rates in solid deuterium and liquid He^3 from high energy protons in D-He^3 fusion plasma have been calculated, and are too high to permit pellet fueling of the interior of the plasma.

ACKNOWLEDGMENT

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NOMENCLATURE

Symbol

a	Pellet radius
e	Magnitude of electron charge
E	Incident particle energy
E_c	Critical energy for high energy particle that is slowing down
E_0	High energy particle birth energy
I	Average target electron ionization energy
m	Electron mass
M	Incident particle mass
M_p	Proton mass
n_e	Plasma electron density
n_I	Plasma ion density
p	Power/unit volume deposited in pellet by high-energy particle
q	Power/unit area deposited near surface of pellet
s_0	Generation rate of high energy charged particle
T_e	Plasma electron temperature
z	Charge of incident particle in units of e
Z	Atomic number of target atom
ε_0	Permittivity of free space
$\ln\Lambda$	Coulomb logarithm
τ_s	Slowing down time for high-energy charged particle born in a fusion reaction

REFERENCE

B.Q. Deng and G.A. Emmert, "Fast Ion Pressure in Fusion Plasmas", UWFDM-718, January 1987.