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HALO PLASMA PHYSICS MODEL FOR MIRROR MACHINES
WITH NEUTRAL BEAM INJECTION

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ABSTRACT

A model for halo plasmas in mirror machines sustained by neutral beam injection is described and applied to the conceptual tandem mirror test facilities TDF and TASKA-M. Halo source terms include alpha particle heating, radial transport, neutral beam attenuation, and reionization of charge exchanged neutrals. The primary sinks are end loss and charge exchange. The neutral beams are found to contribute substantially to first wall surface heat load, but little to halo density or temperature. Reference halo parameters for TDF and TASKA-M are exhibited.
1. INTRODUCTION

Fusion reactor plasmas must be protected from cold gas and impurities. The cold gas can cause loss of energy through charge exchange, while the impurities lead to radiation and to dilution of the fusing plasma. A region of low density and temperature plasma -- called the halo in a mirror machine and the scrape-off layer in a tokamak -- surrounds the core plasma and provides the shielding. This paper presents a model for the halo plasma in a mirror machine with neutral beam injection. Results of applying the model to the tandem mirror engineering test facility conceptual designs TDF [1] and TASKA-M [2] are given.

The model used here is very simple: constant halo density and temperature in a single zone. This suffices because the shielding properties of the halo depend only weakly on temperature and in an integral manner on density. The temperature dependence is weak because the ionization and charge exchange rates vary only slowly with temperature from 20 eV to 2 keV. Density enters the problem through integrals over the radial profile, so it may be approximated by an average value. Thus, since the cold gas source terms are rather uncertain in any case, the model presented here should be adequate for many reactor modeling purposes. Because halo ion loss occurs on an ion-ion collisional time scale and electron-ion thermal equilibration occurs much more quickly, $T_e = T_i = T$ is assumed.

The geometry used for the halo calculations is shown in Fig. 1. The halo extends from the core plasma radius to the largest radius for which the flux tube passes through the magnetic field coils at the ends of the machine. Beyond that radius, magnetic field lines are assumed to intersect the wall at or before the coils, and electron thermal conduction keeps the region too cold to
ionize gas so essentially no plasma forms. In the cold gas zone, diffusion is assumed to distribute the gas uniformly along the z direction, giving uniform charge exchange. All zones are cylinders.

Neutral beams are assumed to penetrate and fuel the core plasma. Neutral beam particles are also deposited in the halo through ionization and charge exchange. Other particle source terms are radial transport from the core plasma and reionization of charge exchanged neutrals. Besides the energy carried into the halo by particles, alpha particles with orbits which intersect the halo are an energy source. The main particle sink is end loss, with a small contribution due to radial transport. Energy is lost primarily through end loss and charge exchange. Radial loss and radiation loss are small terms which are neglected here.

The shielding of the core plasma from cold gas and sputtered atoms will be estimated first. Second, halo particle and power balance will be discussed. Third, halo reference parameters for TDF and TASKA-M will be given.

Units are cgs, except that energies will be in eV and powers will be in watts unless stated otherwise.

2. SHIELDING FROM SPUTTERED BEAM DUMP ATOMS

The beam dump sputtering calculations require the flux of neutral beam particles hitting the beam dumps, the energy of those particles, and the geometry of the configuration. An approximate model for calculating the distribution in energy and angle of sputtered atoms is given in the appendix. The analysis is based on that of Bohdansky [3,4]. The number of sputtered atoms reaching the core plasma may then be calculated by integrating over the solid angle which sees the core. Because this is very geometry dependent,
only results will be quoted, and the reader is referred to the TDF [1] and

3. SHIELDING FROM NEUTRAL GAS

The calculations at hand require the attenuation factor for neutral gas
as a function of integrated line density $\int n_h \, dl$. $n_h$ is the halo plasma
density and $l$ measures distance into the halo. The model used for the TDF and
TASKA-M halo calculations is taken from Ref. 5 and is shown in Fig. 2. That
model includes both charge exchange and ionization in the attenuation factor.
Only a rough estimate of the proportionality factor, $C_A$, between attenuation
and line density is required here. Therefore, $C_A$ may be approximated, using a
value from the linear part of the $T_e$ curve in Fig. 2, as

$$C_A \approx \frac{n_w}{n_h \, n_g(r)}.$$  \hspace{1cm} (1)

$n_w$ is the density of neutral gas at the outer edge of the halo. At the edge
of the core plasma, the neutral gas density is therefore

$$n_{gc} = \frac{n_w}{C_A \, n_h (r_h - r_c)}$$  \hspace{1cm} (2)

where $r_h$ is the outer radius of the halo and $r_c$ is the core plasma radius.

The quality of shielding of neutral gas done by the halo may be measured
by the amount of charge exchange which occurs between the neutral gas and the
core plasma. Inside the core plasma, the density of neutral gas at radius $r$
is approximately

$$n_g(r) = \frac{n_{gc}}{C_A \, \int_r^{r_c} dr \, n_c(r)}$$  \hspace{1cm} (3)
where \( n_c(r) \) is the core plasma density, and it has been assumed that \( C_A \) is still the correct proportionality constant.

The total ionization rate of neutral gas in the core plasma is then

\[
\dot{N}_{igc} = 2\pi L_C \langle \sigma v \rangle_{igc} \int_0^r dr n_c(r)n_g(r)
\]

(4)

where \( \langle \sigma v \rangle_{igc} \) is the ionization rate for neutral gas in the core plasma, and \( L_C \) is the effective central cell length. Total charge exchange, by a similar argument is

\[
\dot{N}_{cxgc} = \frac{\langle \sigma v \rangle_{cxgc}}{\langle \sigma v \rangle_{igc}} \dot{N}_{igc}.
\]

(5)

where \( \langle \sigma v \rangle_{cxgc} \) is the charge exchange rate. Note that the simple assumption of Eq. (1) breaks down if Eq. (3) gives \( n_g(r) > n_{gc} \). A reasonable assumption for the small region involved is that \( n_g(r) = n_{gc} \), which was assumed for TDF and TASKA-M.

4. HALO PARTICLE BALANCE

Since the central cell magnetic fields are axisymmetric, classical radial transport may be assumed. Only hot central cell ions are considered because they are both hotter and denser than the warm ion population. The central cell ion radial transport coefficient is thus

\[
D_r \sim v_i^2 \rho_i^2 = \frac{0.105 n_{Hc}}{E_{Hc}^{1/2} B_C^2} \sin^2 \theta_{inj}
\]

(6)

where \( n_{Hc} \) is the central cell hot ion density, \( \theta_{inj} \) is the neutral beam injection angle, \( E_{Hc} \) is the hot ion energy, \( B_C \) is the central cell magnetic
field, $\tau_{\perp}$ is the $90^\circ$ scattering time, and $\rho_{iH}$ is the hot ion gyroradius. The radial loss time may be approximated by

$$\tau_r \sim \frac{r_c^2}{2D_r}$$

(7)

giving a radial particle loss from the central cell to the halo of

$$\dot{N}_0 = \frac{\tau_r}{\tau_{\text{rpl}}} \frac{n_{Hc}}{V_c}$$

(8)

where $\tau_{\text{rpl}}$ is a radial profile factor, taken to be 0.6 for TDF and TASKA-M, and $V_c$ is the effective central cell volume.

By an argument analogous to that of Section 3 for ionization of neutral gas in the core plasma, ionization of neutral gas in the halo is given by

$$\dot{N}_{ig} = 2\pi L n_c \langle ov \rangle_{ig} \int_r^{r_h} r \, dr \, n_g(r) .$$

(9)

Again, the charge exchange term is

$$\dot{N}_{\text{cxg}} = \frac{\langle ov \rangle_{\text{cxg}}}{\langle ov \rangle_{ig}} \dot{N}_{ig}$$

(10)

and $n_g(r) = n_w$ should be assumed if Eq. (1) gives $n_g(r) > n_w$.

Attenuation of the neutral beams in the halo is modeled by

$$f_{\text{nb}} \sim 1 - \exp(-\lambda_{\text{nb}})$$

(11)

where $f_{\text{nb}}$ is the neutral beam trapping fraction in the halo and the ratio of
path length to the mean free path is given by

$$\lambda_{nb} = 7.15 \times 10^{-7} \left( \frac{m_i}{m_H} \right)^{1/2} \frac{n_n \xi \left( \sigma_v \right)_{inb} + \sigma_v \right)_{cxnb}}{E_{inj}}^{1/2} \]

(12)

where \( \xi \) is the path length on one side of the halo, \( \langle \sigma v \rangle_{inb} \) is the neutral beam ionization rate, \( \langle \sigma v \rangle_{cxnb} \) is the charge exchange rate, \( E_{inj} \) is the neutral beam energy, \( m_i \) is the ion mass, and \( m_H \) is the proton mass. The total trapping rate in the halo on both sides of the core plasma is then given by

$$\dot{N}_{nb} = [f_{nb} + f_{nb}(1 - f_{nb})(1 - f_{tr})] I_{nb}$$

(13)

where \( I_{nb} \) is the incident current and \( f_{tr} \) is the trapping fraction in the core plasma. The fraction of this current which ionizes and fuels the halo is

$$\dot{N}_{nbi} = \frac{\langle \sigma v \rangle_{inb}}{\langle \sigma v \rangle_{inb} + \langle \sigma v \rangle_{cxnb}} \dot{N}_{nb}.$$  

(14)

Similarly, the fraction which charge exchanges with the halo is

$$\dot{N}_{nbcx} = \frac{\langle \sigma v \rangle_{cxnb}}{\langle \sigma v \rangle_{inb} + \langle \sigma v \rangle_{cxnb}} \dot{N}_{nb}.$$  

(15)

Some neutrals from charge exchange events in the halo will reionize in the halo. The model used here is rough, but should suffice for the accuracy of this calculation. The two chief assumptions are that the core plasma is neglected due to its smaller volume, and that slab geometry is used.
The mean free path for reionization of a charge exchanged neutral is

\[ \lambda_{\text{mfp}} = \left[ \frac{E^{1/2}}{7.15 \times 10^{-7} n <\sigma v>_{i} (m_{i}/m_{H})^{1/2}} \right] \]  

(16)

where \( <\sigma v>_{i} \) is the ionization rate and \( E \) is the neutral's energy. The very simple assumption utilized here is that the neutral escapes if its path length, \( \ell \), to escape the halo is shorter than \( \lambda_{\text{mfp}} \). The following conceptual picture is used in the analysis: Defining \( r_{\text{cx}} \) as the radius where the charge exchange event occurs, \( \ell = r_{h} - r_{\text{cx}} \) is greater than \( \lambda_{\text{mfp}} \) if the tip of a vector with magnitude \( \lambda_{\text{mfp}} \) and direction of the neutral atom's velocity lies on a zone of height \( h \) on a sphere of radius \( \lambda_{\text{mfp}} \), as in the geometry shown in Fig. 3a. The fraction escaping, \( f_{\text{esc}} \), is then the area of the zone divided by the total surface area of the sphere. \( f_{\text{ion}} = 1 - f_{\text{esc}} \) is then the fraction ionized. The three possible cases are shown in Fig. 3b and are summarized in Table I.

The source of halo ions due to reionization of charge exchanged neutrals is then

\[ \dot{N}_{\text{icx}} = f_{\text{ion}} (\dot{N}_{\text{nbcx}} + \dot{N}_{\text{cxg}}). \]  

(17)

These particle sources are balanced primarily by end loss. Two regimes may be defined, with the boundary rather arbitrary, as collisionless and collisional. They are delineated by the ratio of ion-ion scattering time, \( \tau_{\perp} \), to bounce time, \( \tau_{\parallel} \). For \( \tau_{\perp} < \tau_{\parallel} \), the halo is collisional and a simple flow time may be used:
\[ (n\pi)^{\text{flow}} = \frac{nR_cL_c/2}{(2T/\pi m_i)^{1/2}} \]  

(18)

where \( T \) is the halo ion temperature, \( R_c \) is an appropriate mirror ratio and the \( \pi \) comes from taking a directed Maxwellian velocity distribution. For \( \tau_{\parallel} < \tau_{\perp} \), the halo is collisionless and

\[ (n\pi)^{\text{scat}} = (n\pi)^{\parallel} \log_{10} R_c \]  

(19)

where the ion-ion-scattering time is

\[ \tau_{\parallel} = 4.9 \times 10^5 \frac{T^{3/2}}{n}. \]  

(20)

End loss is then given by

\[ \dot{N}_{\text{el}} = \frac{n^2}{(n\pi)^{\text{el}}} V_h, \]  

(21)

where \((n\pi)^{\text{el}}\) is the larger of \((n\pi)^{\text{flow}}\) or \((n\pi)^{\text{scat}}\). Finally, total particle balance requires

\[ \dot{N}_{\text{ro}} + \dot{N}_{\text{nbi}} + \dot{N}_{\text{ig}} + \dot{N}_{\text{icx}} = \dot{N}_{r} + \dot{N}_{\text{el}}. \]  

(22)

5. HALO POWER BALANCE

Somewhat surprisingly, halo power balance is not dominated by the energy brought into the halo when a neutral beam particle ionizes or charge exchanges. The reason is that the hot ion from the neutral beam charge exchanges with neutral gas before it slows down appreciably. The pertinent
times are the charge exchange time
\[ \tau_{\text{cxb}} = \frac{1}{n_g \langle \sigma v \rangle_{\text{cxb}}} \] (23)

where \( \langle \sigma v \rangle_{\text{cxb}} \) is the charge exchange rate for the hot "beam" ions in the halo, and the drag time on electrons is
\[ \tau_{\text{dr}} = \frac{(m_i/m_H)T^{3/2}}{3.2 \times 10^{-8} n}. \] (24)

At the low temperatures of interest, energy exchange of hot ions with halo ions is much slower. Total halo power drain on the neutral beams is \( \dot{N}_{\text{nb inj}} \), and the effective power given to the halo may be approximated by:
\[ P_{\text{nb}} = \dot{N}_{\text{nb inj}} \left[ 1 - \exp\left(-\tau_{\text{cxb}} / \tau_{\text{drb}}\right) \right] + \frac{3}{2} \dot{N}_{\text{ng}} T - \frac{3}{2} \dot{N}_{\text{ncx}} T \] (25)

where an appropriately averaged \( n_g \) must be used in \( \tau_{\text{cxb}} \). A linearly averaged value is used for the calculations presented here. \( T_g = 3 \text{ eV} \) is taken as typical of Frank-Condon neutrals in this analysis.

Another possibly important power balance term stems from alpha particle orbits intersecting the halo. Whether significant halo heating occurs depends on the relative thermal equilibration times for alpha particles in the halo and in the core plasma. These equilibration times are proportional to the respective \( n/T_e^{3/2} \) values. For the purposes of this analysis, the fraction of the alpha particle power deposited in the halo from those orbits which intersect the halo is assumed to be given by
\[ P_\alpha = P_{\alpha 0} \left[ 1 - \exp \left( \frac{n}{n_c} \left( \frac{T_{ec}}{T} \right)^{3/2} \right) \right] \]  \hspace{1cm} (26)

where \( P_{\alpha 0} \) is the total power available if all were absorbed. The amount of power absorbed in the halo is then reduced to the geometric question of what fraction of the alpha particle orbits intersects the halo. A slab model and analysis analogous to the reionization model of Section 4 is used. For an alpha particle born at radius \( r \) and angle \( \theta \) with respect to the magnetic field, the possible cases are summarized in Table II and Fig. 4, in which \( f_\alpha (r) \) is the fraction of alpha particles at radius \( r \) which have orbits intersecting the halo, and \( \rho_\alpha \) is the maximum alpha particle Larmor radius. The wall is assumed to be at least 2 \( \rho_\alpha \) from the plasma. The fraction of total alpha particle orbits which intersects the halo is then

\[ f_{\alpha h} = 2 \int_0^{r_c} \frac{r_c}{r^2} \frac{n(r)}{n(0)} \frac{f_\alpha (r)}{r_c} \, dr . \]  \hspace{1cm} (27)

Under this model, then, total power available to the halo is

\[ P_{\alpha 0} = 0.2 \, P_{\text{fus}} f_{\alpha h} , \]  \hspace{1cm} (28)

for \( t_E < t_{E} \) and zero otherwise, where \( P_{\text{fus}} \) is the total fusion power.

Core plasma radial transport brings hot ions into the halo plasma at a rate \( \dot{N}_{r_0} \). These ions quickly undergo charge exchange, and their time history is similar to that of the hot ions from neutral beam charge exchange and ionization. Equations (23) and (24) again apply, with a slightly different reaction rate due to the ion energy difference. Since the loss time,
\[ \tau_L \sim \frac{4.4 \times 10^5 E_{hc}^{3/2}}{n \ln R_c} \]  

(29)

is much longer than the charge exchange time, and loss for the hot ions may be neglected. The total power given to the halo plasma by radially transported core ions is then

\[ P_{ro} = \dot{N}_{ro} \left[ E_{hc} \left[ 1 - \exp \left( -\frac{\tau_{cxc}}{\tau_{drc}} \right) \right] + \frac{3}{2} T_g \right] \]  

(30)

where \( \tau_{cxc} \) and \( \tau_{drc} \) are analogous to \( \tau_{cx} \) and \( \tau_{dr} \). There is an analogous term, \( P_r \), for radial transport out of the halo and into the cold gas region. However, it is very small and is neglected here.

Charge exchange with background gas can contribute an important term to power balance. The charge exchange rate, \( \dot{N}_{cxg} \), was computed in the previous section; the power lost is

\[ P_{cxg} = \frac{3}{2} \dot{N}_{cxg} (T - T_g) \]  

(31)

An effect which may be important for low halo temperatures is the energy of dissociation, \( T_{ig} \). A typical value is 30 eV. Reionization of charge exchanged neutrals gives

\[ P_{icx} = \frac{3}{2} \dot{N}_{icx} (T - T_{ig}) \]  

(32)

Estimating, based on Ref. 6, neglect of line radiation also appears to be a good assumption.
The largest power loss term is generally due to end loss. Using the particle end loss term from the previous section, the end loss power is simply

\[
P_{e\ell} = \frac{3}{2} N_{e\ell} T. \tag{33}
\]

Total halo power balance is then given by

\[
P_{e1} + P_{cxg} = P_{ro} + P_{a} + P_{nb} + P_{icx}. \tag{34}
\]

6. FIRST WALL SURFACE HEAT LOAD

This section discusses the contribution to the first wall surface heat load from processes related to the halo only. These are:

1. Neutral gas charge exchanging with the core plasma or with radially transported core ions in the halo. (Energy \( E_{Hc} \))

2. Ions resulting from the neutral beam by charge exchange with the halo or ionization on the halo and subsequent charge exchange with the neutral gas. (Energy \( \sim E_{inj} \))

3. Ions resulting from halo ions charge exchanging with neutral gas or the neutral beam. (Energy \( \sim 3/2 T \))

The heat load due to the first process may be calculated using the charge exchange rate found in Section 3:

\[
\Gamma_1 = \frac{[\tilde{N}_{cxgc} + \tilde{N}_{ro} \exp(-\tau_{cxc}/\tau_{dc})]E_{Hc}}{2\pi r_w L_c}. \tag{35}
\]

The second term, using the results from Section 5, is approximately
\[ \Gamma_2 = \frac{\dot{N}_{\text{nb}} \text{E}_{\text{inj}} \exp\left(-\tau/c_{\text{xb}}/\tau_{\text{drb}}\right)}{2\pi r L_{\text{wc}}} \] 

(36)

The final term is given by that part of the charge exchanged halo ions which is not reionized:

\[ \Gamma_3 = \frac{3}{2} \frac{(1 - f_{\text{ion}})(\dot{N}_{\text{ncx}} + \dot{N}_{\text{cxg}})T}{2\pi r L_{\text{wc}}} \] 

(37)

The distribution of the surface heat load over the first wall is related to the ratio of the bounce time for trapped neutral beam ions,

\[ \tau_b = \frac{L_c}{V_\|} = \frac{L_c}{(2E_{\text{inj}}/m_i)^{1/2} \sin \theta_{\text{inj}}} \] 

(38)

to the charge exchange time

\[ \tau_{\text{cx}} = \frac{1}{n_g \langle \sigma v \rangle_{\text{cxnb}}} \] 

(39)

For \( \tau_b \ll \tau_{\text{cx}} \), which generally applies, the surface heat load is distributed evenly over the whole first wall.

7. APPLICATION TO TDF AND TASKA-M

The model described in the earlier sections was originally developed for the conceptual tandem mirror engineering test facility designs TDF [1] and TASKA-M [2]. Since the neutral beam parameters were given by requirements for sustaining the core plasma, the only "knob" to turn in choosing reference cases was the base pressure, which gives the neutral gas density at the first wall. Also, as shown in TASKA-M, it is convenient to have the halo do all of the vacuum pumping in the central cell region, so essentially no freedom in
picking parameters existed. In TDF, central cell vacuum pumping was done by separate pumps, leading to a somewhat different operating regime for the halo.

The main constraint on the choice of a reference case is that the halo must shield the core plasma. It is also generally desirable to keep the surface heat load as low as possible and to minimize attenuation of the neutral beams. The pertinent TDF and TASKA-M input parameters are given in Table III. Reference cases applicable to those studies are given in Tables IV and V. The actual cases given in the TDF and TASKA-M reports differ somewhat from the present cases due to refinements in the model. Table IV contains general parameters, while Table V gives particle and power balance parameters. Note that the final TASKA-M beam dumps used copper instead of molybdenum as used in the halo analysis, but such a change has little effect on the final results.

TDF halo density and temperature as a function of base pressure are shown in Fig. 5. The choice of operating point at a base pressure of $5 \times 10^{-5}$ torr was motivated by the desire to keep vacuum pumping to a minimum while simultaneously minimizing neutral beam attenuation.

In TASKA-M, calculations [2] showed that the halo acted as a very efficient vacuum pump, so no extra central cell pumps were used. The base pressure value chosen here is $1 \times 10^{-5}$ torr. TASKA-M halo density and temperature as a function of base pressure are shown in Fig. 6.

TASKA-M will be used to illustrate the general parametric dependencies for this class of devices: For cases with base pressures, $P_b$, below about $5 \times 10^{-5}$ torr, the halo is in the collisionless regime. The input power is mainly due to alpha particle heating of the halo and is approximately constant. Since the particle source to the halo is primarily due to ionization
and charge exchange (with subsequent ionization) of neutral gas, it is roughly proportional to the base pressure. Constant input power and a lower particle source combine to give a rising temperature as base pressure falls. The approximately constant density in this regime is due to the strong increase in confinement time with temperature. When $P_b$ is greater than $5 \times 10^{-5}$ torr, the halo approaches the collisional regime, and the confinement time is almost independent of density. Thus, since the particle source is rising with base pressure, the density rises. Also, attenuation of the neutral beams becomes a significant contributor to power balance, with power to the halo rising with density. Because both power and particle sources are rising approximately linearly with density, the halo temperature remains essentially constant.

The most important conclusion to be drawn from Figs. 5 and 6, however, is that considerable leeway exists in the halo operating regimes. If, for example, the uncertainty in cold gas sources leads to a reference base pressure as much as a factor of six higher than that used in TASKA-M or a factor of three lower than in TDF, the shielding properties of the halo remain essentially unchanged. Thus, we may have good confidence that the halo will shield the core plasma and vacuum pump central cell cold gas, despite the rough nature of the model.

8. SUMMARY

A model for the halo plasma in mirror machines sustained by neutral beam injection has been developed and applied to the conceptual tandem mirror test facilities TDF and TASKA-M. The resulting halo parameters are reasonable in an intuitive sense. This model suffices to define halo parameters adequate for the design of a fusion test facility. However, presently operating tandem mirror experiments have relatively large amounts of impurities in the neutral
beams, have no alpha particle heating, and are often dominated by neutral gas effects [8], so the model will not apply without substantial modification.

Interesting aspects of the model are that alpha particles can provide substantial halo heating and that the neutral beams have little effect on the halo. However, due to absorption by the halo and subsequent charge exchange on neutral gas, the neutral beams contribute greatly to the surface heat load.

Simple models for alpha particle heating of the halo and for reionization of neutrals in the halo were also presented.
APPENDIX: BEAM DUMP SPUTTERING ANALYSIS

The amount of sputtering due to shine-through neutral beam atoms impacting the beam dumps is estimated using the sputtering yield analysis of Bohdansky [3,4]. The number of sputtered atoms which reaches the core plasma may then be calculated by geometric arguments. Both calculations are rough, but they suffice since shielding requirements will be defined by the pessimistic assumption that all sputtered atoms reaching the core are absorbed by the core.

The sputtering calculations require the flux of neutral beam particles hitting the beam dumps, the energy of those particles, and the geometry of the configuration. The sputtering yield for normal incidence on a given substance will be designated $Y_0$. Sputtering yield depends on the incident beam angle as [3]

$$Y(\phi) = Y(0) \cos^{-f} \phi$$

(A.1)

where $f$ ranges from one to two. $f$ is one when incident atoms penetrate so deeply that the resulting, scattered atoms have randomized direction. $f = 1$ will be used here. Beam dumps are generally inclined at shallow angles to the neutral beams in order to minimize the surface heat load, so $\phi$ is large. Sputtering yield is proportional to $E^{-2}$ and peaks in energy at about $E = 2E_D/3$, where $E_D$ is the surface binding energy [3]. For very high energy incident particles (~ 90 keV), the sputtered atoms have a $\cos \theta \cos \zeta$ distribution, where $\theta$ and $\zeta$ are 0° normal to the surface. Thus, the distribution in energy and angle of sputtered atoms at a given location may be reasonably ap-
proximated if the neutral beam radial profile and absorption rate in the core plasma are known. The quantity of sputtered material which reaches the core plasma may then be estimated by simple geometric and halo absorption arguments. The probability that a sputtered atom with energy \( E \) penetrates the halo is assumed to be \( \exp(-\lambda) \), where \( \lambda \) is the ratio of path length in the halo to mean free path, given by

\[
\lambda = 7.15 \times 10^{-7} \frac{n_h \langle \sigma v \rangle_i \mu^{1/2}}{E^{1/2}},
\]  

(A.2)

where \( \mu \) is the ratio of the beam dump atomic mass to the proton mass, \( \ell \) is the path length, \( n_h \) is the halo density, and \( \langle \sigma v \rangle_i \) is the ionization rate. For TASKA-M and TDF,

\[
\langle \sigma v \rangle_i = \frac{10^{-5} (T_e/I)^{1/2}}{I^{3/2} (6 + T_e/I)} \exp \left( \frac{-I}{T_e} \right)
\]  

(A.3)

was used [7], where \( T_e \) is the halo electron temperature, \( I \) is the ionization potential for the beam dump atoms, and the reaction rate has been averaged over Maxwellian electrons.

The number of sputtered atoms per second which reaches the core plasma is therefore given by

\[
\dot{N}_S = \dot{N}_{bd} Y \cdot 4E_b^2 \int d\Omega \int dE \frac{E \cos \theta \cos \zeta}{E_b} e^{-\lambda}
\]  

(A.4)

where \( \dot{N}_{bd} \) is the particle source to the beam dump, assuming an appropriate average over a beam localized on its axis, and \( \int d\Omega \) is over the solid angle which sees the core plasma.
Since beam dump atoms are high-Z material and sputtered atoms hit the core plasma inside the central cell potential dip, the end loss time is very long. Classical radial transport, therefore, sets the loss rate. The radial loss time is approximately

$$\tau_r \sim \frac{r_c^2}{2D_{\perp}}$$  \hspace{1cm} (A.5)$$

where

$$D_{\perp} \sim v_{\perp} \rho_{\perp}^2 \sim 2.3 \times 10^{-11} \frac{n_{HC}}{E_{HC}^{1/2}}$$  \hspace{1cm} (A.6)$$

is the classical radial diffusion coefficient, $r_c$ is the central cell radius, $v_{\perp}$ is the 90° scattering time, $\rho_{\perp}$ is the gyroradius, $n_{HC}$ is the core hot ion density, and $E_{HC}$ is the core hot ion energy. The radial loss rate is thus

$$\dot{N}_r \sim f_{rpl} \frac{n_h}{\tau_r} V_c$$  \hspace{1cm} (A.7)$$

where $f_{rpl}$ is a radial profile factor and $V_c$ is the effective central cell volume.

Particle balance, given by

$$\dot{N}_r = \dot{N}_s,$$  \hspace{1cm} (A.8)$$

then sets the equilibrium density, $n_s$, of sputtered impurities in the core plasma. Note, however, that most of the sputtered atoms which reach the core plasma edge will be quickly ionized and lost out the ends, since the axial potentials are expected to be small at large radii. This analysis, therefore, gives an overestimate of the number of atoms which actually become trapped in
the core plasma. Since the energy of the core plasma is high enough to almost completely strip the sputtered atoms [6], the fusion power reduction factor is

\[ F_s = 1 - \left(1 - \frac{Z_sn_s}{n_c}\right)^2 \] (A.9)

where \( Z_s \) is the resulting ions' charge.
Acknowledgements

I am pleased to acknowledge helpful conversations with J.E. Osher, G.A. Emmert and T.C. Simonen. The author also wishes to thank a referee for suggesting the importance of charge exchange to Eq. (30). This research was supported by U.S. DOE contracts DE-AC02-82ER54013 and DE-AS02-78ET52048, and by contract 130/DI/597092 with Kernforschungszentrum, Karlsruhe, FRG.

References


Table I. Cases for the Reionization of Charge Exchanged Neutrals Model

<table>
<thead>
<tr>
<th>Case</th>
<th>Range of $\lambda_{mfp}$</th>
<th>Fraction Ionized</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\lambda_{mfp} &lt; r_h - r_{cx}$</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>$r_h - r_{cx} &lt; \lambda_{mfp} &lt; r_h + r_{cx}$</td>
<td>$\frac{1}{2} \left( 1 + \frac{r_h - r_{cx}}{\lambda_{mfp}} \right)$</td>
</tr>
<tr>
<td>3</td>
<td>$r_h + r_{cx} &lt; \lambda_{mfp}$</td>
<td>$\frac{r_h}{\lambda_{mfp}}$</td>
</tr>
</tbody>
</table>

Table II. Cases for the Halo Heating by Alpha Particles Model

A. $r_c > 2 \rho_\alpha$

<table>
<thead>
<tr>
<th>Case</th>
<th>Range of $r$</th>
<th>$f_\alpha(r)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>$0 &lt; r &lt; r_c - 2 \rho_\alpha$</td>
<td>0</td>
</tr>
<tr>
<td>A2</td>
<td>$r_c - 2 \rho_\alpha &lt; r &lt; r_c$</td>
<td>$\frac{1}{2} \left( 1 - \frac{r_c - r}{2 \rho_\alpha} \right)$</td>
</tr>
</tbody>
</table>

B. $r_c < 2 \rho_\alpha$

<table>
<thead>
<tr>
<th>Case</th>
<th>Range of $r$</th>
<th>$f_\alpha(r)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>$0 &lt; r &lt; 2 \rho_\alpha - r_c$</td>
<td>$\frac{r_c}{2 \rho_\alpha}$</td>
</tr>
<tr>
<td>B2</td>
<td>$2 \rho_\alpha - r_c &lt; r &lt; r_c$</td>
<td>$\frac{1}{2} \left( 1 - \frac{r_c - r}{2 \rho_\alpha} \right)$</td>
</tr>
</tbody>
</table>
Table III. Pertinent TDF and TASKA-M Input Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>TDF</th>
<th>TASKA-M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutral beam parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total current, A</td>
<td>1080</td>
<td>233</td>
</tr>
<tr>
<td>Average particle energy, keV</td>
<td>60</td>
<td>90</td>
</tr>
<tr>
<td>Injection angle, degrees</td>
<td>65</td>
<td>45</td>
</tr>
<tr>
<td>Trapping fraction ($f_{tr}$)</td>
<td>0.79</td>
<td>0.92</td>
</tr>
</tbody>
</table>

| Beam dump parameters                            |      |         |
| Surface material                                | Mo   | Mo*     |
| Angle of dumps to beam (off normal), degrees   | 69   | 75      |

*Cu was used in the final engineering design.
Table IV. Halo Reference Case General Parameters for TDF and TASKA-M

<table>
<thead>
<tr>
<th>Parameter</th>
<th>TDF</th>
<th>TASKA-M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, cm$^{-3}$</td>
<td>$2.8 \times 10^{12}$</td>
<td>$3.3 \times 10^{12}$</td>
</tr>
<tr>
<td>Temperature, eV</td>
<td>276</td>
<td>660</td>
</tr>
<tr>
<td>Radii, cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Core</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Halo</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>Wall</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>First wall surface heat, W/cm$^2$</td>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>Neutral gas</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base pressure at wall, torr</td>
<td>$5.0 \times 10^{-5}$</td>
<td>$1.0 \times 10^{-5}$</td>
</tr>
<tr>
<td>Density at first wall, cm$^{-3}$</td>
<td>$1.7 \times 10^{12}$</td>
<td>$3.3 \times 10^{11}$</td>
</tr>
<tr>
<td>Neutral beam atoms absorbed in halo</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Charge exchange time, s</td>
<td>$3.9 \times 10^{-4}$</td>
<td>$2.7 \times 10^{-3}$</td>
</tr>
<tr>
<td>Drag time, s</td>
<td>$4.9 \times 10^{-2}$</td>
<td>0.40</td>
</tr>
<tr>
<td>Bounce time, s</td>
<td>$3.7 \times 10^{-6}$</td>
<td>$2.7 \times 10^{-6}$</td>
</tr>
<tr>
<td>Reionization of charge exchanged neutrals</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reionization fraction</td>
<td>0.14</td>
<td>0.068</td>
</tr>
<tr>
<td>Mean free path, cm</td>
<td>121</td>
<td>265</td>
</tr>
</tbody>
</table>
Table V. Halo Reference Case Particle and Power Balance Parameters for TDF and TASKA-M

<table>
<thead>
<tr>
<th>Parameter</th>
<th>TDF</th>
<th>TASKA-M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confinement parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>End loss $n_e$, $\text{cm}^{-3}\text{s}$</td>
<td>$6.1 \times 10^8$</td>
<td>$5.1 \times 10^9$</td>
</tr>
<tr>
<td>Particle gain, $s^{-1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radial transport, $\dot{N}_{ro}$</td>
<td>$2.4 \times 10^{20}$</td>
<td>$2.0 \times 10^{19}$</td>
</tr>
<tr>
<td>Neutral beam ionization, $\dot{N}_{nbi}$</td>
<td>$4.8 \times 10^{19}$</td>
<td>$8.1 \times 10^{18}$</td>
</tr>
<tr>
<td>Neutral gas ionization, $\dot{N}_{ig}$</td>
<td>$4.3 \times 10^{21}$</td>
<td>$4.7 \times 10^{20}$</td>
</tr>
<tr>
<td>Reionization of CX neutrals, $\dot{N}_{icx}$</td>
<td>$1.3 \times 10^{21}$</td>
<td>$9.5 \times 10^{19}$</td>
</tr>
<tr>
<td>Particle loss, $s^{-1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>End loss, $\dot{N}_{el}$</td>
<td>$5.9 \times 10^{21}$</td>
<td>$5.9 \times 10^{20}$</td>
</tr>
<tr>
<td>Power gain, kW</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radial transport, $P_{ro}$</td>
<td>0.2</td>
<td>0.02</td>
</tr>
<tr>
<td>Alpha particles, $P_{\alpha}$</td>
<td>484</td>
<td>296</td>
</tr>
<tr>
<td>Reionization of CX neutrals, $P_{icx}$</td>
<td>37</td>
<td>14</td>
</tr>
<tr>
<td>Neutral beams, $P_{nb}$</td>
<td>6</td>
<td>0.4</td>
</tr>
<tr>
<td>Power loss, kW</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neutral gas CX, $P_{cxg}$</td>
<td>322</td>
<td>218</td>
</tr>
<tr>
<td>End loss, $P_{el}$</td>
<td>206</td>
<td>93</td>
</tr>
<tr>
<td>Neutral beam attenuation in halo, %</td>
<td>1.6</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Fig. 1. Model geometry used for the halo calculations. The plasma is modeled as concentric cylinders.
Fig. 2. Attenuation of neutral gas vs. line density ($\int n_h \, dl$).
Fig. 3. a) Geometry for the reionization model.
b) Possible cases.
Fig. 4. Possible cases for halo heating by alpha particles.
Fig. 5. TDF halo density and temperature vs. base pressure.
Fig. 6. TASKA-M halo density and temperature vs. base pressure.