Alpha Particle Containment

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Alpha Particle Containment

The coordinate system shown below describes the toroidal confinement system considered.

\[ \phi \] is defined as the toroidal coordinate and \( R \) locates the particle birth position from the major axis as shown. \( R_0 \) and \( a \) are the major and minor radii of the plasma, respectively. The conservation of canonical angular momentum requires,

\[
(Ze)R A_\phi + m v R = p_\phi = \text{constant} \quad (1)
\]

\( Z \equiv \) charge of the particle
\( A_\phi \equiv \) toroidal component of vector potential
\( m \equiv \) mass of the particle
\( v_\phi \equiv \) toroidal component of particle velocity.

Assuming \( B_\phi \gg B_\theta \),

\[ v_\phi \approx v_\parallel \]
Further, assuming a constant plasma current density and circular flux surfaces centered at \( R_0 \), equation (1) can be averaged over a gyro orbit to obtain,

\[
\mathbf{r}^2 + \frac{4m v_{\parallel}}{Ze \mu_0} \frac{R}{JR_0} = \text{constant.} \tag{2}
\]

Finally, the total energy of the particle,

\[
E = \frac{1}{2} mv^2, \tag{3}
\]

and the adiabatic invariant,

\[
\mu \equiv \frac{1}{2} m v_{\perp}^2 / B_\phi, \tag{4}
\]

are constant along the initial orbit of the particle. Combining equations (2) - (4) and evaluating the constant in terms of the particle's initial conditions (subscript \( b \)),

\[
(\frac{r}{a})^2 = \left( \frac{R_b}{a} \right)^2 + \frac{4\pi mn}{Ze\mu_0} \frac{R_b}{a} \frac{\cos \chi}{AI} \left[ \frac{1}{AI} \left( \frac{4\pi mn}{Ze\mu_0} \right) \left[ \frac{R}{a} \left( \frac{R}{a} - \frac{R_b}{a} \sin^2 \chi \right) \right]^{\frac{1}{2}} \right] \tag{5}
\]

where the upper signs apply when the particle is produced costreaming with the plasma current, i.e.,

\[
\mathbf{J} \cdot \mathbf{v} > 0 .
\]

In equation (5),

- \( A \equiv \text{plasma aspect ratio} \)
- \( I \equiv \text{total plasma current (amperes x 10}^6\text{)} \)
- \( \chi \equiv \text{initial pitch angle of the particle} \).
For an alpha particle produced in a deuterium-tritium reaction,

\[ \frac{4\pi n_p}{Z e \nu} = 2.7. \]

The initial orbit of the alpha particle, projected into the minor cross section, is given by,

\[ \left( \frac{r}{a} \right)^2 = \left( \frac{r_b}{a} \right)^2 + \frac{2.7}{1A} \left( \frac{R}{a} \right) \cos \chi \pm \frac{2.7}{1A} \left[ \frac{R}{a} \left( \frac{R}{a} - \frac{R_b}{a} \sin^2 \chi \right) \right]^{1/2}. \] 

(6)

It is noted that the alpha particle orbit is a function of aspect ratio and total plasma current only.

An alpha particle is considered contained when \( r < a \) everywhere on the orbit. Alpha particles which undergo excursions such that \( r > a \) are considered lost to a limiter or perhaps an efficient divertor. Equation (6) was solved numerically for several cases.

In the first case, the total plasma current required to contain all alpha particles produced within a specified plasma radius was determined as a function of aspect ratio. The results are shown on Figure 1. Note that the production rate of alpha particles per unit volume is,

\[ \dot{n}_\alpha = n_0 n_p \langle \sigma v \rangle \frac{r_f}{f}, \]

so that the usual assumption of plasma profiles which are peaked at the center of the discharge suggest maximum alpha production in the central plasma zone. Therefore, a total plasma current of 4–5 MA is required for efficient alpha containment.

In the second case, for aspect ratios of 3 and 4 and various total currents, the fraction of alpha particles lost, as a function of birth radius, was determined. Alpha production is assumed isotropic in velocity space and the results are flux surface averages so that the loss fractions are functions of birth radius only. The results are shown on Figures 2 and 3. Note that from an energy deposition point of view, 2–3 MA provides containment of at least 65–75 percent of the alpha particles produced between the plasma center and \( r/a \sim 0.5 \).
Conclusion:

Alpha particle containment in a fusion reactor operating at I>5 MA will be very efficient. However, an ignition experiment will probably operate in the critical current range relative to alpha containment which adds one more consideration to sizing the device.

General Reference
