Numerical Simulations of ICRF Heated Tokamak Plasmas

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by

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ABSTRACT

NUMERICAL SIMULATIONS OF ICRF HEATED TOKAMAK PLASMAS

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Under the supervision of Professor J.E. Scharer

Several computer models are used to examine ICRF heating in PLT and the conceptual tokamak reactor NUWMAK. A 0-D transport code which uses trapped particle scaling finds that \( \sim 240 \text{ MJ} \) of RF energy is required to ignite the 660 MW NUWMAK reactor.

The complete multi-species 3 x 3 hot plasma dispersion tensor, accurate for any temperature, for ion cyclotron harmonics up to five is derived. The dispersion relation which is obtained is then coupled to a Fokker-Planck code and a 1-D transport code.

Using a 2-D in velocity space, time-dependent Fokker-Planck code, in which charge exchange, radial losses, radiation and Ohmic heating are included, we find that over 90% of the RF power is absorbed through fundamental minority damping by the protons (\( n_{H}/n_{e} \sim 0.1 \)). The spatially-averaged deuterium temperature doubles while the protons become nonMaxwellian with a high energy tail of 18 keV. In NUWMAK, both deuterium and tritium remain Maxwellian with \( T_{D} \approx T_{e} \). There is little ICRF Q enhancement when fundamental or second harmonic damping is the
dominant heating process.

A WKB slab model is coupled to a 1-D time-dependent transport code to investigate the spatial aspects of ICRF heating. In PLT, over 90% of the RF power is absorbed near the center by the protons, over a heating width of 12-14 cm. In both PLT and NUWMAK, a plane fast wave launched from the low field side is almost completely absorbed before the mode conversion layer is reached. The occurrence of mode conversion, near the ion cyclotron resonance layer, does not alter the density and temperature profiles. Due to the high reactor densities and temperatures, the dominant heating processes in the ICRF startup of NUWMAK are electron Landau and transit time damping when the RF frequency $f = n f_{\text{CD}}$ near the plasma center with $n = 1, 2, 3$. Using "PLT empirical" scaling, 150-300 MJ of RF energy is needed to ignite NUWMAK.

For PLT, both the Fokker-Planck and space-time simulations agree with the experiment. For NUWMAK, direct electron heating dominates and no high energy ion tails are produced. In both devices the RF power is deposited close to the plasma core. Therefore, ICRF heating appears to be an attractive auxiliary heating scheme for tokamaks.
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CHAPTER 1
INTRODUCTION

1.A. Overview

Auxiliary heating should be required in a tokamak reactor since Ohmic heating alone cannot supply enough energy for a plasma to reach ignition. Auxiliary heating is now used to investigate the effects of a higher plasma temperature than that provided by Ohmic heating alone on plasma transport and confinement properties.

There are two general types of auxiliary heating currently being investigated, neutral beams and RF heating. Studies comparing the advantages and disadvantages of these two types of heating have been done by Stix\(^{(1-2)}\), Scharer, Conn, and Blackfield\(^{(3)}\), and others\(^{(4-6)}\).

Under the category of RF heating, several frequency regimes have been proposed\(^{(4-10)}\). From the standpoint of both experimental results and the availability of power sources, the Ion Cyclotron Range of Frequencies (ICRF) heating seems most promising. In this thesis, we investigate fundamental minority, second harmonic, and higher ion harmonic fast wave heating.

We shall briefly review recent experimental results in the next section. In Chapter 2, we review the 0-D, or point model fluid description of second and third harmonic heating in a reactor. In Chapter 3, we derive the complete \(3 \times 3\) hot plasma dispersion tensor which forms the basis of the Fokker-Planck and 1-D slab model.
calculations. In Chapter 4, we examine both PLT and NUWMAK\(^{(11-12)}\)
for anisotropy and high energy tail formation due to the presence of
RF. Chapter 5 focuses on ICRF heating using a WKB slab model in
connection with a 1-D multi-species transport code. We look at
fundamental minority heating in PLT and NUWMAK and second and third
harmonic heating in NUWMAK. Results obtained from the three codes,
0-D and 1-D fluid and 2-D velocity space models, are compared in
Chapter 6. Finally, in Chapter 7 we discuss the areas of theory and
computational analysis which need more development.

1.B. Recent ICRF Experimental Results

Some of the earliest experiments involving ICRF heating in a
tokamak were carried out on the ST machine at Princeton\(^{(13-17)}\) from
1972-1974. Further experiments were performed on the ATC\(^{(10-21)}\) and
PLT\(^{(22-25)}\) tokamaks, also at Princeton. ICRF heating is currently
being studied on MICROTOR and MACROTOR\(^{(26)}\) at UCLA and on a tokamak
at Cal Tech\(^{(27)}\).

In France, extensive investigation of ICRF heating is being
done on TFR\(^{(28-31)}\). The Russians have studied ICRF in T0-1, TM-Vch,
and T-4\(^{(22-40)}\). The ERASMUS tokamak\(^{(41)}\) in Belgium also uses ICRF.
Finally, the DIVA tokamak\(^{(42-43)}\) in Japan has ICRF heating.
A review of ICRF heating experiments prior to 1979 on these tokamaks
may be found in several sources\(^{(4-6,44)}\).

Since 1978, significant experimental results have been ob-
tained on DIVA, T-4, TFR and PLT. In DIVA\(^{(43)}\), second harmonic
(f = 2f_{c_D}) ICRF heating experiments have been performed on a deuterium plasma with a variable proton minority concentration ($5\% \leq \frac{n_H}{n_D} \leq 40\%$). The plasma in this machine is quite clean, having $Z \sim 1$.

Without electrostatic shielding and a balanced drive antenna, the overall ion heating efficiency was 40\% at most. The remaining power was coupled to the plasma edge. When the antenna was electrostatically shielded from the plasma by a Faraday shield and driven in a balanced RF mode, the ion heating efficiency approached 100\%. Net RF power up to 100 kW was coupled to the plasma, increasing the ion temperature from 160 eV to 450 eV. The dominant ion energy loss mechanisms appeared to be through neoclassical thermal conduction and electron-ion Coulomb collisions. An RF power density up to $1.4 \text{ W/cm}^3$ was achieved without adversely affecting plasma confinement.

On T¼(40), good agreement has been obtained between the experimentally measured hydrogen minority velocity distribution and Stix's quasilinear theory(45). In addition, with $B_o = 15$ kG, $\bar{n}_e = 1.5 \times 10^{13} \text{ cm}^{-3}$ and $P_{RF} = 72$ kW, the deuterium or majority species' temperature rose from 150 eV to 260 eV. The $m = 0$ mode was excited with RF frequency, $f = 23$ MHz. Since the minority concentration was low ($n_H/n_D \sim 2.3\%$), the dominant heating mechanism appeared to be fundamental minority heating of the protons. The deuterons were heated through Coulomb collisions with the high energy proton tail. Using Stix's notation, $Z = 2.2$, $\zeta = 12.8$ and $\langle p \rangle \sim 3.5 \times 10^{-2} \text{ W/cm}^3$ appeared to match the measured proton
distribution \( f(v) \). Since \( \langle p \rangle \) is low, this implies that there may be substantial energy losses by RF heated protons escaping through loss cones.

In TFR(31), the hydrogen concentration in the deuterium plasma was varied from 3% to 45% to investigate the effect of varying minority concentrations on second harmonic ICRF heating. High density \( (n_e(0) = 1.2 - 1.4 \times 10^{14} \text{ cm}^3) \) and low density \( (n_e(0) = 5 \times 10^{13} \text{ cm}^3) \) experiments were performed for RF power levels up to 250 kW with pulse durations from 30 to 60 ms and \( f \sim 60-61 \text{ MHz} \) \( (B_o = 41 \text{ kG} - 42 \text{ kG}) \).

In the high density experiments, \( \sim 85\% \) of the RF power delivered by the generator was coupled to the plasma. Using different power levels, the peak deuterium temperature increased linearly with power

\[
\Delta T_D(0) \sim \frac{0.7 \text{ eV/kW}}{P_{RF} \text{ kW}} \quad \text{or} \quad \Delta T_D(0) = \frac{5P_{RF} \text{ kW}}{<n_e> (10^{13} \text{ cm}^{-3})} \quad (1-1)
\]

where \( P_{RF} \) is the amount of RF power absorbed in the plasma. A high energy proton tail \( (T_{\text{eff}} \sim 2 \text{ keV}) \) was observed. No energy tail was seen in the deuterium distribution.

In the high density case, no significant heating efficiency difference was observed between the \( m = 0 \) and \( m = -1 \) modes. In the low density experiments \( (m=0) \) the heating efficiency was halved

\[
\Delta T_D(0) \sim \frac{2.5 P_{RF} \text{ kW}}{<n_e> (10^{13} \text{ cm}^{-3})} \cdot \quad (1-2)
\]
For high density operation $\tau_E \sim 20$ ms, while the low density $\tau_E$ is 8-10 ms. The shorter confinement time leads to greater energy transport losses, thereby reducing the overall heating efficiency.

In the above experiments, $n_H/n_D < 5\%$. In these cases, fundamental minority heating of the hydrogen species appears to be the dominant heating mechanism. By varying both the magnetic field and proton concentration it was shown that the radial position of the ion-ion hybrid layer determines, to a large extent, the radial position of the RF heating. Consequently, this influences the overall heating efficiency. An analysis of the heating mechanisms is currently underway.

For low hydrogen concentrations, the ion-ion hybrid resonance layer

\begin{equation}
\omega^2 = \frac{\chi_D f_D + \chi_H f_H}{\chi_D f_D + \chi_H f_H} \omega_{CD} \omega_{CH} \tag{1-3}
\end{equation}

where

\begin{align*}
\chi_D &\equiv n_D/n_e, \quad f_D = m_e/M_D \\
\chi_H &\equiv n_H/n_e, \quad f_H = m_e/M_H
\end{align*}

lies near the cyclotron layer, both being near the plasma center. In low proton concentration experiments the RF power was deposited in the center, resulting in high heating efficiencies. As the proton concentration is increased ($n_H/n_D \sim 15\%$), the ion-ion hybrid layer moves near the plasma edge, while the cyclotron layer stays near the
center. In this case the heating efficiency became comparable to the best result, when both resonance layers were near the center.

The TFR experiments have shown that power levels up to 250 kW can be coupled to a predominantly deuterium plasma using an all-metal loop. The average ion temperature was raised 150 eV. Finally, the position of the ion-ion hybrid resonance layer seems to play an important role in determining the overall heating efficiency.

Recent PLT experiments \(^{(24-25)}\), which will be analyzed in detail in Chapters 4-6, will now be briefly summarized. Initial low density, low power \((\bar{n}_e \sim 1-1.5 \times 10^{13} \text{ cm}^{-3}, P_{RF} = 30-35 \text{ kW})\) resulted in stronger than expected damping of the fast magnetosonic wave with \(\Delta T_D(0)\leq 80 \text{ eV}\). Though the deuterium spectra showed no apparent energetic tail, hydrogen charge exchange measurements showed strong proton \((n_H/n_D \sim 3\%)\) heating. The strong wave damping and hydrogen heating can again be explained from the presence of fundamental minority heating of the fast wave.

Within the past year, PLT has upgraded the amount of power coupled to 350 kW for an RF pulse length of 100 ms \(^{(25)}\). For higher densities \((\bar{n}_e \sim 3 \times 10^{13} \text{ cm}^{-3})\), the average ion temperature at the center increased 600 eV from 600 eV to 1200 eV. Although no deuteron high energy tail was seen, a high energy \((T_{eff} \sim 15 \text{ keV})\) proton tail was. The heating approached saturation to within 10% during the RF pulse indicating a \(\tau_{RF} \sim 40 \text{ ms}\).

When a minority concentration of \(^3\text{He}\) was released into the deuterium plasma, the overall heating efficiency doubled.
\( \Delta T_D(0) \sim 500 \text{ eV for } P_{RF} \sim 150 \text{ kW with } n_e \sim 2.1 \times 10^{13} \text{ cm}^{-3} \). By heating at the fundamental resonance frequency of \(^3\text{He}\), the magnetic field could be raised from 16.4 kG to 24.6 kG. Therefore, the plasma current could be raised from 400 kA to 500 kA, increasing the Ohmic power. The increased magnetic field leads to better plasma confinement, while the increased Ohmic heating and lowered charge exchange loss improves the overall heating efficiency. Therefore, the energetic \(^3\text{He}\) ions are confined better and can deposit more of their energy in the rest of the plasma.

Our goal is to develop a realistic model of ICRF heating using the fast magnetosonic wave which will predict the plasma behavior during the ICRF startup of a fusion reactor. To have confidence in the model, we should be able to simulate present day experiments. In Chapters 4-6, computer simulated PLT results will be compared with the most recent experimental findings. The next chapter contains a review of the initial NUWMAK reactor studies\(^{11-12}\) which used the O-D fluid time-dependent code, FOint Radiofrequency Transport (PORT).
CHAPTER 1 REFERENCES


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CHAPTER 2

REVIEW OF 0-D FLUID MODEL NUWMAK RESULTS

2.A. 0-D Model

There have been several reactor studies where neutral beams have been the main auxiliary heating system\(^{(1-7)}\). Recently RF heating, particularly ICRF, has become more attractive. The overall heating efficiencies of both systems are comparable. At Wisconsin, we have begun to examine reactor systems where the dominant heating source is ICRF\(^{(8-10)}\). The potential advantages of ICRF are utilized in the reactor designs while potential problems are brought out.

To design an RF heating system, an estimate of the amount of power and wave frequency is necessary. For the NUWMAK reactor study\(^{(9-10)}\) the Ppoint Radiofrequency heating Transport code, Pfort, provided this. Using the computer code Pfort, based on a spatially averaged, time dependent model\(^{(8-13)}\), the following electron and ion energy balance equations are solved

\[
\frac{3n_e}{3t} = \frac{4.28 \times 10^{-11}}{n_i(T_i - T_e)} + \frac{1}{1.5r} \frac{3}{3r} (r n_e \times e \frac{3T_e}{3r})
\]

\[-\frac{1}{r} \frac{3}{3r} (r n_e V e T_e) + 4.17 \times 10^{-15} \left\{ n_D n_T < \sigma v > U_{ae} \right\} \]

(2-1)
\[ \frac{\partial n_i}{\partial t} = -4.28 \times 10^{-11} \frac{n_e n_i (T_i - T_e)}{T_e^{3/2}} + \frac{1}{1.5r} \frac{\partial}{\partial r} \left( r n_i \chi_i \frac{\partial T_i}{\partial r} \right) \]

\[ - \frac{1}{r} \frac{\partial}{\partial r} \left( r n_i V_i T_i \right) + 4.17 \times 10^{15} \left\{ n_D n_T \langle \sigma V \rangle U_{ai} \right\} + \frac{p_i^{\text{RF}}}{r} \quad (2-2) \]

where \( T_{e(i)} \) is the electron (ion) temperature (eV) \( (T_D = T_T = T_i) \) due to the short ion-ion equilibration time); \( n_{e(i)} \) is the electron (ion) density \( (\text{cm}^{-3}) \) \( (n_i = n_D + n_T) \); \( V_{e(i)} \) is the electron (ion) fluid velocity \( (\text{cm}/\text{ms}) \); \( r \) is the plasma radius \( (\text{cm}) \); \( t \) is time \( (\text{ms}) \); \( \chi_{e(i)} \) is the electron (ion) thermal diffusivity \( (\text{cm}^2/\text{sec}) \); \( U_{ae(i)} \) is the fraction of alpha energy deposited in the electrons (ions); \( \bar{E} \) is the toroidal electric field \( (\text{V} \cdot \text{cm}^{-1}) \); \( \bar{J} \) is the toroidal current density \( (\text{A} \cdot \text{cm}^{-2}) \); \( P_B \) is the Bremsstrahlung loss term \( (W) \); \( P_S \) is the synchrotron radiation loss term \( (W) \); \( P_{\text{RF}}^{\text{e(i)}} \) is the electron (ion) RF heating term \( (W) \).

Equations \( 2-1 \) and \( 2-2 \) are obtained from the second moment of the Boltzmann equation \( (14-16) \) averaged over a magnetic flux surface (see Chapter 5). The heating terms due to the RF and the slowing down of alphas are added. The radiation loss terms are also added. Since \( Z_{\text{eff}} = 1 \) is assumed, there are no charge exchange or impact ionization loss terms. The ion-electron equilibration is calculated from classical theory while the Ohmic heating term uses
neoclassical theory.

The Bremsstrahlung radiation term may be written as \(^{(14,17)}\)

\[
P_B = 4.8 \times 10^{-31} Z_{\text{eff}} n_e^2 T_e^{1/2} \text{ (W/cm}^3\text{)}
\]  \(2-3\)

with \(T_e\) in keV. The synchrotron term is \(^{(18)}\)

\[
P_S = 1.55 \times 10^{-14}(1 - r_w)^{3/2} [5. + .17(5-A)]^3 \frac{n_e^2 B_o^{5/2}}{a^2 \chi} T_e^{2.1} \text{ (W/cm}^3\text{)}
\]  \(2-4\)

where \(A\) is the aspect ratio; \(r_w\) is the wall reflectivity; \(T_e\) is in keV; \(B_o\) is the toroidal B field in Tesla; \(a\) is the minor radius in cm.

PORT assumes that a density feedback control exists which keeps the total ion density constant, though the ratio \(n_D/n_T\) may vary in time. The electron density is solved from the charge neutrality condition.

The fusion alphas, produced at 3.52 MeV, are assumed to slow down instantaneously. The amount of energy deposited to each species as the alphas slow down is calculated using classical theory \(^{(14,19)}\).

Since PORT is spatially independent, the conduction and convection gradients are eliminated by assuming that

\[
\frac{2}{3} \frac{\partial}{\partial r} (\rho n V \times \frac{\partial T}{\partial r}) + \frac{n_T}{\tau_c}
\]

\(2-5\)

where \(\tau_c \equiv a^2/\chi\), and that
\[
\frac{1}{r} \frac{\partial}{\partial r} (rnVT) \rightarrow \frac{nT}{\tau_D}
\]  \hspace{1cm} (2-6)

where \( \tau_D \equiv a^2/D \).

\( \tau_c \) is a "characteristic" thermal diffusion time and \( \tau_D \) is a "characteristic" particle diffusion time. For the NUWMAK calculations, \( \chi \) and \( D \) were calculated using trapped particle scaling \((14,20)\).

To eliminate the spatial dependency in density and temperature, the following expressions for the average density and temperature are used

\[
\bar{n} = \frac{2}{\pi} \int_0^a n(r) r \, dr
\]  \hspace{1cm} (2-7)

and

\[
\bar{T} = \frac{\int_0^a n(r) T(r) \, r \, dr}{\int_0^a n(r) \, r \, dr}
\]  \hspace{1cm} (2-8)

where

\[
n(r) = 2n_0 \left(1 - \frac{r^2}{a^2}\right) + 0.01 \, n_0
\]

and

\[
T(r) = 2 \, T_0 \left(1 - \frac{r^2}{a^2}\right) + 0.01 \, T_0.
\]  \hspace{1cm} (2-9)

The dispersion relation used in PORT in the calculation of the RF heating terms is somewhat limited. Only \( \omega = 2 \, \omega_{CD} = 3 \omega_{CT} \) or \( \omega = 2 \, \omega_{CT} \) heating for the ions could be calculated. Landau and
transit time damping are the assumed heating processes for the electrons.

For the $\omega = 2 \omega_{CD} = 3 \omega_{CT}$ case, the ion RF heating term is (9-10,21)

$$p_{RF}^i = \sum_{l=2}^{3} \frac{|E_+|^2}{2 \omega_{cio} \omega_{ci}^2} \varepsilon_o \int R_{pl} \left[ \sum_{l=2}^{3} \frac{\omega_{pl}^2(r)}{\omega_{ci}^2(r)} \right] \frac{<k_i^2 > \nu_{thi}^2 (r)}{2 \omega_{ci}^2(r)} dr$$

(2-10)

where $l = 2$ for deuterium and 3 for tritium.

The electron RF heating term is (9-10,21)

$$p_{RF}^e = \frac{<k_i^2 > |E_+|^2 \varepsilon_o \pi \frac{a}{2}}{2 k_n a^2} \int \frac{\omega_{be}^2(r)}{\omega_{ce}^2(r)} \nu_{the}(r) (1 + x^2(r)) e^{-\frac{\omega_{cio}^2}{k_n^2} \nu_{the}^2} r dr$$

(2-11)

where

$$x^2(r) = \frac{3 \pi \frac{a}{2} <k_i^2 > \nu_{the}(r)}{8 k_n \omega_{ci}^2(r)}$$

(2-12)

and $<k_i^2>$ is $k_i(r)$ spatially averaged over the heating zone. For the ions, this heating width is over the cyclotron resonant layer (9-10,21)

$$\Delta R = \frac{2 k_n \nu_{thi} R}{\omega_{cio}}$$

(2-13)

For electron Landau and transit time damping, the heating zone is over the entire plasma cross section.
Assuming a spatially averaged RF electric field $E_+$

\[
P_{RF}^t = \text{PRFI}' |E_+|^2
\]

(2-14)

\[
P_{RF}^e = \text{PRFE}' |E_+|^2 .
\]

The normalization for $|E_+|$ is obtained by assuming that all of the RF power is coupled to the plasma. This assumption yields a minimum RF heating system requirement. $|E_+|$ is normalized so that

\[
|E_+|^2 = \frac{\text{Total RF Power assumed coupled to Plasma}}{\text{PRFI}' + \text{PRFE}'} .
\]

(2-15)

By assuming a spatially averaged value for $|E_+|$ and $k^2(r)$, we are implicitly assuming that the RF energy is deposited near the plasma center where $\omega \sim 2\omega_{CD} \sim 3\omega_{CT}$. Although this may be a good assumption in present day ICRF heating experiments, Chapter 5 shows that for reactors, this may not occur.

2.B. Numerical Results

To ignite the NUWMAK conceptual reactor (parameters given in Table 2-1), 175 MW of RF power for 1 sec. must be provided. The ion and electron time dependent temperature curves are shown in Fig. (2-1). Also plotted in Fig. (2-1) is the amount of alpha fusion power deposited in the electrons. The ignition point is reached on the steep rise in the curves. Once ignition is achieved, both the temperatures and alpha power production continue to
increase after the RF is turned off. An equilibrium between the alpha power and the energy transport is obtained near \( t = 1.5 \text{ sec} \), indicated by the levelling of the temperature curves.

The electron and ion energy balance terms are plotted in Figs. (2-2-3). Early in the startup phase \( T_i > T_e \) since most of the RF power heats the ions directly. Therefore, the ion-electron equilibration term is positive (heating) for the electrons and negative (cooling) for the ions. However, since a majority of the alpha power is deposited in the electrons, \( T_e \) becomes greater than \( T_i \) and the ion-electron equilibration term cools the electrons and heats the ions. This interchange in the dominant heating role between the RF and alpha power determines which temperature is higher \( (T_i > T_e \text{ when } P_{RF} > P_{\alpha}) \).

In Fig. (2-4) we see that over 98% of the RF power is deposited in the ions through second and third harmonic damping with second harmonic damping on the deuterons being dominant. Since the electron absorption term is small, only a small amount of power is deposited outside of the cyclotron resonance layer.

As the plasma temperature increases, slightly more electron Landau and transit-time damping occurs. In the small gyroradius expansion, the power deposited to the ions goes as

\[
P_{RF}^i \sim (k_i^2 V_i^2)^{l-1} \sim \lambda_i^{l-1}.
\]

Consequently, as \( T_i \) increases, more power flows to the tritium.
In Fig. (2-5) the magnitude of the spatially-averaged $|E_+|$ field is shown. As the plasma temperature increases, the conductivity increases. To keep the RF power constant, the electric field magnitude must drop. In Fig. (2-6), the deuterium and tritium resonance widths are plotted. Since $\Delta R \propto T_i^{3/2}$, the widths increase in time. Finally, the magnitudes of $\langle k_\perp \rangle \equiv (|\langle k_\perp^2 \rangle|)^{1/2}$ are shown in Fig. (2-7). Since $\Delta R$ for the deuterons does not equal $\Delta R$ for the tritons, $\langle k_\perp \rangle_D \neq \langle k_\perp \rangle_T$.

PORT is limited to two different ICRF harmonics, n=2 or 3. The effects of resonant alphas and impurities being present in the plasmas are not included. In addition, the small $\lambda$ or gyroradius expansion used may begin to break down as the plasma reaches ignition. The 3.52 MeV alphas cannot be included in a small gyroradius expansion since $\lambda_\alpha \propto 0(1)$. For these reasons, a general dispersion relation is derived in the next chapter. With this dispersion relation, the spatial effects which had been neglected will be examined in Chapter 5.
**Table 2-1**

**NUWMAK REACTOR PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a = 125 cm</td>
<td>( n_e = 1.95 \times 10^{14} \text{ cm}^{-3} )</td>
</tr>
<tr>
<td>R = 500 cm</td>
<td>( n_e = n_T = 0.975 \times 10^{14} \text{ cm}^{-3} )</td>
</tr>
<tr>
<td>A = 4</td>
<td>( \frac{n_{\text{edge}}}{n_{\text{max}}} = \frac{T_{\text{edge}}}{T_{\text{max}}} = .01 )</td>
</tr>
<tr>
<td>S = 1.33</td>
<td>( T_i = T_e = 1.0 \text{ keV} )</td>
</tr>
<tr>
<td>I = 6.48 MA</td>
<td>( f = 92 \text{ MHz} )</td>
</tr>
<tr>
<td>B_0 = 60 kG</td>
<td>( k_m = 0.1 \text{ cm}^{-1} )</td>
</tr>
<tr>
<td>P_{Thermal} &amp; 2300 MW</td>
<td>( k_\perp \sim 1.6 \text{ cm}^{-1} )</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

(2-1) \( T_e, T_i \) and alpha power deposited in the electrons for a 50\%D-50\%T NUWMAK plasma with \( P_{RF} = 175 \text{ MW} \).

(2-2) Electron energy balance heating (solid lines) and cooling (dashed lines) terms.

(2-3) Ion energy balance heating (solid lines) and cooling (dashed lines) terms.

(2-4) RF power absorbed by deuterium, tritium, electrons and ions \( (P_{RF}^I = P_{RF}^D + P_{RF}^T) \).

(2-5) Spatially-averaged \( |E_+| \) RF electric field with \( P_{RF} = 175 \text{ MW} \).

(2-6) Time dependent variation in ion cyclotron resonance heating width.

(2-7) \( R_{\perp}(r) \) averaged over the resonance heating width for the ions and over the entire cross section for the electrons.
Fig. (2-1)

Fig. (2-2)

Fig. (2-3)
CHAPTER 2 REFERENCES

1. B. Badger, UWFDM-68 (March, 1974).


CHAPTER 3

HOT PLASMA FAST WAVE DISPERSION RELATION

3.A. Hot Plasma Dispersion Tensor

In cold plasma theory, both the fast and slow ion cyclotron waves may propagate parallel to an externally applied magnetic field. From space and access considerations of the RF launching structure, the wave should be launched from the low field edge of the plasma (see Fig. (3-1)). Any wave generated in a tokamak must propagate across magnetic field lines and up a density gradient. In a tokamak a slow wave, when propagating from the high field edge towards the plasma center, must travel up a density gradient. Near the inside plasma edge, the perpendicular cyclotron resonance is reached. Here the slow wave mode may convert to an electrostatic ion cyclotron or Ion Bernstein wave which propagates back towards the high field edge\(^1\). The electrostatic wave is strongly damped by electron Landau and transit time processes. The resulting surface heating may lead to enhanced radiation and particle losses. A slow wave launched from the low field side is evanescent and cannot penetrate into the plasma. Therefore, the slow wave does not appear to be an attractive mode for core heating in a tokamak.

The fast wave can propagate across magnetic field lines. In a cold plasma with a single ion species, the fast wave near the ion
Fig. (3-1) Cross-sectional view of the conceptual NUWMAK reactor.
cyclotron resonance layer is right-hand circularly polarized. Since the ions rotate about field lines in the opposite sense, little ion heating would be expected to occur. However, the presence of two ion species, together with hot plasma effects, produces an enhancement of the left-hand component of the RF electric field. For fundamental and second ion harmonic heating with several ion species, $|E_+|/|E_-| \sim O(1)$ (see Chapters 4-5).

The dispersion relation determines the region of wave propagation, evanescence, mode coupling and ultimately the RF power deposition. To calculate a general, multi-species, hot plasma dispersion relation, a local approximation is used. This implies that each point in the plasma receives no information from adjacent spatial regions. Density, temperature and magnetic field gradients are ignored. The effects of density and temperature gradients have been examined by Mau$^{(2)}$ and found to be small. Therefore, we assume that each point represents an isotropic unbounded plasma with density $n(r_o)$, temperature $T(r_o)$, and magnetic field $B(r_o)$. We also assume a slab model geometry with plane RF electric fields of the form

$$E = E_0 e^{i(k_\perp \cdot \hat{x} + k_y \cdot \hat{z}) - \omega t}$$

with $k_y = 0$; $\hat{z}$ is the direction along $B_0$; $\hat{x}$ is along the minor radius at the mid-plane.
Using the Vlasov–Maxwell equations and integration along unperturbed particle orbits, we obtain the following $3 \times 3$ generalized hot plasma dispersion tensor for slab geometry\(^{(2-5)}\)

\[
\begin{bmatrix}
K_{xx} - n_{\parallel}^2 & K_{xy} & K_{xz} + n_{\parallel} n_{\perp} \\
K_{yx} & K_{yy} - n^2 & K_{yz} \\
K_{zx} + n_{\parallel} n_{\perp} & K_{zy} & K_{zz} - n_{\perp}^2
\end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0
\]

(3-2)

where $\bar{\kappa} = \frac{c^2}{\omega^2} \bar{\epsilon}; n = \frac{c \bar{\kappa}}{\omega}$; and $\bar{\epsilon}$ is the dielectric tensor for a uniform, unbounded plasma

\[
\frac{c^2}{\omega^2} (\bar{k} \times (\bar{k} \times \bar{E}) + \bar{\epsilon} \cdot \bar{E}) = 0
\]

(3-3)

and

\[
K_{xx} = 1 + \sum_{\alpha} \frac{\omega_{\alpha}^2}{2 \omega_{\alpha}} e^{-\lambda_{\alpha}} \sum \frac{n^2 I_n}{\lambda_{\alpha}} \theta_n^\alpha
\]

\[
K_{yy} = 1 + \sum_{\alpha} \frac{\omega_{\alpha}^2}{2 \omega_{\alpha}} e^{-\lambda_{\alpha}} \sum \frac{n^2 I_n}{\lambda_{\alpha}} \theta_n^\alpha - \frac{2 k^2}{\lambda_{\alpha}} \frac{2 \theta_n^\alpha}{I_n} \lambda_{\alpha} (I_n - I_n) \theta_n^2
\]
\[ K_{zz} = 1 - \sum_{\alpha} \frac{\omega^2}{\omega k_{n} \theta_{\alpha}} e^{-\lambda_{\alpha} \sum_{n} \xi_{n}^{\alpha} \phi_{n}^{\alpha}} \]

\[ K_{xy} = -K_{yx} = -i \sum_{\alpha} \frac{\omega^2}{2\omega k_{n}} e^{-\lambda_{\alpha} \sum_{n} (I_{n}^{n} - I_{n}^{n'}) \theta_{n}^{\alpha}} \]

\[ K_{xz} = \frac{\omega^2}{\lambda_{\alpha} k_{n} \omega_{ca}} \sum_{n} k_{x} \frac{nI_{n}^{n} \phi_{n}^{\alpha}}{\lambda_{\alpha}} = K_{zx} \]

\[ K_{yz} = -K_{zy} = \sum_{\alpha} \frac{\omega^2}{\omega k_{n} \omega_{ca}} e^{-\lambda_{\alpha} \sum_{n} k_{x} (I_{n}^{n'} - I_{n}^{n}) \phi_{n}^{\alpha}} \quad (3-4) \]

where \( \theta_{n}^{\alpha} = \frac{2}{V_{\alpha}} \theta_{\alpha} \)

\[ \phi_{n}^{\alpha} = Z_{\alpha} (\xi_{n}^{\alpha}) \]

\[ \xi_{n}^{\alpha} = \frac{\omega - n\omega_{ca}}{k_{n} V_{\alpha}} \theta_{\alpha} \]

\[ \lambda_{\alpha} = \frac{1}{2} \frac{k_{1} V_{\alpha}^2}{\omega_{ca}} = \frac{1}{2} \frac{k_{1} \rho_{\alpha}^2}{\omega_{ca}} \]

\[ I_{n}^{\alpha} = \frac{I_{n}^{\alpha}}{I_{n}^{\alpha}} (\lambda_{\alpha}) \]

3.B. Fast Wave Dispersion Relation

Previously, a small argument expansion in \( \lambda \ll 1 \) was used to obtain the fast wave dispersion relation. In Eqs. (3-4), both
the Bessel functions and the exponentials involving $\lambda$ are expanded in a power series. For example

$$K_{XX} + A_0 + A_1 k^2 + A_2 k^4 + A_3 k^6 + \ldots$$

Equation (3-2) has a unique solution when

$$\det[D] = 0.$$  \hspace{1cm} (3-6)

A dispersion polynomial in powers of $k^2$ is obtained. If the dispersion polynomial is inverse Fourier transformed, we obtain a differential equation of the following form

$$D_0 + D_1 k^2 + D_2 k^4 + D_3 k^6 + \ldots = 0$$  \hspace{1cm} (3-7)

$$D_0' + D_1 \frac{\partial^2}{\partial x^2} + D_2 \frac{\partial^4}{\partial x^4} + D_3 \frac{\partial^6}{\partial x^6} + \ldots = 0.$$  \hspace{1cm} (3-8)

Equation (3-8) describes the coupling between different propagating modes within the plasma. This differential equation may be solved to obtain coupling coefficients between the fast and Ion Bernstein branches at the ion cyclotron and ion-ion hybrid resonance layers (See Chapter 5 and Refs. (1, 6-19)).

Equation (3-7) and hence Eq. (3-8) breaks down as $\lambda$ approaches unity. For 3.52 MeV alphas in NUWMAK
\[ \lambda_\alpha \propto \frac{5.2 \times 10^3 T_i}{B^2} \]  
\( (T_i \text{ in eV}) \)  
\( (B \text{ in Gauss}) \)  
\( k_\perp \sim 1 \text{ cm}^{-1} \)  
\( (3-9) \)

\[ \lambda_\alpha \propto 5 > 1 . \]  
\( (3-10) \)

For 30 keV deuterons in NUWMAK

\[ \lambda_D \propto \frac{1.04 \times 10^4 T_i}{B^2} \]  
\( (T_i \text{ in eV}) \)  
\( (B \text{ in Gauss}) \)  
\( k_\perp \sim 1 \text{ cm}^{-1} \)  
\( (3-11) \)

\[ \lambda_D \propto .1. \]  
\( (3-12) \)

If we want to investigate the effects of alphas and hot ions on the RF power absorption and deposition, the small \( \lambda \) expansion should not be used.

Therefore, Eq. (3-6) is solved numerically using the full 3 x 3 hot plasma dispersion tensor (Eq. (3-4)). The dispersion relation includes electrons, deuterons, tritons, alphas and carbon in NUWMAK, and electrons, deuterons, protons and oxygen in PLT calculations. In the Fokker-Planck code FORCE (see Chapter 4), the dispersion relation finds \( k_\perp \) at a single point \( (r = 0) \). In the space-time codes STICR-I and STICR-II (see Chapter 5), a local \( k_\perp(r) \) is calculated at 117 points across the entire plasma cross section.
In the dispersion relation, five harmonics are summed over 
\((n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5)\). Therefore, the RF frequency can 
be between 0 and \(4f_{ci}\) and \(k_\perp(r)\) is accurate at any temperature. 
Though each species may have a different temperature, we have 
assumed that each distribution is an isotropic Maxwellian.

To test the accuracy of the dispersion relation code given 
in Appendix A, we have examined the same reactor plasma parameters 
as Stix\(^{20}\). In Table 3-1, we find very close agreement with 
Stix's results. In the calculations, we have assumed a real RF 
frequency, \(\omega\), and parallel wavenumber, \(k_p\), determined by the 
launching structure. The perpendicular wavenumber, \(k_\perp\), obtained is 
complex. Stix neglects the complex part, which is good approxima-
tion in the case he examines since

\[
\frac{|\text{Im}k_\perp|}{|\text{Re}k_\perp|} \ll .1. \quad (3-13)
\]

Besides comparison with Stix's case, we compare the hot 
plasma \(k_\perp^2(r)\) with the cold plasma result\(^{1,21}\)

\[
\frac{8}{\pi} \frac{n_x^4}{n_x} - (S - n_x^2) \frac{n_x^2}{n_x} + (R - n_x^2) (L - n_x^2) = 0 \quad (3-14)
\]

where

\[
R = \sum_i \frac{\omega_{pi}^2}{\omega_{ci}(\omega + \omega_{ci})} \quad L = -\sum_i \frac{\omega_{pi}^2}{\omega_{ci}(\omega - \omega_{ci})} \quad (3-15)
\]
\( S = \frac{R + L}{2} = \frac{1}{\omega_{ci}} \sum_i \frac{\omega_i^2}{\omega^2 - \omega_i^2} \)

\[ n_x^2 = \frac{k_{\perp}^2 c^2}{\omega^2}, \quad n_z^2 = \frac{k_{||}^2 c^2}{\omega^2}. \tag{3-15} \]

Away from the resonance zone, we find \( n_z^2 \) \( \frac{(R - n_z^2)(L - n_z^2)}{S - n_z^2} \)

\[ n_{\perp}^2 \leq \frac{(R - n_z^2)(L - n_z^2)}{S - n_z^2} \tag{3-16} \]

where \( n_z^2 = \frac{k_{\perp}^2 c^2}{\omega^2} \). The \( S = n_z^2 \) appears to be a singular or resonant zone. However, near this point, the approximations used in deriving Eq. (3-16) break down and one should use Eq. (3-14).

\( R = n_z^2 \) and \( L = n_z^2 \) describe cold plasma cutoff surfaces. Hot plasma effects allow propagation across these surfaces. Chapter 5 shows that the hot plasma \( k_\perp \) is close to the cold plasma value except near the cyclotron resonance where cold plasma predicts a singularity.

In conclusion, we have seen that the hot plasma dispersion relation, which is numerically solved, produces values of \( k_\perp \) which compare well with published and cold plasma results. Having confidence in the accuracy, we next proceed to calculate the effect that ICRF has on the ion and electron velocity space distributions in both a present day experiment (PLT) and a conceptual reactor (NUWMAK).
### Table 3-1
Reactor Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_T$ (cm$^{-3}$)</td>
<td>$5 \times 10^{13}$</td>
</tr>
<tr>
<td>$T_i$ (keV)</td>
<td>4</td>
</tr>
<tr>
<td>$T_e$ (keV)</td>
<td>4</td>
</tr>
<tr>
<td>$B_o$ (kG)</td>
<td>40</td>
</tr>
<tr>
<td>$R$ (cm)</td>
<td>274</td>
</tr>
<tr>
<td>$a$ (cm)</td>
<td>60</td>
</tr>
<tr>
<td>$f$ (MHz)</td>
<td>$30.5 = f_{coD}$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>STIX</th>
<th>BLACKFIELD</th>
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<tbody>
<tr>
<td>$k_n$ (cm$^{-1}$)</td>
<td>.511</td>
<td>.500</td>
</tr>
<tr>
<td>$k_1^2$ (cm$^{-2}$)</td>
<td>.0642</td>
<td>(.0641, 1.06 $\times 10^{-4}$)</td>
</tr>
<tr>
<td>$k_{lcpl, \text{cold plasma}}^2$</td>
<td>.0644</td>
<td>.0644</td>
</tr>
<tr>
<td>$K_{xx}$</td>
<td>$-1.41 \times 10^3$</td>
<td>$(-1.403 \times 10^3, .513)$</td>
</tr>
<tr>
<td>$K_{yy}$</td>
<td>$-1.41 \times 10^3$</td>
<td>$(-1.406 \times 10^3, 5.515)$</td>
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<td>$K_{xy}$</td>
<td>$-2.12 \times 10^3$</td>
<td>$(-.725, -2.109 \times 10^3)$</td>
</tr>
<tr>
<td>$K_{yz}$</td>
<td>(3810, 445)</td>
<td>(3814, 577)</td>
</tr>
<tr>
<td>$K_{zz}$</td>
<td>($-6.65 \times 10^5, 5.6 \times 10^6$)</td>
<td>($-8.66x10^5, 5.77x10^6$)</td>
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<tr>
<td>$\Delta K_{yy}$</td>
<td>5.14</td>
<td>5.03</td>
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<tr>
<td>$\Delta K_{yz}$</td>
<td>3810</td>
<td>3814</td>
</tr>
<tr>
<td>$\Delta K_{zz}$</td>
<td>$5.65 \times 10^6$</td>
<td>$5.77 \times 10^6$</td>
</tr>
<tr>
<td>$</td>
<td>E_y/E_z</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>E_+/E_-</td>
<td>$</td>
</tr>
<tr>
<td>$E_+/1E_y$</td>
<td>$.218$</td>
<td>$.220$</td>
</tr>
<tr>
<td>$P_{TT}/P_{CT}$</td>
<td>$-$.998</td>
<td>$-.981$</td>
</tr>
<tr>
<td>$P_{L}/P_{CT}$</td>
<td>$-.514$</td>
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</tr>
<tr>
<td>$P_{TT}/P_L$</td>
<td>1.92</td>
<td>1.90</td>
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</table>
CHAPTER 3 REFERENCES


CHAPTER 4

ANISOTROPY AND TAIL FORMATION IN ICRF HEATED TOKAMAK PLASMAS

4.A. The Fokker-Planck Equation

ICRF heating has produced velocity space anisotropy in ion distributions in several experiments to date (see Chapter 1). In this chapter we examine the ICRF produced anisotropy predicted by our model to occur in PLT and compare our results with the experiment. Finding good agreement, we proceed to examine the reactor regime to see if high energy ion tails are formed which enhance the fusion output, thereby reducing the auxiliary heating requirement.

To examine the time evolution of particle distributions, we start with the Boltzmann transport equation\(^{1-3}\). This equation describes the statistical transport of particles in a plasma. A Fokker-Planck collision operator is used which takes into account small angle Coulomb scatterings between particles (two-body scattering). Upon adding a source term, due to neutral beam or alpha particle production, the following Fokker-Planck Equation is obtained

\[
\frac{\partial f_\beta}{\partial t} + \mathbf{v} \cdot \frac{\partial f_\beta}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m_\beta} \cdot \frac{\partial f_\beta}{\partial \mathbf{v}} = \frac{\partial f_\beta}{\partial t} \left( \mathbf{c} + S_\beta \right) \quad (4-1)
\]
for species "β", where $f_β(r, v, t)$ is the distribution function in the six dimensional phase space $(r, v)$ and $\frac{3f_β}{3t}$ is the Fokker-Planck collision operator in spherical coordinates. $S_β(r, v, t)$ is the source function and $\vec{F}$ is the force acting on species "β".

The multispecies, two dimensional in velocity space, O-D in real space Fokker-Planck Radiofrequency heating Code, FORCE, is an improved version of HYBRID-III. HYBRID-III, which included quasilinear ICRF heating terms, was an outgrowth of HYBRID-II developed at Lawrence Livermore Laboratories. Since FORCE is spatially independent

$$\frac{3f_β}{3r} = 0.$$ \hspace{1cm} (4-2)

Any force which acts on species "β" is assumed to arise from an external magnetic field about which $f_β$ possesses azimuthal symmetry. Therefore,

$$\frac{\vec{F}}{m_β} \cdot \frac{3f_β}{3v} = 0.$$ \hspace{1cm} (4-3)

Equation (4-1) becomes

$$\frac{3f_β}{3t} = \frac{3f_β}{3v} + S_β$$ \hspace{1cm} (4-4)

and Eq. (7-12) takes the following form

$$\frac{3f}{3t} = n_o \left[ \frac{1}{v} \frac{3}{3v} \left( \vec{A} - \vec{B} \frac{3f}{3v} + \vec{C} \frac{3f}{3θ} \right) + \frac{1}{v^2 \sin^2 θ} \left( \frac{3f}{3v} \vec{B} \frac{3f}{3v} + \vec{C} \frac{3f}{3θ} \right) \right] + S.$$ \hspace{1cm} (4-5)
upon dropping the "β" subscript, where \( \Gamma = \Gamma_β = \frac{4\pi Z_β^4}{m^2} A^4 \) and \( n_0 = n_0β \) is a normalizing density; \( \bar{v} \) is a normalizing velocity; and the expressions \( \bar{A}, \bar{B}, \bar{C}, \bar{D}, \bar{E}, \bar{F} \) are functions of the distribution and independent variables \((v,θ)\). To this Fokker-Planck equation, quasi-linear ICRF heating terms are added.

4.B. Quasi-Linear ICRF Theory

Quasi-linear theory describes the evolution of particle distributions under the action of a spectrum of uncorrelated waves, an inherently stochastic heating process. Stix\(^{13}\) has argued that this formalism is applicable to the study of ICRF heating when the plasma is "sufficiently collisional". This means collisions are so frequent that wave-particle coherence is destroyed between successive transits of the particle through the resonant heating zone. Appendix B shows that both PLT and NUWMAK satisfy the "sufficiently collisional" condition.

Several assumptions are used in ICRF quasi-linear theory.\(^{14}\) These are listed in Table 4–1. The last assumption implies that there is no strong dependence of the spatially averaged distribution function on the azimuthal velocity about the constant magnetic field, hence

\[
\frac{3f}{3\Phi} = 0. \quad (4-6)
\]

This assumption does allow the distribution to be strongly anisotropic, providing the time variation of the average particle distribution is small during one particle duration.
Beginning with the Vlasov equation,\(^{(2-3,14)}\) with \(f^+(\vec{r},\vec{v},t)\) describing the ions and \(f^-(\vec{r},\vec{v},t)\) the electrons, we have

\[
\frac{\partial f^+}{\partial t} + \vec{v} \cdot \nabla f^+ = \frac{e}{m+C} (c\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f^+}{\partial \vec{v}} = 0. \tag{4-7}
\]

We now assume that \(f^+, \vec{E}\) and \(\vec{B}\) can be expressed as the sum of a spatially independent part and a small rapidly fluctuating part due to the RF wave. Let \(\delta f^+\) be the rapidly fluctuating part of the distribution

\[
\int d^3r \, \delta f^+ = 0. \tag{4-8}
\]

Then

\[
f^+(\vec{r},\vec{v},t) = g^+(\vec{v},t) + \int \frac{d^3k}{(2\pi)^3} \delta f^+(\vec{v},t) e^{i\vec{k} \cdot \vec{x}} \tag{4-9}
\]

with

\[
g \equiv \lim_{V \to \infty} \frac{1}{V} \int d^3r f^+(\vec{r},\vec{v},t) \tag{4-10}
\]

where \(V\) is the volume of integration after Fourier analyzing \(f\) in space.

Fourier analyzing the magnetic and electric fields in space

\[
\vec{E}(\vec{r},t) = \vec{E}_0 \hat{z} + \int \frac{d^3k}{(2\pi)^3} \vec{E}_k(t) e^{i\vec{k} \cdot \vec{x}}
\]

\[
\vec{B}(\vec{r},t) = \int \frac{d^3k}{(2\pi)^3} \vec{B}_k(t) e^{i\vec{k} \cdot \vec{x}}. \tag{4-11}
\]
Upon transforming from Cartesian to cylindrical coordinates, assuming real wave frequencies, and expanding $g$ in the power of $1/n^2$ to first order, we obtain the Kennel and Engelmann quasilinear diffusion equation (I4) with $q^+ \rightarrow f$

\[
\frac{\partial f_\beta}{\partial t} = \lim_{v \to \infty} \sum \frac{e^2}{n \pi m_\beta} \int \frac{d^3k}{V(2\pi)^3} \mathbf{L}_n^v \delta(\omega - k_n v_n - n\Omega_\beta) \left| \theta_{n,k} \right|^2 L_f_\beta (4-12)
\]

where

\[
\mathbf{L} = (1 - \frac{k_n v_n}{\omega}) \frac{1}{v_\perp} \frac{2}{\partial v_\perp} + \frac{k_n}{\omega} \frac{2}{\partial v_\parallel}
\]

and

\[
\theta_{n,k} \equiv \frac{e^{i\psi}}{2} (E_x - iE_y) J_{n+1} + \frac{e^{-i\psi}}{2} (E_x + iE_y) J_{n-1} + \frac{v_n}{v_\perp} (E_x) J_n (4-13)
\]

with

\[
J_n \equiv J_n (k_n v_\perp / \Omega_\beta).
\]

Equation (4-12) is for a spectrum of waves and contains a phase function setting $\psi = 0$, assuming a monochromatic wave spectrum and transforming to spherical coordinates, Stix's quasi-linear diffusion equation is obtained (13)
\[
\frac{\partial \beta}{\partial t} = \frac{\pi e^2}{8m_0^2} |E_x + iE_y|^2 \left( \frac{1}{v} \frac{\partial}{\partial v} \right) J^2 n_0 \delta(n_C - \omega + k,,v,,) (v^2 \sin^2 \theta \frac{\partial \beta}{\partial \theta} - \\
\sin \theta(\frac{k,,v}{\omega} - \cos \theta) \frac{\partial \beta}{\partial \theta} + \frac{1}{v \sin \theta} \frac{\partial}{\partial \theta} (J_{n+1} \delta(n_C - \omega + k,,v,,)) \\
(-\sin^2 \theta(\frac{k,,v}{\omega} - \cos \theta) \frac{\partial \beta}{\partial \theta} + \sin \theta(\frac{k,,v}{\omega} - \cos \theta)^2 \frac{\partial \beta}{\partial \theta} ) \right) \\
+ \frac{\pi e^2}{8m_0^2} |E_x - iE_y|^2 \left( \frac{1}{v^2} \frac{\partial}{\partial v} \right) J^2 n_0 \delta(n_C - \omega + k,,v,,) (v^2 \sin^2 \theta \frac{\partial \beta}{\partial \theta} - \\
\sin \theta(\frac{k,,v}{\omega} - \cos \theta) \frac{\partial \beta}{\partial \theta} + \frac{1}{v \sin \theta} \frac{\partial}{\partial \theta} (J_{n+1} \delta(n_C - \omega + k,,v,,)) \\
(-\sin^2 \theta(\frac{k,,v}{\omega} - \cos \theta) \frac{\partial \beta}{\partial \theta} + \sin \theta(\frac{k,,v}{\omega} - \cos \theta)^2 \frac{\partial \beta}{\partial \theta} ) \right) \\
+ \frac{\pi e^2}{8m_0^2} |E_x|^2 \left( \frac{1}{v} \frac{\partial}{\partial v} \right) J^2 \cot \theta \delta(n_C - \omega + k,,v,,) (v^2 \sin^2 \theta \frac{\partial \beta}{\partial \theta} - \\
\sin \theta(\frac{k,,v}{\omega} - \cos \theta) \frac{\partial \beta}{\partial \theta} + \frac{1}{v \sin \theta} \frac{\partial}{\partial \theta} (J^2 \cot \theta \delta(n_C - \omega + k,,v,,)) \\
(-\sin^2 \theta(\frac{k,,v}{\omega} - \cos \theta) \frac{\partial \beta}{\partial \theta} + \sin \theta(\frac{k,,v}{\omega} - \cos \theta)^2 \frac{\partial \beta}{\partial \theta} ) \right) \right) .
\]

Rewriting Eq. (4-14) in the same form as Eq. (4-5), the following RF diffusion terms may be added to the terms in Eq. (4-5) (13, 15-16)
\[ \bar{A}_{RF} = 0 \]

\[ \bar{B}_{RF} = C \frac{v^2 \sin^2 \theta}{\bar{c}} \]

\[ \bar{C}_{RF} = -C(v \sin \theta \left( \frac{k_x}{\omega} - \cos \theta \right)) \]

\[ \bar{D}_{RF} = 0 \]  \hspace{1cm} (4-15)

\[ \bar{E}_{RF} = -C(v \sin^2 \theta \left( \frac{k_x}{\omega} - \cos \theta \right)) \]

\[ \bar{F}_{RF} = C((v \sin \theta \left( \frac{k_x}{\omega} - \cos \theta \right)^2 \]  

where

\[ C = \left| (E_x + iE_y) J_{n-1} \right|^2 + \left| (E_x - iE_y) J_{n+1} \right|^2 + |E_{y 0}\|^2 \csc^2 \theta \right| \]

\[ \frac{\delta(n \Omega_B - \omega + k_x, v_y)}{32 \pi a^2 n B^2 \Omega_B (r=0) \xi} \]  \hspace{1cm} (4-16)

To eliminate the delta function we integrate over a magnetic flux surface of radius "a" to obtain the following expression for C (see Appendix C and refs. (13,15-16)).

\[ C \rightarrow \left\{ \frac{|E_x J_{n-1}|^2 + |E_x J_{n+1}|^2 + |E_{y 0}|^2 \csc^2 \theta}{32 \pi a^2 n B^2 \Omega_B (r=0) \xi} \right\} \]  \hspace{1cm} (4-17)

where

\[ \xi = \left( 1 - \frac{R_a}{a} \right)^2 \left( \frac{n \Omega_B (r=0)}{\omega + k_x, v_y} - 1 \right)^{1/2} \]
In Eq. (4-14), the fast wave $k_\perp$ and the RF electric fields are used. FORCE uses Eqs. (3-4 and 6) to find these. Since FORCE is 0-D in real space, mode conversion is neglected and only the smallest or fast wave root is used.

Equation (3-2) is solved to obtain the ratios $E_{f}^y/E_{x}^f$ and $E_{z}^f/E_{x}^f$. $E_{x}^f$ is assumed real to fix the phase of the electric field. Taking the second moment of the quasi-linear operator, the following power balance equation is obtained

$$\int \frac{1}{2} mv^2 \frac{\partial f}{\partial t} d^3v = \frac{\partial E}{\partial t} = P_{RF}^- \quad (4-18)$$

The RF electric field is normalized so that all the RF power is absorbed to the plasma:

$$P_{RF}^- \equiv \text{Input RF Power} = \sum_{\beta} \int \frac{1}{2} mv^2 D_{RF}^\beta d^3v \quad (4-19)$$

where $D_{RF}^\beta$ is the RF quasi-linear Diffusion Operator, Eq. (4-14).

The RF dispersion relation assumes that each species is an isotropic Maxwellian. To take into account nonMaxwellian effects, an equivalent Maxwellian temperature is used:

$$T_\beta^- \equiv \frac{2}{3} \int \frac{1}{2} m_{\beta} v^2 f_{\beta}^\star(\vec{v}) d^3v \int f_{\beta}^\star(\vec{v}) d^3v \quad (4-20)$$

The effects of Coulomb collisions and the RF quasi-linear operator on the time evolution of the electron, deuteron, proton, and oxygen particle distributions will now be examined. A global
picture is used except in the RF calculations where \( \omega = n \omega_{cio} \). PLT parameters are given in Table 4-2.

The fast wave \( k_k^2 \) increases slightly in time. The real part is close to the cold plasma value as shown in Fig. (4-1). For PLT \( \frac{\text{Im}k_k^2}{\text{Re}k_k^2} < 0.1 \) so the imaginary part may be neglected.

The spatially averaged RF electric field components are shown in Figs. (4-2-4). Due to the presence of hydrogen in PLT, there is an increase in the \( E_\parallel \) contribution to the wave field

\[
\frac{|E_\parallel|}{|E_\perp|} \sim 1
\]

while the \( E_\perp \) component is much smaller

\[
\frac{|E_\perp|}{|E_\parallel|} \sim 3 \times 10^{-4}
\]

(4-21)

(4-22)

The plasma conductivity increases with increasing temperature. To keep the RF power level constant, the magnitude of the wave field decreases. The RF power flow to the ions increases in time since \( \text{Im}k_k^2 \) increases. In Fig. (4-5) the RF power to deuterium, hydrogen and electrons is plotted. Over 90% of the power is absorbed by the protons through fundamental minority damping. There is very little second harmonic damping by the deuterons. Though the electrons initially absorb 40 kW of power, as the ion temperature increases this amount drops to 10 kW.

In the PLT experiments, \(^{(17)}\) the plasma reached a steady state 40 ms after the RF was turned on. All the species appeared
to be isotropic. Hydrogen was the only species containing a high
energy tail, with an effective temperature of 15 keV. (17-18)

In PLT, the bulk plasma ions increase 200 eV in temperature
while the electrons do not appear to be heated. (17,19)

Since the Fokker-Planck collision operator conserves energy,
the entire plasma cannot reach an equilibrium when only a source
term is present. Some of the distributions may reach a steady
state if collisional losses with cooler species balance with the
direct RF heating term and collisions with hotter species.

Computationally, PLT does not reach equilibrium during the
RF pulse. The total system energy increases in time. Since
there are no overall plasma energy loss mechanisms

$$\Delta E_{\text{plasma}} = E_{RF} = 350 \text{ kW x 100 ms.} \quad (4-23)$$

Figure (4-6) shows that none of the distributions reach steady
state. The electrons are seen to increase in temperature from
650 eV at t = 0 to 1120 eV when the RF is shut off at t = 100 ms.
Since $T_e < T_H$ at t = 100 ms, the electrons continue to heat to a
temperature of 1200 eV at t = 130 ms. The distributions reach a
steady state 30 ms after the RF is turned off. The deuterium
temperature increases 930 eV during the RF pulse, over 4 times
the experimental value. After the RF is shut off, the deuterons
continue to heat due to hotter protons slowing down on the
deuterons, reaching a steady state value of 1890 eV. The oxygen
closely follows the deuterium temperature.
Since the protons receive over 300 kW of RF power, $T_H$ is the highest temperature during the RF pulse with $\frac{T_H}{T_D} \sim 2.1$ at $t = 100$ ms.

Initially $T_e > T_H$, $T_D$, $T_0$ so energy flows from the electrons to the ions and $T_e$ drops (Fig. (4-7)). After 10 ms, $T_e < T_H$, power flows from the hotter protons to the electrons and $T_e$ increases until well after the RF is turned off. Only the protons cool after the RF is shut off, $T_H > T_e, T_D, T_0$. All species reach the same temperature $\sim 1.2$ keV, 30 ms after the RF is off.

FORCE assumes that electrons are isotropic since the electron isotropization time is short (20)

$$\tau_{iso}^e \sim \frac{T_e^{3/2}}{8.2n_e^{1/4} \ln A} \sim 10^{-9} \text{ sec.} \quad (4-24)$$

The electrons absorb a small amount of RF power, so they remain Maxwellian. The deuterons and fully stripped oxygen impurities, like the electrons, remain isotropic Maxwellians.

Since the protons absorb over 90% of the RF power, we find a significant deviation in the distribution from the initial Maxwellian. According to theory (13,21-24) in the absence of other processes a high energy tail, stronger in the $v_\parallel$ direction, should appear. A large high energy proton tail and anisotropy can be found on examining Figs. (4-8-10). In addition to the overall heating, the hydrogen distribution becomes elongated in the $v_\parallel$ direction. The amount of anisotropy increases in time, reaching a peak value at $t=100$ ms of

$$\frac{T_{_{H}}}{T''_{H}} = \frac{E_{_{H}}}{2E''_{H}} = 1.3 \quad (4-25)$$
as shown in Fig. (4-8).

From Figs. (4-9-10) we see a high energy tail forms which is somewhat stronger in the $v_\perp$ direction. From Figs. (4-11-12), we see that the hydrogen distribution relaxes back to an isotropic Maxwellian within 30 ms after the RF is off.

An estimate of the effective tail temperature can be obtained from examining the log $f$ versus energy plots. The initial Maxwellian distribution is a straight line. Since

$$ f_\beta \propto e^{-E/kT_\beta} \quad (4-26) $$

we can define an effective tail temperature

$$ T_{\text{eff}} = \frac{E_2 - E_1}{\log f(v_1) - \log f(v_2)} \quad (4-27) $$

Figure (4-10) shows in the tail

$$ T_{\text{eff}} \sim 40 \text{ keV}. \quad (4-28) $$

This effective tail temperature is too high when compared to the experimental value.

The isotropization time for hydrogen in PLT is (20)

$$ t_{\text{iso}}^{H,D} \sim \frac{m_H^{1/2} m_D^{3/2} A_H^2}{2 \sqrt{\pi} e^{A_H/2} n_H^{1/2} \ln A_{HD}} \left[ -3 + (A_H + 3) \tan^{-1} \frac{A_H^{1/2}}{A_H^{1/2}} \right] \quad (4-29) $$

$$ \sim 3 \text{ ms} \left( \frac{T_D}{T_H} \sim 3 \right). \quad (4-30) $$

The thermal equilibration time for hydrogen on deuterium for PLT is (20)

$$ \tau_{\text{eq}}^{H,D} = \frac{(m_D T_D + m_H T_H)^{3/2}}{1.8 (m_H^{1/2} n_H^{1/2})^{1/2} n_D \ln A_{HD}} \times 10^{19} \text{ sec} \quad (4-31) $$

$$ \sim 5 \text{ ms} \ (T_D = 2 \text{ keV}, T_H = 1 \text{ keV}) \quad (4-32) $$
while the slowing down time is

\[
\tau_{\text{slow}} = \tau_{\text{n}} \ln \frac{(E_{\text{Ho}}^{3/2} + E_{\text{crit}}^{3/2})}{(T_e^{3/2} + T_{\text{crit}}^{3/2})}
\]  

(4-33)

where

\[
\tau_{\text{n}}^{-1} = 1.5 \times 10^{-19} \frac{n_e^2 \ln n_e}{A_{\text{plasma}} T_e^{3/2}}
\]  

(4-34)

\(
\sim 12 \text{ ms (} E_{\text{Ho}} = 10 \text{ keV, } T_e = 1 \text{ keV}).
\)

(4-35)

On the time scale of the experiment (\(\sim 100 \text{ ms}\)), one expects to observe a hot anisotropic hydrogen distribution with a high energy tail. In comparison to the experiment, the overall ion and electron temperature increases calculated by FORCE are too large. In addition, the hydrogen distribution is too anisotropic with too large a tail. There are several factors which might cause this discrepancy.

The RF power level has been taken to be

\[
<P_{\text{RF}}^{\text{RF}}> = \frac{350 \text{ kW}}{\text{Volume}} = 0.067 \text{ W/cm}^3.
\]  

(4-36)

The RF effects would be lessened if \(P_{\text{RF}}^{\text{RF}}\) was smaller. However, from space-time calculations, we find \(P_{\text{RF}}^{\text{RF}}\) at the center to be larger than Eq. (4-36). Therefore, the RF induced high energy tail should be larger in the center, not smaller.

In the previous figures, the RF heating was averaged over a magnetic flux surface with a minor radius of 10 cm. As this radius increases, fewer ions can satisfy the Doppler shifted resonance condition.
\[ \omega + \kappa v \nu_i(r) - n \omega_{ci}(r) = 0. \] (4-37)

If the total RF absorption is kept constant, as "a" increases, the amount of electron absorption must increase. This increased electron absorption would decrease the amount of hydrogen anisotropy and tail formation. The electron absorption is shown in Fig. (4-13) for three different radii, a = 1 cm, 10 cm, 20 cm. The decrease in hydrogen absorption is shown in Fig. (4-14). Figure (4-15) shows there is little change in the amount of hydrogen anisotropy as "a" increases from 10 cm to 20 cm. There is also little change in the tail formation. Chapter 5 shows for the RF heating zone, a \( \leq 10 \) cm, the actual amount of anisotropy and tail formation should be even larger.

Evidently, there must be other processes which greatly modify the ICRF power balance in PLT. These processes should also lead to steady state distributions during the RF pulse. Some possible processes are examined later in this chapter. For now, realizing that FORCE tends to overestimate the RF heating effects, we proceed to examine ICRF heating in the reactor regime. By including only RF and Coulomb collisions, an upper limit on the RF induced effects can be obtained.

**4.C-2. Fundamental Minority Heating of Deuterium in NUWMAK**

In this section and the next, we examine the effects of ICRF
heating on particle distributions in the NUWMAK conceptual tokamak reactor \(^{(27-28)}\). The space-time transport codes STICR-I and STICR-II assume that ions and electrons are nearly Maxwellian and all ions have the same temperature. In this section we examine these assumptions for minority heating of deuterium in NUWMAK.

The NUWMAK reactor parameters for a \(10\% \text{D-90\%T}\) plasma are given in Table 4-3. We again assume a global picture using the average densities and temperatures and a RF power density

\[
\frac{\langle P_{\text{RF}} \rangle}{\text{Volume}} = \frac{150 \text{ MW}}{\text{cm}^3} = 0.71 \text{ W/cm}^3.
\] (4-38)

From Chapter 5, we will find that the plasma ignites with 150 MW of RF power provided for 2.0 sec. By neglecting all loss mechanisms in this and the next section (charge exchange, radiation, finite radial confinement), an upper limit of the RF effects on the plasma will be obtained.

Figure (4-16) plots the hot and cold plasma \(k_1^2\) as a function of time. \(\text{Im} k_1^2\) is again small and could be neglected. Figures (4-17-19) show the value of the spatially-averaged RF electric field is the same order of magnitude as computed for PLT.

The presence of a minority concentration of deuterons again produces a large \(E_+\) component

\[
\frac{|E_+|}{|E_-|} \sim 0.1.
\] (4-39)
$E_n$, as in Filt, is again small

$$\frac{|E_n|}{|E|} \sim 1.5 \times 10^{-3}.$$ \hspace{1cm} (4-40)

In Fig. (4-20) the solid lines indicate RF power absorption when only collisions are present. Initially, the deuterons absorb over 90\% of the RF power.

We have assumed a $10^8 - 10^9$ cm$^{-3}$ density of fully stripped carbon impurity at the center. This impurity, having the same cyclotron frequency as deuterium, absorbs the remaining RF energy (initially 3 MW). Since the concentration is quite low, and no losses are present, the carbon ions become quite hot and nonMaxwellian. As the carbon ions heat, they absorb more power until at $t = 2.0$ sec, they absorb $\sim 35$ MW. In Fig. (4-21), we see $E_C \sim 500$ keV with $\frac{E_C}{E_T} \sim 80$. The impurities become highly anisotropic with

$$\frac{T_{LC}}{T_{nC}} \sim 28.$$ \hspace{1cm} (4-41)

Fig. (4-22) shows that a large high energy tail component appears and then disappears within 500 ms after the RF is off.

This computational result should not occur in NUWMAK. If we include a loss term due to finite radial confinement ($\tau_E^C \sim 5$ sec), we find that carbon remains cooler and absorbs less power (shown by the dash line in Fig. (4-20). The carbon absorbs less than 6 MW at
t = 2.0 sec, becomes cooler, and
\[
\frac{n_{iC}}{T_{iC}} \approx 1.8.
\]  

(4-42)

From Chapter 5, we find over 60% of the RF power damped by the electrons before the fast wave reaches the cyclotron resonance layer at the plasma center. The carbon impurities at the center should absorb a significantly lower amount of power and, therefore, remain Maxwellian. If the resonant zone is moved to the plasma edge, the impurities may absorb a large amount of power and become highly anisotropic. If their transport out of the plasma is enhanced, ICRF with \( \omega = \omega_C \) (r = outside edge) may provide a good barrier against impurity penetration to the plasma center.

When the RF is shut off, the system energy continues to increase due to fusion produced alphas which slow down in the plasma. The steep rise in the fusion energy curve near the RF shutoff (see Fig. (4-23)) indicates ignition. At the time the RF is turned off
\[
Q = \frac{\text{FUSION POWER}}{\text{RF POWER}} \approx 1.27.
\]  

(4-43)

The electron temperature increases from an Ohmically heated value of 2.0 keV to 6.3 keV at \( t = 2.0 \) sec, reaching a value of 7.2 keV at 2.5 sec (Fig. (4-24)). The electrons remain Maxwellian during the entire startup phase, being heated only through collisional processes (ohmic heating being small at high temperatures).

Figure (4-24) shows the initial tritium temperature of 2.0 keV increases to 6.2 keV at the time the RF is turned off. At
t = 2.5 sec, $T_T \sim 7.0$ keV. During the RF phase $T_T \propto T_e$. The alphas slow down preferentially on the electrons. After the RF is off and there is a great number of fusion events occurring, the electrons absorb more fusion energy than the tritons so the electrons become slightly hotter.

The tritons remain an isotropic Maxwellian. This is again expected since the tritons are only heated by collisions with hotter species. On the other hand, the deuterons absorb over 90% of the RF power. One may expect, in analogy to the protons in PLT, that deuterium becomes nonMaxwellian and hotter than the other species (excepting the alphas).

From Fig. (h-24) we find that during the RF phase $T_D \propto T_T$. At $t = 2.0$ sec, $\frac{T_D}{T_T} = 1.03$. Both the plots of $f(v_\parallel)$, $f(v_\perp)$ (Figs. (h-25-26)) show that deuterium ions remain isotropic and no high energy tail develops. Although the deuterons absorb almost 150 MW of power, they remain Maxwellian. These results, which differ with PLT, can be explained.

First, the amount of RF energy per minority heated particle is smaller. In PLT, we have

$$\frac{\langle P_{RF} \rangle}{n_H} \sim 3.5 \times 10^{-14} \text{ W/particle}$$

while in NUWMAK
\[ \frac{P_{\text{RF}}}{n_D} \sim 2.3 \times 10^{-14} \text{ W/particle.} \]  \tag{4-45}

The amount of energy available to drive an individual ion into the tail is less in NUWMAK. In addition, the various Maxwellian producing characteristic times (Eqs. (4-29,31,33)) are shorter in NUWMAK.

\[ \tau_{\text{iso}} \sim 0.4 \text{ ms} \left( \frac{T_1}{T_\text{n}} = 3 \right) \]  \tag{4-46}

\[ \tau_{\text{eq.}} \sim 2.7 \text{ ms} \left( T_D = 6 \text{ keV}, T_T = 4 \text{ keV} \right) \]  \tag{4-47}

\[ \tau_{\text{slow}} \sim 0.7 \text{ ms} \left( E_{D_0} = 10 \text{ keV}, T_e = 4 \text{ keV} \right). \]  \tag{4-48}

These faster times, together with the lower \( P_{\text{RF}} \) per particle value leads to isotropic Maxwellian deuterons. The short equilibration time causes the tritium and electron temperatures to closely follow the deuterium temperature.

One reason ICRF appeared to be an attractive heating source was its potential for producing high energy ions which would enhance the fusion output. Earlier calculations \(^4\) showed \( Q \) enhancement for ICRF heating would be less than 11%. If charge exchange, radiation, Ohmic heating and radial losses are included, any \( Q \) enhancement would be decreased. When space-time results are examined, we can see there
should be very little, if any, Q enhancement for minority heating of deuterium in NUWMAK.

Unlike PLT, it appears a one ion temperature fluid model, as in STICR-II, is a good approximation for NUWMAK if the alphas are not included. Initially, we have assumed a Maxwellian alpha population with \( n_\alpha = 8 \times 10^8 \text{ cm}^{-3} \) is present in NUWMAK with a temperature of 2.0 keV. Since the alphas are born at 3.52 MeV, a high energy tail is formed, shown in Fig. (4-27). The distribution averaged alpha energy is quite high, shown in Fig. (4-28). As the plasma heats, the alphas slow down on an increasingly hotter background, producing an increasing average alpha energy. The alpha density is only \( 2.75 \times 10^{11} \text{ cm}^{-3} \) by the time the RF is turned off, so the alphas do not absorb any RF power.

STICR-II assumes the alphas instantaneously deposit their energy in the background plasma. The alpha temperature is then the same as the other ions. This is the case for the majority of alphas. However, since there are a significant number of alphas always in the process of slowing down, indicated by the tail, STICR-II overestimates the amount of fusion energy during the startup phase, although the difference is small.

In the NUWMAK reactor, we find

\[
k_{\text{i}} \rho_{\text{i}} \sim \frac{k_{\text{i}} (2E_{\text{i}} m_{\text{i}} c^2)^{1/3}}{q_{\text{i}} B_{0}} \quad (4-9)
\]
\[ k_\perp \rho_a \sim \frac{1}{4} (k_\perp \sim 1 \text{ cm}^{-1}) \] (4-50)

for alphas and

\[ k_\perp \rho_c \sim \frac{1}{5} (k_\perp \sim 1 \text{ cm}^{-1}) \] (4-51)

for carbon. In this case, the small gyroradius expansion is not appropriate and one needs to use the complete 3 x 3 determinant as done in FORCE.

Except for alphas and possibly impurities, the ions and electrons remain Maxwellian for minority heating in NUWMAK. A fluid model should be accurate in NUWMAK and is used in Chapter 5. We now examine in our idealized picture a similar plasma to that used in Chapter 2.


Since fundamental minority heating in the reactor regime does not produce high energy tails, we now proceed to the next higher harmonic. Since

\[ P_{RF} \propto (k_\perp v_\perp)^{n-1} \propto (k_\perp \rho_\perp)^{n-1} \] (4-52)

second harmonic heating might produce a larger tail.

For this case, we have a 50\%D-50\%T plasma with parameters given in Table 4-3. In second harmonic heating \( k_\perp^2 \) is larger as
shown in Fig. (4-29). This larger value of \( k_\perp^2 \) should also help produce a larger ion tail.

By increasing the concentration of deuterium to 50%, the \( E_+ \) component of the wave field is increased. Figs. (4-30-32) show

\[
\frac{|E_+|}{|E_-|} \sim \frac{1}{3}
\]  

(4-53)

while the \( E_- \) component is still very small

\[
\frac{|E_-|}{|E_-|} \sim 9 \times 10^{-4}.
\]  

(4-54)

Second harmonic absorption is weaker than fundamental damping. To produce the same amount of RF heating we find the RF electric field must be larger

\[
\frac{|E_+|}{|E_-|}\text{ second harmonic} \sim 4.4 \text{ and } \frac{|E_-|}{|E_-|}\text{ second harmonic} \sim 1.5.
\]  

(4-55)

The increased concentration of deuterium produces more fusion alphas for the same plasma temperature. In Chapter 5 we find only 100 MW for 1.5 sec is needed to ignite the reactor. For the same temperature and total ion density, the amount of increased fusion power is

\[
\frac{P_a^{50-50}}{P_a^{10-90}} = \frac{\frac{1}{2}}{\frac{1}{10}} = 2.8.
\]  

(4-56)

Figure (4-33) shows that 97 MW of RF power is deposited in
the deuterons. Tritium, through third harmonic damping, absorbs only 0.7-1.0 MW, while 2 MW is deposited in the electrons. In contrast with the previous case, the carbon impurities absorb a negligible amount of RF power (≈100 kW) as do the alphas.

Figure (4-34) shows ignition is again reached. The total system energy increases after the RF is turned off and there is again a steep rise in the fusion energy curve. At $t = 1.5$ sec

$$Q \approx 1.33.$$  \hfill (4-57)

In Fig. (4-35), the initial electron temperature of 2.0 keV increases to 4.6 keV at $t = 1.5$ sec and reaches a value of 5.3 keV at $t = 2.0$ sec. The electrons again remain Maxwellian during the startup phase.

Figure (4-35) shows that the tritium and carbon temperatures are now equal. At the RF shutoff point, $T_T = T_C = 4.6$ keV. After ignition, the electrons are again slightly hotter. At $t = 2.0$ sec, $T_T = T_C = 5.2$ keV. Tritium remains an isotropic Maxwellian, while in contrast with the minority heating case, the carbon impurity is also an isotropic Maxwellian. Second harmonic damping on impurities is small since RF power absorption is proportional to the density. With the carbon density low, the impurities are not heated.

The alphas again have a much higher average energy, shown in Fig. (4-36) and there is again a high energy component due to
thermalizing alphas (Fig. (4-37)).

From Figs. (4-38-39) we see the deuterons, as before, remain an isotropic Maxwellian. Although 97% of the RF power is deposited in the deuterons we again find $T_D \propto T_T \left( \frac{T_D}{T_T} \right) \approx 1.01$ at $t = 1.5$ sec, in Fig. (4-35)). The characteristic slowing down, isotropization, and equilibration times calculated in Eqs. (4-46-48) together with the decreased amount of RF power per particle heated

$$\frac{<P_{RF}>}{n_D} \sim 3.1 \times 10^{-15} \text{ W/particle} \quad (4-58)$$

again counteract any anisotropization or tail formation in the deuterons. If the energy losses are included along with spatial effects, we can again expect that any Q enhancement will be negligible. If the shorter characteristic times for NUWMAK are ignored, to get the same RF power per particle heated as in PLT, one would need to supply 1000 MW. If the shorter characteristic times are included, almost 10 GW of RF power would be needed to get the same amount of anisotropy and tail formation as in PLT.


From examination of fundamental and second harmonic heating of NUWMAK in our model, we find that deuterons, tritons, and electrons all remain isotropic Maxwellians. Although the deuterons absorb over 90% of the RF power, the reactor equilibration time
is so short that $T_D \approx T_T$. The alphas are not Maxwellian in either case. However, since they do not absorb a significant amount of RF power, the Maxwellian assumption used in STICR-II should not affect the RF calculations. The assumption of alphas slowing down instantaneously does not change the auxiliary heating requirement since the amount of alpha energy in the tail during the RF startup is small.

The carbon impurities remain Maxwellian in the second harmonic heating case. In fundamental heating, we found that impurities could become quite hot and non-Maxwellian in the absence of any energy loss mechanisms. When finite radial confinement losses are included, these effects are greatly reduced. When these results are coupled with spatial effects, found in Chapter 5, we conclude that impurities should remain Maxwellian with $T_D \approx T_T \approx T_C$.

We can conclude from our FORCE reactor calculations that a fluid model with a single ion energy equation should be accurate for ion harmonic heating with the resonance zone near the plasma center. There is little tail produced $Q$ enhancement. However, to be sure of our conclusions, we need to return to the PLT results, include possible plasma processes which tend to inhibit RF heating and see if FORCE agrees with the experiment.
4.D. PLT-Additional Heating and Cooling Processes

In the previous examination of PLT, when only collisions and RF heating were considered, the results obtained from FORCE differed substantially from the experiment. Either the RF model is in error, or other processes must be included. In this section, we examine several heating and cooling mechanisms which when included, provide good agreement. We then conclude that if "PLT" scaling holds in the reactor regime, our results, which found no anisotropy or tail formation, should be valid for an ICRF reactor startup.

First, there is radial particle and energy transport due to finite particle ($\tau_P$) and energy ($\tau_E$) confinement. To model radial losses (29)

\[
\int \frac{3f_\beta}{3t} d^3v = \frac{3n_\beta}{3t} \to - \frac{n_\beta}{\tau_P}
\]

and

\[
\int \frac{1}{2} m_\beta v^2 \frac{3f_\beta}{3t} d^3v = \frac{3E_\beta}{3t} \to - \frac{E_\beta}{\tau_E}.
\]

Dimensionally, this transport is modelled by inputting a $\tau_P$ and $\tau_E$ for each species consistent with the PLT experiment. Equation (4-5) together with eqs. (4-16,18) now become

\[
\frac{3f_\beta}{3t} = \left( \frac{3f_\beta}{3c} + S_\beta + D_\beta RF - \frac{r_\beta}{\tau_P} + \frac{1}{2v^2} \frac{3}{3v} \frac{r_\beta v^3}{\tau_E} \right) + S_\beta + D_\beta RF - \frac{r_\beta}{\tau_P} + \frac{1}{2v^2} \frac{3}{3v} \frac{r_\beta v^3}{\tau_E}
\]
or

\[
\frac{1}{\nu \sin^2 \theta} \left\{ \frac{3}{\nu} \frac{3}{2} \left[ A' f + B' \nu \frac{3}{\nu} \frac{3}{2} f \right] + C' \frac{3}{\nu} \frac{3}{2} f \right\} + \frac{1}{\nu^2} \frac{3}{\theta \nu} \left[ D' f + E' \nu \frac{3}{\nu} \frac{3}{2} f + F' \nu \frac{3}{\nu} \frac{3}{2} f \right] + S - \frac{f}{\tau_T} + \frac{1}{2} \frac{2}{\nu^2} \frac{3}{\nu} \frac{3}{2} f \frac{3}{\nu} \frac{3}{2} f \frac{3}{\nu} \frac{3}{2} f \right\}. \quad (4-62)
\]

Recycling in PLT keeps the ion densities constant during the RF pulse. To model this, we set all the particle confinement times to infinity. Later, when a neutral gas density is added, impact ionization creates a source of ions. The various ion particle confinement times are then readjusted to keep the ion density constant.

To find the electron energy confinement time we use the fact that in PLT, the electrons reach a quasi-equilibrium during the Ohmic heating phase before the RF is on

\[
\frac{\partial E_e}{\partial t} + \nabla \cdot \left[ Q'_e + \Gamma_e E_e \right] = P_{ei} + P_{OH} \quad (4-63)
\]

where \( P_{OH} \) is Ohmic heating; \( P_{ei} \) is electron-ion rethermalization; \( \nabla \left[ Q'_e + \Gamma_e E_e \right] \) represent conduction and convection losses. Setting

\[
\nabla \left[ Q'_e + \Gamma_e E_e \right] = \frac{E_e}{\tau_E} \quad (4-64)
\]

at equilibrium, Eqs. (4-63 and 64) become

\[
\frac{-E_e}{\tau_E} = P_{OH} + P_{ei}. \quad (4-65)
\]

Since \( P_{ei} \ll P_{OH} \), using Eq. (4-65) we find
\[ \tau_E^e = \frac{E_e}{P_{OH}} \sim 45 \text{ ms} \]  

(4-66)

for PLT with \( I = 350 \text{ kA} \), in agreement with the measured value.

In determining the hydrogen energy confinement time, we use the result from Chapter 5 which finds over 90\% of the RF power goes to heating the protons in the plasma center. To model this spatial heating we set

\[ \tau_E^H = C_{RF} \times \tau_E^D \quad C_{RF} = 2 \]  

(4-67)

where \( \tau_E^D \) represents the bulk ion energy confinement time. The correction factor will be shown to be noncritical. Unless \( \tau_E^H \ll \tau_E^D \), not seen experimentally, the hydrogen radial energy loss is small.

To find \( \tau_E^D \) and \( \tau_E^O \), we use a parameter study of tokamak experiments by Hugill and Sheffield\(^{(30)}\). The four parameter fit, reproduced in Fig. (4-40) is

\[ \tau_E \sim 2.7 n_e^{-0.61} a^{1.57} B^{0.88} A_i^{0.65} \text{ (ms)} \]  

(4-68)

where \( n \) is in \( 10^{13} \text{ cm}^{-3} \); \( a \) is in m; \( B \) is in kG; \( A_i \) is the atomic mass number. Although PLT was not used in deriving the empirical fit, they found the experimental value to be close to the fit.

For PLT

\[ \tau_E^D \sim 30 \text{ ms} \]  

(4-69)

\[ \tau_E^O \sim 110 \text{ ms} \]
using the parameters in Table 4-4. Equation (4-67) becomes

$$\tau_{E}^{H} = 60 \text{ ms}.$$  \hspace{1cm} (4-70)

With only radial losses included in Eq. (4-62), FORCE was unable to simulate the experiment (4). Other processes besides radial transport need to be included. We now examine additional processes which affect the electrons.

4. D-1. Electrons in PLT

Besides Landau and transit time damping, Coulomb collisions and radial transport, the electrons are Ohmically heated and radiate energy away. On colliding with a hydrogen or deuterium neutral atom, an electron may lose 13.6 eV. Electron impact ionization increases the ion density and cools the electrons. By adding these three effects, we may define an effective electron energy confinement time

$$P_{OH} - P_{Z} - P_{RF} = \frac{E_{e}}{\tau_{E}}$$  \hspace{1cm} (4-71)

where $P_{OH}$ is the Ohmic heating term

$$P_{OH} = \eta_{SP} J^{2}$$  \hspace{1cm} (4-72)

and where $\eta_{SP}$ is the Spitzer resistivity.
$P_Z$ is the radiation loss term (see Chapter 5)

$$P_Z = n_e n_Z L_Z$$  \hspace{1cm} (h-73)

where $L_Z$ is a polynomial fit to the reaction rate (31)

$$\log_{10} L_Z = \sum_{i=0}^{5} A_i (\log_{10} T_e)^i.$$  \hspace{1cm} (h-74)

The $A_i$ are numerical coefficients. $P_{EP}$ is the electron impact ionization loss term (32)

$$P_{EP} = n_e (n_{D^0} + n_{H^0}) \langle \sigma v \rangle_{EP} (-13.6 \text{ ev})$$  \hspace{1cm} (h-75)

with

$$\ln \langle \sigma v \rangle_{EP} = \sum_{i=0}^{5} B_i (\ln E)^i$$  \hspace{1cm} (h-76)

also a polynomial fit with the assumption that ionization cross sections for $H^0$ and $D^0$ are equal.

Since FORCE preserves charge neutrality, an effective $\tau_p^e$ is not needed. However, both a $\tau_p^*$ and a $\tau_e^*$ must be calculated for the hydrogenic species in PLT.
4.D-2. Hydrogenic Species in PLT

Impact ionization and charge exchange may cause the proton and deuteron densities to vary. We need to model an effective particle $\tau_p^*$ as well as energy $\tau_E^*$ confinement times for each species.

Let $R$ designate the event rate for either ionization or charge exchange. A deuterium density balance equation can then be written

$$\frac{\partial n_D}{\partial t} = \left( R_{H,D^0}^{\text{cx}} - R_{D^+}^{\text{H}^0} \right) + R_{E,D^0}^0 + R_{p,D^0}^0 \frac{n_D}{\tau_p^*} \quad (4-77)$$

where $R_{\text{cx}}^{H,D^0}$ is the rate at which a proton collides with a deuterium neutral, $D^0$, to produce a hydrogen neutral, $H^0$, and a deuteron, $D^+$,

$$R_{\text{cx}}^{H,D^0} \equiv n_H n_{D^0} \langle \sigma v \rangle_{\text{cx}} \quad (4-78)$$

$R_{\text{cx}}^{H,D^0}$ determines the rate at which a deuteron charge exchanges with a hydrogen neutral, becoming a deuterium neutral with a resulting decrease in the deuteron density

$$R_{\text{cx}}^{D^+H^0} \equiv n_D n_{H^0} \langle \sigma v \rangle_{\text{cx}} \quad (4-79)$$

$R_{E,D^0}^0$ is the rate at which electrons ionize neutral deuterium

$$R_{E,D^0}^0 \equiv n_e n_{D^0} \langle \sigma v \rangle_{\text{EH}} \quad (4-80)$$
while $R_{PD}^O$ is the rate at which hydrogenic ions ionize deuterium \(^{(32)}\)

$$R_{PD}^O \equiv (n_H + n_D)n_{D^O}^{<\sigma v>}_{PH}. \quad (4-81)$$

In an analogous fashion, an effective $\tau_P$ for hydrogen can be written

$$\frac{3n_H}{3t} = \frac{R_{D^OH^O}}{cx} - \frac{R_{D^OH^O}}{cx} + \frac{R_{D^OH^O}}{EH} + \frac{R_{D^OH^O}}{PH} \approx \frac{n_H}{\tau_P^{*H}} \quad (4-82)$$

where

$$R_{D^OH^O}^{D^OH^O} \equiv n_D n_{H^O}^{<\sigma v>}_{cx}$$

$$R_{D^OH^O}^{D^OH^O} \equiv n_D n_{H^O}^{<\sigma v>}_{cx}$$

$$R_{D^OH^O}^{D^OH^O} \equiv n_D n_{H^O}^{<\sigma v>}_{EH}$$

$$R_{D^OH^O}^{D^OH^O} \equiv n_D n_{H^O}^{<\sigma v>}_{PH} \quad (4-83)$$

In the energy equation we again assume that 13.6 eV is lost per ionization event. In a charge exchange event, the ion and neutral effectively exchange energies. Therefore, for deuterium

$$\frac{3E_D}{3t} = \frac{E_{D,D^O}}{cx} - \frac{E_{D,D^O}}{cx} - \frac{E_{D,D^O}}{cx} - \frac{E_{D,D^O}}{DH^O} - \frac{E_{D,D^O}}{DH^O} \approx \frac{E_{D,D^O}}{\tau_E} \quad (4-84)$$

and for hydrogen

$$\frac{3E_H}{3t} = \frac{E_{H,H^O}}{cx} - \frac{E_{H,H^O}}{cx} - \frac{E_{H,H^O}}{cx} - \frac{E_{H,H^O}}{DH^O} - \frac{E_{H,H^O}}{DH^O} \approx \frac{E_{H,H^O}}{\tau_E} \quad (4-85)$$
where
\[ p_{\alpha \beta}^{\circ, \circ} = n_{\alpha} n_{\beta}^{\circ} <\sigma v>_{\circ, \circ} (E_{\alpha} - E_{\beta}^{\circ}) \]
\[ P_{\alpha \beta}^{\circ} = n_{\alpha} n_{\beta}^{\circ} <\sigma v>_{PH} (\text{13.6 eV}) \] (4.86)

for \( \alpha = D, H \)
\[ \beta^{\circ} = D^{\circ}, H^{\circ} . \]

In the evaluation of the charge exchange and ionization cross sections, we have again assumed there is no difference between protons and deuterons. The general hydrogenic species charge exchange is
\[ \ln<\sigma v>_{cx} = \sum C_{i} (\ln(E))^{i} \] (4.87)

while the ion impact cross section is
\[ \ln<\sigma v>_{PH} = \sum D_{i} (\ln(E))^{i}. \] (4.88)

The Fokker-Planck equation can now be written
\[ \frac{\partial f}{\partial \tau} = n_{\circ} \left\{ \frac{1}{2} \frac{\partial}{\partial v} \left[ A' v + B' \frac{\partial f}{\partial v} + C' \frac{\partial f}{\partial \theta} \right] + \right. \]
\[ \left. \frac{1}{v^{2} \sin^{2} \theta} \frac{\partial}{\partial \theta} \left[ D' v + E' \frac{\partial f}{\partial \theta} + F' \frac{\partial f}{\partial \theta} \right] \right\} \]
\[ - \frac{f}{\tau_{P}} + \frac{1}{2v} \frac{\partial}{\partial v} \left[ \frac{v^{3}}{\tau_{E}} - \frac{f}{\tau_{P}} + \frac{1}{2v} \frac{\partial}{\partial v} \left( \frac{v^{3}}{\tau_{E}} \right) \right] . \] (4.89)
$S_{\beta} \to 0$ and $x^e$ $x^o$ $x^o$ $x^o \to \infty$ since there are no external sources, charge neutrality holds, and the impurities have no sources or sinks.

In the evaluation of the cross sections used in Eq. (4-89), all species were assumed to be isotropic Maxвелlians with temperatures determined from the previous time step. Since the time step is .01 ms, the distributions do not change greatly over one step and this approximation should be valid. From Fig. (4-41) for PLT with $T_e$ $T_i \approx 1.0$ keV, charge exchange is the dominant cross section and proton impact is negligible. We now use FORCe to examine ICRF heating in PLT.

4.D-3. PLT Results

For the average neutral gas density in PLT, we have assumed $n_o = 2.2 \times 10^8$ cm$^{-3}$ composed of 10% H$^+$ and 90% H$^0$ both at a temperature of 5 eV. This density results in a hydrogen charge exchange loss comparable to the experiment. A list of parameters for PLT, with these additional processes included, can be found in Table 4-4. When Figs.(4-1 and 42) are compared, we see these additional processes have a minimal effect on the dispersion relation, since the fast wave $k^2$ is weakly dependent on temperature. Figures (4-43-45) show the electric field components are not significantly changed either.

Again we see in Fig. (4-46) that fundamental damping is the strongest RF heating mechanism; 310-330 kW of RF power is absorbed
by the protons. From these plots we see the RF heating process is
unaffected by the various additions to the Fokker-Planck equation.
However, the particle distributions are greatly altered.

In contrast with our earlier result, Fig. (4.47) shows the
plasma now reaches a steady state, with \( \tau_{RF} \approx 40-60 \text{ ms} \), comparable
to the experiment. The total system energy at \( t = 100 \text{ ms} \) is now
lower due to the losses, \( 3.4 \times 10^{13} \text{ keV/cm}^3 \) compared to \( 8.4 \times 10^{13} \)
keV/cm\(^3\). Figure (4.48) shows all the distributions reach equilibria
40-60 ms after the RF pulse is turned on. In good agreement with
PLT, the spatially averaged electron temperature does not increase
significantly during the RF pulse. The FORCE code obtains
\( \Delta T_e \approx 40 \text{ eV} \).

FORCE has the average deuterium or bulk plasma ion tempera-
ture increasing 150 eV, in agreement with PLT. The average proton
energy increase is now 450 eV as compared to 2240 eV calculated
previously. At \( t = 100 \text{ ms} \), \( \frac{T_H}{T_D} \approx 1.8 \).

Figure (4.49) shows the importance of Ohmic heating, radial
and radiation losses on the electron temperature. The amount of
power in Ohmic heating (\( \approx 550 \text{ kW} \)) and radial transport (\( \approx 460 \text{ kW} \))
is larger than the total RF power (350 kW). Radiation losses
cancel the direct RF electron absorption term. Since the Ohmic
heating term is larger than the radial transport term, a rise in
the electron temperature might be expected. However, \( T_e > T_D, T_o \)
and the electrons lose energy via collisions. The high energy
protons which slow down on the electrons do not provide enough power to counteract the electron collisional losses. After the RF is turned off, $T_e$ drops. The drop in $T_e$ causes an increase in the Ohmic power since $P_{\text{OH}} \propto T_e^{-3/2}$ while the radial losses decrease since $P_{\text{Rad}} \propto T_e$. The net result is little electron heating.

Deuterium radial ($\sim 360 \text{ kW}$) and charge exchange ($\sim 190 \text{ kW}$) losses are large as shown in Fig. (4-50). Second harmonic absorption is minimal ($< 2.5 \text{ kW}$). For the average deuterium temperature to rise 150 eV, over 550 kW of power must be received from collisions with electrons and high energy protons.

From Fig. (4-52) we see the low concentration of hydrogen results in small charge exchange ($\sim 30 \text{ kW}$) and radial losses ($\sim 20 \text{ kW}$). The dominant terms in the hydrogen energy balance are RF heating and Coulomb collisions with the cooler species. Setting $\tau_E^H = \tau_E^D$ would raise the radial losses to $\sim 45 \text{ kW}$, still a small amount. Unless $\tau_E^H << 60 \text{ ms}$, which has not been observed, the RF correction factor in Eq. (4-67) has a small effect. The small radial and charge exchange losses and the large amount of direct RF heating cause the protons to become much hotter than the background deuterons and electrons.

The initial Maxwellian electrons and deuterons again remain Maxwellian during the RF heating phase. The deuterons, although being heated during the RF pulse, remain isotropic.
The protons, which absorb most of the RF, become non-Maxwellian. Figure (4-51) shows the protons becoming slightly anisotropic during the RF pulse. The amount of anisotropy at \( t = 100 \text{ ms} \) has now decreased, \( \frac{T_\perp}{T_\parallel} \sim 1.17 \) as compared to \( \frac{T_\perp}{T_\parallel} \sim 1.29 \) predicted before. This anisotropy again disappears within 30 ms after the RF pulse is off.

The hydrogen temperature is now lower than in our idealized picture, dropping from 2.55 keV to 0.76 at \( t = 100 \text{ ms} \).

In comparison with the earlier model, Figs. (4-53-54) show that a smaller high energy tail forms which quickly disappears once the RF is off. Figure (4-55) shows the effective hydrogen tail temperature, 18 keV, agrees with the experiment.

4.E. Conclusions

We have seen that besides RF heating and collisions, Ohmic heating, radial, charge exchange and radiation losses are important in interpreting the RF heating experiments in PLT. When all these processes are included, FORCE obtains good agreement with the experiment.

Only the protons become non-Maxwellian with an effective tail temperature of 15-20 keV. The protons absorb over 90% of the available RF power through fundamental minority damping. The protons are slightly anisotropic and quickly isotropize once the RF is off.
The deuterons, which make up the bulk of the plasma, double in temperature. They remain Maxwellian, being heated by collisions, not by the RF. The electrons are not heated substantially. The oxygen impurities have little affect on RF absorption. All these results agree well with the PLT experiment.

In the reactor regime, our idealized model has shown that there is little Q enhancement. Both the deuterons and the tritons are Maxwellian with $T_D \approx T_T$. Stix\(^{(13)}\) finds tail formation because he neglects other processes such as radial transport and charge exchange. In addition, his reactor model has a lower plasma density which leads to longer slowing down and isotropization times. Finally, Stix assumes much higher power densities than are needed in NUWMAK. For $\zeta = 10$

\[
\frac{P_{RF}}{n_D} = 6.5 \times 10^{-14} \text{ W/particle} \quad (4-90)
\]

while for $\zeta = 100$

\[
\frac{P_{RF}}{n_D} = 6.5 \times 10^{-13} \text{ W/particle.} \quad (4-91)
\]

For NUWMAK, $\zeta$ should be less than 3.

We have seen that alphas do not absorb much power. The impurities should not absorb much RF power when the resonance zone is near the plasma center. Any additional processes which could be added to our idealized model would decrease further any RF
induced non-Maxwellian effects.

The space-time codes STICR-I and STICR-II discussed in the next chapter assume isotropic, nearly Maxwellian distributions with only a single ion temperature. FORCE has shown this to be an accurate picture. A Fokker-Planck formulation is not needed. However, FORCE has shown, as has the experiment, that for PLT this assumption is not valid. All the species, except the protons, are isotropic Maxwellian with \( T_D = T_0 \). However, the protons are non-Maxwellian with \( \frac{T_H}{T_D} \sim 1.7 \) at \( t = 100 \text{ ms} \). Since the protons are only slightly anisotropic, the fluid equations should still be valid.

However, FORCE neglects several effects which the space-time codes include. The Fokker-Planck description neglects all spatial effects, using only a spatially-averaged density and energy for each species. FORCE also assumes \( \omega = n \omega_{ci} \) (r=0) everywhere (RF heating is concentrated near the center). FORCE neglects both mode conversion processes if present, and absorption outside of the cyclotron resonance layer. These spatial effects may lead to a further reduction in any ICRF induced effects in both NUWMMAK and PLT.

Although the PLT space-time code STICR-I has only one ion temperature, we are able to incorporate into the RF calculations a separate hydrogen temperature
\[ T_H = C_H \times T_1 \]  \hspace{1cm} (4-92)

where \( C_H \) is an input variable greater than 1. We shall see that \( C_H \) plays an important role in determining the mode conversion threshold in PLT.
Table 4-1
Quasi-Linear RF Assumptions

1) Nonrelativistic, collisionless, infinite, spatially uniform plasma exists in the absence of wave excitation.

2) There is a constant, static magnetic field, $B_0$, with no curvature oriented along the Z axis.

3) No static electric field exists.

4) Direct nonlinear coupling between waves can be neglected.

5) Spatially averaged distribution functions change slowly compared to the gyroperiod and characteristic times of the wave motion.

6) The constant magnetic field is strong enough that particles gyration frequencies are larger than the damping rate of the waves.
### Table 4-2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<td>Minor radius (cm)</td>
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<td>n_D (cm⁻³)</td>
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<tr>
<td>n_H (cm⁻³)</td>
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<td>n_o (cm⁻³) (oxygen)</td>
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<tr>
<td>(\tilde{n}_T) (cm⁻³)</td>
<td>2.7 x 10¹⁴</td>
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<tr>
<td>Initial (\tilde{n}_c) (cm⁻³)</td>
<td>1.0 x 10¹¹</td>
</tr>
<tr>
<td>Initial (\tilde{n}_\alpha) (cm⁻³)</td>
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<tr>
<td>Initial (T_e) (keV)</td>
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<tr>
<td>(\tau_p)</td>
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<tr>
<td>(\tau_E)</td>
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<td>(Z_{\text{eff}})</td>
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<td>(K_n) (cm⁻¹)</td>
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<td>Minor radius of heating (cm)</td>
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<tr>
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<tr>
<td>RF power (MW)</td>
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<td>RF pulse (sec)</td>
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<td>(\Delta t_{\text{step}}) (ms)</td>
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<td>(P_D/N_D) (x10⁻¹⁴W)</td>
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<td>Variable</td>
<td>Fundamental</td>
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<td>--------------------------</td>
<td>-------------</td>
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<td>$P_{RF}^{e}$ (MW)</td>
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<td>$P_{RF}^{D}$ (MW)</td>
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<td>$P_{RF}^{T}$ (MW)</td>
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<td>E_\alpha</td>
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<tr>
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<td>Minor radius (cm)</td>
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<tr>
<td>$\bar{n}_e$ (cm$^{-3}$)</td>
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<td>$\bar{n}_D$ (cm$^{-3}$)</td>
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<td>$\bar{n}_H$ (cm$^{-3}$)</td>
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<td>$\bar{n}_Z$ (cm$^{-3}$) (oxygen)</td>
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<td>Initial $T_D = T_H = T_Z$ (eV)</td>
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<td>$T_O$ (eV)</td>
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<td>RF power (kW)</td>
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Table 4-4 (cont.)

At RF Shutoff

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<td>$P^e_{RF}$ (kW)</td>
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<td>$P^D_{RF}$ (kW)</td>
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<td>$P^H_{RF}$ (kW)</td>
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<tr>
<td>$P^Z_{RF}$ (kW)</td>
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<td>$P_{OH}$ (kW)</td>
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<td>$P_z$ (kW)</td>
<td>~17</td>
</tr>
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<td>$P_{EP}$ (kW)</td>
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</tr>
<tr>
<td>$P^e_{radial}$ (kW)</td>
<td>~460</td>
</tr>
<tr>
<td>$P^D_{cx}$ (kW)</td>
<td>~186</td>
</tr>
<tr>
<td>$P^D_{radial}$ (kW)</td>
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<td>$P^H_{cx}$ (kW)</td>
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<td>$P^H_{radial}$ (kW)</td>
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<td>$P^Z_{radial}$ (kW)</td>
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<td>$\Delta E_e$ (eV)</td>
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<td>$\Delta E_D$ (eV)</td>
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<td>$\Delta E_H$ (eV)</td>
<td>685</td>
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<td>255</td>
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<tr>
<td>$</td>
<td>E_+</td>
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<td>E_-</td>
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<td>$</td>
<td>E_n</td>
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</tbody>
</table>
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Fig. (4-6)

Fig. (4-7)
Fig. (4-8)

HYDROGEN
PLT 350 kW
FOKKER-PLANCK
CODE

Fig. (4-9)

Fig. (4-10)
Fig. (4-11)

Fig. (4-12)
Fig. (4-13) 

Fig. (4-14) 

Fig. (4-15)
Fig. (4-25)

Fig. (4-26)
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Fig. (4-28)
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Fig. (4-31)

Fig. (4-32)
Fig. (4-36)

Fig. (4-37)
Correlation function = 0.92

![Figure 4-40](image)

\[ \tau_E = 2.7 \times 10^{61} \alpha^{57} B^{68} A_0^{65} \text{ (ms)} \]

![Figure 4-41](image)

**Rate per Atom (cm³/s)**

**Particle Energy (eV)**

- Charge exchange
- Proton ionization
- Electron ionization
Fig. (4-51)

Fig. (4-52)
Fig. (4-53)

Fig. (4-54)
Fig. (4-55)
REFERENCES

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CHAPTER 5

SPATIAL ASPECTS OF ICRF HEATING IN TOKAMAK PLASMAS

5.A. 1-D Fluid Model

In this chapter we investigate the spatial aspects of ICRF heating in PLT and NUWMAK using a multi-species, 1-D transport code with a WKB slab model of RF heating. Spatial RF power depositions and temperature and density profiles are obtained. We also find an estimate for minority concentrations, which determine the mode conversion threshold. The amount of auxiliary power needed to ignite NUWMAK is calculated. Finally, we examine various RF heating scenarios which minimize the effects of complex and unresolved physical processes (mode conversion and ion-ion hybrid resonance effects).

Starting with the Boltzmann equation (1-4)

\[ \frac{\partial f_\beta}{\partial t} + \nabla \cdot \nabla f_\beta + \frac{e_\beta}{m_\beta} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} f_\beta = C_\beta \]  \hspace{2cm} (5-1)

where $C_\beta$ is the Fokker-Planck Collision operator. On taking the zeroth moment of Eq. (4-1) we obtain the density balance equation

\[ \frac{\partial n_\beta}{\partial t} + \nabla \cdot \mathbf{v}_\beta = 0 \]  \hspace{2cm} (5-2)
where \( n_\beta = \int f_\beta \, d^3v \), and \( \overline{P} \) represents a particle flux

\[
\overline{P}_\beta = n_\beta \overline{V}_\beta = \int \overline{V} \, f_\beta \, d^3v.
\] (5-3)

In obtaining Eq. (5-2) we find

\[
\int (\overline{E} + \overline{v} \times \overline{B}) \cdot \overline{V} \, f_\beta \, d^3v = 0
\] (5-4)

and

\[
\int \overline{C}_\beta \, d^3v = 0
\]

since Coulomb scattering conserves particles. Taking the first moment of Eq. (5-1)

\[
n_\beta m_\beta \overline{V}_\beta + n_\beta m_\beta \overline{V} \cdot \overline{V} \, f_\beta = n_\beta e_\beta (\overline{E} + \overline{V} \times \overline{B}) - \overline{V} \cdot \overline{P}_\beta
\]

\[
+ m_\beta \int \overline{V} \, \overline{C}_\beta \, d^3v
\] (5-5)

where \( \overline{P} \) is the pressure tensor

\[
\overline{P}_\beta = m_\beta \int (\overline{V} - \overline{V}_\beta) \cdot (\overline{V} - \overline{V}_\beta) \, f_\beta \, d^3v.
\] (5-6)

Assuming a spherically symmetric velocity distribution for species "\( \beta \)" (see Chapter 4), the pressure tensor can be
represented by a diagonal tensor with a single scalar pressure $p$

$$p_\beta = \frac{m_\beta}{3} \left\{ (\overline{v} - v_\beta)^2 f_\beta \, d^3v \right\} = n_\beta \, T_\beta.$$  \hfill (5-7)

The Fokker-Planck Collision operator conserves momentum in collisions between similar particles, hence

$$\int C_\beta \, \overline{v} \, d^3v = 0.$$  \hfill (5-8)

The second moment of Eq. (5-1) yields the following energy balance equation

$$\frac{\partial}{\partial t} \left( n_\beta \, \frac{m_\beta}{2} \, v_\beta^2 + \frac{3}{2} \, n_\beta \, T_\beta \right) = n_\beta \, e_\beta \, E \cdot \overline{v}_\beta - \overline{v} \cdot \overline{H}_\beta + \frac{m_\beta}{2} \left\{ v^2 \, C_\beta \, d^3v \right\}$$  \hfill (5-9)

where

$$\overline{H} = \frac{3}{2} \, m_\beta \left\{ \overline{v} \, v^2 \, f_\beta \, d^3v \right\}$$  \hfill (5-10)

and

$$\frac{\partial}{\partial t} \left( \frac{3}{2} \, n_\beta \, T_\beta \right) = - \, \overline{v}_\beta \cdot (\frac{3}{2} \, T_\beta \, \overline{v}_\beta + \overline{Q}_\beta) - n_\beta \, T_\beta \, \overline{v} \cdot \overline{v}_\beta$$

$$+ n_\beta \, e_\beta \, E \cdot \overline{v}_\beta + \frac{m_\beta}{2} \left\{ v^2 \, C_\beta \, d^3v \right\}$$  \hfill (5-11)
where \( \overline{Q}_\beta \) is the heat flow vector

\[
\overline{Q}_\beta = \frac{m_\beta}{2} \int \overline{v} \cdot (\overline{v} - \overline{v}_\beta)^2 f_\beta d^3v. \tag{5-12}
\]

In \( \overline{Q}_\beta \), we neglect energy flow due to nonisotropic particle distributions. Chapter 4 shows this to be valid for NUWMAK. In PLT, this approximation should be good for all species except possibly the protons. Experimentally \( T_H \sim T''_H \tag{5-6} \).

In Eq. (5-11) \( \overline{v} \cdot \left( \frac{3}{2} T_\beta \overline{v}_\beta \right) \) represents energy flow due to diffusing particles while \( \overline{v} \cdot \overline{Q}_\beta \) represents energy flow due to thermal conduction. The term, \(-n_\beta T_\beta \overline{v} \cdot \overline{v}_\beta \) represents a compressive energy flow which is usually small and therefore neglected. The Ohmic heating term is \( n_\beta e_\beta \overline{E}_\beta \cdot \overline{v} \) while \( \frac{m_\beta}{2} \int \overline{v}^2 c_\beta d^3v \) describes the rate of energy transfer between collision of unlike particles.

When the Fokker-Planck Collision operator is used, Eqs. (5-2,5 and 9) form a closed system and neoclassical transport is obtained\(^{(1,7-10)}\).

By averaging over a magnetic flux surface, one-dimensional particle and energy fluid transport equations are obtained. Whereas previous work assumed that deuterium and tritium did not transport separately\(^{(11-12)}\), and impurities were neglected\(^{(13,14)}\), the Space-Time Ion Cyclotron Resonance heating code STICR-II can calculate separate deuterium, tritium, impurity, and alpha particle transport in NUWMAK.
\[ \frac{\partial}{\partial t} n_D = -\frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_D) - n_D n_T \langle \sigma v \rangle + S_D \]

\[ \frac{\partial}{\partial t} n_T = -\frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_T) - n_D n_T \langle \sigma v \rangle + S_T \]

\[ \frac{\partial}{\partial t} n_\alpha = -\frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_\alpha) + n_D n_T \langle \sigma v \rangle \]

\[ \frac{\partial}{\partial t} n_z = -\frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_z) \] (5-13)

where particle sources and sinks due to alpha fusions\(^{(15-16)}\), charge exchange, and impact ionization have been added. In PLT, hydrogen is present instead of tritium and oxygen is the dominant impurity. STICR-I solves the following density equations

\[ \frac{\partial}{\partial t} n_D = -\frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_D) + S_D \]

\[ \frac{\partial}{\partial t} n_H = -\frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_H) + S_H \]

\[ \frac{\partial}{\partial t} n_o = -\frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_o) \] (5-14)
where the only sources and sinks are from charge exchange and impact ionization.

Adding sources and sinks to the electron energy balance equation we find

\[
\frac{2}{3t} \left( \frac{3}{2} \frac{n_i T}{e} \right) = n_D n_T \langle \sigma v \rangle E \alpha_{ae} - \frac{2}{r} \left[ r(Q_e + \frac{3}{2} \frac{T}{e} \right) \]

\[ - P_{\text{EP}} - P_{\text{ei}} - P_z + P_{\text{OH}} + P_{\text{RF}}^e \]

(5-15)

where \( n_D n_T \langle \sigma v \rangle E \alpha \) is the alpha energy production term; \( E \alpha = 3.52 \text{ MeV} \) for NUWMAK and 0 for PLT; \( U_{ae} \) is the fraction of alpha energy deposited in the electrons \(^{(1,17)}\); \( P_{\text{ei}} \) is the electron-ion rethermalization term; \( P_z \) is the radiation loss term; \( P_{\text{EP}} \) is the electron impact ionization loss term; \( P_{\text{OH}} \) is the Ohmic heating term; and \( P_{\text{RF}}^e \) is the RF heating term from Landau and transit time damping.

Both STICR-I and STICR-II assume that all ions have the same temperature. The ion energy balance equation with sources and sinks is

\[
\frac{2}{3t} \left( \frac{3}{2} \frac{n_i T}{e} \right) = n_D n_T \langle \sigma v \rangle E \alpha_{ai} - \frac{2}{r} \left[ r(Q_i + \frac{3}{2} \frac{T_i}{i} \right) \]

\[ + P_{\text{ei}} - P_{\text{IP}} - P_{\text{cx}} + P_{\text{RF}} \]

(5-16)
where $E_a = 3.52$ MeV for NUWMAK and 0 for PLT; $U_{ai} = 1 - U_{ae}$ is the fraction of alpha energy deposited in the ions; $P_{cx}$ is the charge exchange term; $P_{IP}$ is ion impact ionization loss term; and $P_{RF}^i$ is the ion RF heating term.

Since charge neutrality is assumed in NUWMAK

$$n_e = n_D + n_T + 2n_A + <z>n_Z$$  \hspace{1cm} (5-17)

whereas in PLT

$$n_e = n_D + n_H + 8n_O.$$  \hspace{1cm} (5-18)

The particle transport coefficient $\Gamma$ can be written as

$$\Gamma = - \sum_b D_{bb} \nabla F_b$$  \hspace{1cm} (5-19)

while the energy transport coefficient $Q$ can be expressed as

$$Q_e = - \sum_b x_{eb} \nabla F_e$$

$$Q_i = - \sum_b x_{ib} \nabla F_i$$  \hspace{1cm} (5-20)

where $F_b$ is either density, ion or electron temperature and $b = e, D, T, a, C$ for NUWMAK or $b = e, D, H, O$ for PLT.

The conductivity and diffusion terms in Eqs. (5-15,16) now include the collisional effects in Eq. (5-9). Previously, trapped
particle scaling\(^{(1,7,13-14,18)}\) was assumed to describe the plasma transport. However, trapped particle scaling has yet to be seen in any tokamak to date. With collisional effects incorporated into \(D_{\text{bb}}\) and \(\chi_{\text{bb}}\), "empirical" scaling can be used. Although "PLT empirical" scaling is not fully understood from a theoretical standpoint, computationally this scaling has produced good agreement with both PLT Ohmic heating and neutral beam heating experiments. Having shown that ICRF heating does not affect the bulk plasma distribution, we have assumed this scaling holds for the RF experiments.

After obtaining agreement with the present day PLT RF heating experiments\(^{(6)}\), we next assume that "PLT empirical" scaling predicts particle and energy transport in NUWMAK. The transport scaling, although not affecting the RF power deposition, greatly influences the auxiliary heating requirement.

For high density and temperature plasmas, "PLT empirical" scaling is\(^{(19)}\)

\[
D_B = \frac{1.25 \times 10^{17}}{n_e} \text{ cm}^2/\text{sec} + \text{neoclassical}^{(9)}
\]

\[
\chi_i = \text{neoclassical}^{(9)}
\]

\[
\chi_e = \frac{1.0 \times 10^{17}}{n_e} \text{ cm}^2/\text{sec} + \text{neoclassical}^{(9)}
\]

\[
\text{for } \frac{1.0 \times 10^{17}}{n_e} < 2 \times 10^5 \text{ cm}^2/\text{sec}
\]

\[
= 2 \times 10^5 \text{ cm}^2/\text{sec} + \text{neoclassical}^{(9)}
\]

\[
\text{for } \frac{1.0 \times 10^{17}}{n_e} \geq 2 \times 10^5 \text{ cm}^2/\text{sec}.
\]
In STICR-I and STICR-II, the classical electron-ion equilibration term is used \(^{(1)}\)

\[
P_{ei} = \frac{3n_e m_e}{\tau_{ei} m_i} (T_e - T_i)
\]  

(5-22)

where \(\tau_{ei}\) is the electron-ion equilibration time \(^{(20)}\)

\[
\tau_{ei} = \frac{3m_e^{1/2} T_e^{3/2}}{4(2\pi)^{1/2} e^4 n_i \ln\Lambda}
\]  

(5-23)

with \(m_i = \sum_j \frac{m_i n_i}{n_e}\). This expression for \(P_{ei}\) is used since this provides good agreement with experiments \(^{(1)}\).

The Ohmic heating term is calculated using neoclassical theory

\[
P_{OH} = E_\phi J_\phi
\]  

(5-24)

\[
E_\phi = \eta_{nc} J_\phi
\]  

(5-25)

where \(\eta_{nc}\) is the neoclassical resistivity \(^{(11,13)}\)

\[
\eta_{nc} = \eta_{sp} (1 - 1.95(r/R)^{1/2} + .95(r/R))^{-1}
\]  

(5-26)

\(\eta_{sp}\) is the Spitzer resistivity \(^{(21)}\).
Because PLT and NUWMK are assumed to be transparent to their own radiation, a coronal equilibrium model\textsuperscript{(22)} is used. Radiation in this model results from Bremsstrahlung, radiative recombination and line transitions. Using coronal equilibrium, the radiation loss term is\textsuperscript{(22)}

\[ P_z = n_e n_z L_z \]  
\[ L_z = \sum_{i=0}^{5} A_i \left( \log_{10} T_e \right)^i \]  
\[ (T_e \text{ in keV}) \]

and \( A_i \) are coefficients of a polynomial least square fit to the radiation cooling rate.

For charge exchange and impact ionization, analytical or polynomial expressions were fitted to the experimental data\textsuperscript{(23)}. These expressions, when integrated over Maxwellian distributions, were fitted to polynomials to obtain the cooling rates. For charge exchange

\[ P_{\text{cx}} = n_i n_o <\sigma v>_{\text{cx}} (E_i - E_o) \quad (o \equiv \text{neutral atom}) \]

where

\[ <\sigma v>_{\text{cx}} = \sum_{i=0}^{5} B_i (\ln E)^i \]  
\[ (E \text{ in eV}) \]
with $E$ in eV; $n_o$ is the neutral gas density; $E_i$ is the incoming ion energy; and $E_o$ is the neutral gas energy. For ion impact ionization

$$P_{IP} = n_i n_o <σv>_{IP} (-13.6 \text{ eV}) \tag{5-31}$$

while electron impact ionization has

$$P_{EP} = n_e n_o <σv>_{EP} (-13.6 \text{ eV}) \tag{5-32}$$

where

$$<σv>_{IP} = \sum_{i=0}^{5} C_i (ln \quad E)^i$$

$E$ in eV

$$<σv>_{EP} = \sum_{i=0}^{5} D_i (ln \quad E)^i. \tag{5-33}$$

The incoming ion or electron is assumed to lose 13.6 eV in ionizing a neutral atom. In the charge exchange calculation, the initial or first generation charge exchange event has a neutral atom energy of 5 eV.

The radiation, charge exchange and impact ionization terms are calculated at every grid point along the minor cross section.

Electron and ion impact are a source of ions in Eqs. (5-13,14)
\[ S_D = n_i n_{D^0} \langle \sigma v \rangle_{IP} + n_e n_{D^0} \langle \sigma v \rangle_{EP} \]

\[ S_T = n_i n_{T^0} \langle \sigma v \rangle_{IP} + n_e n_{T^0} \langle \sigma v \rangle_{EP} \quad \text{(for NUWMAK)} \]

\[ S_H = n_i n_{H^0} \langle \sigma v \rangle_{IP} + n_e n_{H^0} \langle \sigma v \rangle_{EP} \quad \text{(for PLT).} \quad (5-34) \]

When the neutral gas has a different concentration of hydrogenic species than the plasma, charge exchange can alter the ion concentration. For PLT

\[ S_D = n_i n_{H^0} \langle \sigma v \rangle_{cx} - n_e n_{H^0} \langle \sigma v \rangle_{cx} \]

\[ S_H = n_i n_{H^0} \langle \sigma v \rangle_{cx} - n_e n_{H^0} \langle \sigma v \rangle_{cx} \quad (5-35) \]

while for NUWMAK

\[ S_D = n_i n_{T^0} \langle \sigma v \rangle_{cx} - n_e n_{T^0} \langle \sigma v \rangle_{cx} \]

\[ S_T = n_i n_{T^0} \langle \sigma v \rangle_{cx} - n_e n_{T^0} \langle \sigma v \rangle_{cx} \quad (5-36) \]

STICR-I and STICR-II assume that a feedback control exists which keeps both hydrogenic ion densities constant. Particles are added or removed using a flat profile, which leaves the temperature profiles unchanged. In PLT\(^{(5-6)}\), recycling and wall
sputtering keep the plasma density fairly constant over the RF pulse. By adding particles without affecting the temperature profiles, we can simulate PLT and obtain good agreement.

Unlike PLT, we have no source of impurities; consequently, Z decreases. In PLT experiments, the impurity density actually increases during the RF pulse. Since radiation losses appear to be small, this discrepancy between impurity densities has a small effect. Since the PLT electron temperature rise is uncertain at this time\(^{(6)}\), it is difficult to compare our electron temperature profiles to the experiment. The radiation and impurity transport models should not affect either the ion temperature profiles or the RF power deposition.

We now examine the ICRF heating model which calculates the RF heating terms in Eqs. (5-15,16).

5.B. 1-D WKB Slab Model of ICRF Heating

The full 3 x 3 hot plasma dispersion tensor (Eq. (3-4)) is used to obtain a local complex \( k_\perp \) for a given real \( \omega \) and \( k_\parallel \). The plasma minor cross section is divided into 117 points, neglecting curvature and edge effects (see Fig. (5-1)). At each point \( r_i \), \( n(r_i) \), \( T(r_i) \), and \( B(r_i) \) are used to calculate \( k_\perp(r_i) \). All densities and temperatures are assumed to be symmetric in minor radius. Mau\(^{(24)}\) has shown density and temperature gradient effects on the dispersion relation to be small.

To find the power deposition for each species "s" at each
point \( r_1 \), the weak damping formula is used \(^{(25-26)}\)

\[
P_{RF}^p(r) = \omega \bar{E}(r)^* \cdot \bar{K}_{\beta}^a(r) \cdot \bar{E}(r)/2 \text{ (W/cm}^3)\tag{5-37}\]

which is valid when \(|\frac{\text{Im} k}{\text{Re} k}| \ll 1\). The validity of Eq.(5-37) is examined in Appendix D. The weak damping formula holds over the entire cross section in NUMMAK. For PLT, it breaks down over the cyclotron resonance layer. Near this layer, mode conversion may also occur. We address this problem later.

A local description is again used to calculate the RF electric field component ratios \( E_y/E_x \), \( E_z/E_x \), at each point along the minor cross section

\[
\bar{D}(r) \cdot \bar{E}(r) = 0. \tag{5-38}\]

An incoming fast wave may mode convert into an electrostatic Ion Bernstein wave \(^{(27-28)}\). The RF dispersion relation

\[
\det|\bar{D}(r)| = 0 \tag{5-39}\]

has an infinite number of solutions. The smallest root corresponds to the fast wave root \((k_{\perp f})\). The next smallest root is associated with an electrostatic ion cyclotron mode called an Ion Bernstein wave \((k_{\perp I,B})\). Equations (5-38-39) are solved at each point using both roots
\[ \det |\vec{D}(r)| = 0 \quad \vec{D}_{f}(r) \cdot \vec{E}_{f}(r) = 0 \]

and

\[ \det |\vec{D}_{I.B.}(r)| = 0 \quad \vec{D}_{I.B.}(r) \cdot \vec{E}_{I.B.}(r) = 0 \]  \hspace{1cm} (5-40)

Equation (5-40) yields \( k_{\perp f}, k_{\perp I.B.} \) and the ratios \( \frac{E_{y}^{f}}{E_{x}^{f}}, \frac{E_{z}^{f}}{E_{x}^{f}} \), \( \frac{E_{y}^{I.B.}}{E_{x}^{I.B.}}, \frac{E_{z}^{I.B.}}{E_{x}^{I.B.}} \) where \( f \) designates the fast wave and \( I.B. \) the Ion Bernstein mode. So far, each point has been calculated independently. We now must "connect" these points to form a 1-D picture. To accomplish this a WKB type plane wave equation for \( E_{x}^{\gamma}(r) \) is used

\[ \frac{d^{2}E_{x}^{\gamma}}{dr^{2}} + k_{\perp}^{2}(r)E_{x}^{\gamma}(r) = 0 \]  \hspace{1cm} (5-41)

where \( \gamma = f, I.B. \). The solution to Eq. (5-41) has the following form (30)

\[ E_{x}^{\gamma} \propto \frac{1}{(k_{\perp})^{1/2}} e^{i \int \vec{k}_{\perp} \cdot dr} \]  \hspace{1cm} (5-42)

The validity of this solution is investigated in Appendix D and found to hold over the entire cross section NUWMAK. In PLT, the WKB solution also breaks down in the vicinity of the cyclotron resonance.
It is not obvious that a slab model holds for PLT, a much smaller device than NUWMAK. However, the temperature profiles obtained by STICR-I agree with the experiment. Although a 2-D or 3-D cylindrical model (27) would be more accurate, the differences may not affect the temperature profiles.

We now need to find the magnitude of the electric field.

We first fix the phase by assuming that the fast wave has a real $E^f_x$ at the low field edge

$$E^f_x(r = \text{edge}) = C(1., 0.). \tag{5-43}$$

To find $C$, we normalize the electric field assuming all the wave energy launched is absorbed by the plasma

$$P_{RF} = \frac{\omega}{2} \sum_{\beta} \sum_{\gamma} E^f(r_1) \cdot \bar{K}_{\beta}^f(r_1) \cdot \bar{E}^f(r_1) + E^{I.B.}_I(r_1) \cdot \bar{K}_{\beta}^{I.B.}(r_1) \cdot \bar{E}^{I.B.}(r_1) \tag{5-44}$$

where we sum over the RF energy deposited at each point $r_1$, for each species, for both the fast and Ion Bernstein modes. Writing the normalized electric field as

$$\tilde{E}^\gamma = C \tilde{E}^\gamma \tag{5-45}$$

where $\tilde{E}$ is the unnormalized field, $C$ is

$$C \equiv \frac{P_{RF}}{\frac{\omega}{2} \sum_{\beta} \sum_{\gamma} E^f(r_1) \cdot \bar{K}_{\gamma}^f(r_1) \cdot \bar{E}^f(r_1) \cdot \bar{E}^{I.B.}(r_1)} \tag{5-46}$$
In PLT and NUWMAK, a plane fast wave is launched from the low field edge. At each point \( r \), as the wave propagates inwards, \( k_{f}^{2}(r) \) and \( k_{I.B.}^{2}(r) \) are calculated. If \( \text{Re}k_{f}^{2} \approx \text{Re}k_{I.B.}^{2} \) to within 10\%, over a finite mesh width, mode conversion is assumed to occur. Jacquinot\(^{(29,31)}\) has obtained the following approximate expression for the mode conversion threshold in PLT:

\[
\eta_{H} \geq \frac{n_{D}k_{n}V_{th}(H)}{\omega} \left( \frac{\eta}{3} + \frac{\xi_{n}c^{2}}{\omega_{PD}} \right)
\]

(5-47)

This equation agrees with the computational threshold.

When mode conversion does not occur, \( E^{I.B.} = 0 \) and Eq. (5-44) is evaluated using the fast wave mode. If mode conversion does occur, reflection, transmission and mode conversion coefficients are calculated at the mode conversion surface (see Fig. (5-2)). Upon reaching the mode conversion surface, part of the incoming fast wave is reflected back towards the low field side with an initial electric field \( \overline{RE} \), where \( R \) is the reflection coefficient and \( \overline{E} \) the incident electric field. Some of the incoming wave is transmitted through the mode conversion layer with an electric field \( \overline{TE} \), where \( T \) is the transmission coefficient. In NUWMAK, the remaining wave energy is assumed to mode convert into an Ion Bernstein wave which propagates towards the high field side with an initial electric field \( \overline{CE} \), with \( C \) being the mode conversion coefficient.
For PLT, Swanson has shown (28) that some of the wave energy may be deposited in the mode conversion layer. To model this, STICR-I assumes that in the mode conversion layer the electric fields are constant and the absorbed energy is equally divided between the two branches. We shall see that mode conversion appears to play a minor role in both PLT and NUWMMAK.

5.C. Mode Conversion Coefficients - NUWMMAK

By inverse Fourier transforming a small ion gyroradius expansion of Eq. (5-38), a fourth order differential equation can be obtained to describe coupling between the fast and Ion Bernstein branches (27)

\[
\frac{d^4 E}{dr^4} + \lambda r \frac{d^2 E}{dr^2} + (\lambda r + \gamma) E = 0
\]  
(5-48)

where \( \lambda, \gamma \) are constants. Using Laplace's method and analytical continuation of the contours of integration, the WKB solution for the wave electric field on each side of the mode conversion layer can be connected using various connection formulae. The following transmission, reflection and mode conversion coefficients are obtained for small \( k_n \):

\[
|T| = e^{-\pi n/2}
\]

\[
|R| = |1 - e^{-\pi n}|
\]

\[
|C|^2 = 1 - |T|^2 - |R|^2 = e^{-\pi n} - e^{-2\pi n}
\]  
(5-49)
where

\[ n = \frac{R_o}{2} \frac{\omega_{PD}}{c} \beta (1 + \frac{n_H}{n_e} \beta^{-1}) \]  \hspace{1cm} (5-50)

Equations (5-49-50) are valid for a fast wave launched from the low field side.

5.D. Mode Conversion Coefficients—PLT

Swanson\(^{28}\) has derived mode conversion coefficients for fundamental minority heating in PLT. Instead of Eq. (5-48), Swanson obtains fourth order coupled differential equations which describe mode coupling. Swanson finds the following mode conversion coefficients

\[ T = e^{-\eta} \]

\[ R = -(1 - e^{-2 \eta} + I_{22}) \]

\[ C = -e^{-\eta}(1 - e^{-2 \eta}) (1 + I_{02}) \]  \hspace{1cm} (5-51)

where

\[ \eta = \frac{\pi \omega_{PD} R_o}{4c(1+p^2)^2} a^3 (a^2 \beta_D + \frac{n_H}{n_e}) \]  \hspace{1cm} (5-52)

and

\[ a^2 = (1 + p^2) (1 - 3 p^2) / (1 + 3p^2) \]

\[ p = \frac{k_H V_A}{\omega} \]  \hspace{1cm} (5-53)
for $\frac{n_H}{n_e}$ small. Since rotational transform effects are small\(^{(28)}\), they are neglected.

In Eq. (5-51) $I_{02}$ and $I_{22}$ must be solved numerically.

Swanson has found $C = (C_{r2}, C_{i2})$ and $R = (R_{r2}, R_{i2})$ for $\frac{n_H}{n_e} = 6.2 \times 10^{13} \text{ cm}^{-3}$, $B_o = 36 \text{ kG}$, $\beta_D = 0.01$, $\frac{n_H}{n_e} = 0$ (see Fig. 5-3)).

For those parameters and $k_\nu = 0.1 \text{ cm}^{-1}$,

$$|C| \% 0.01$$
$$|R| \% 0.05$$

so

$$I_{22} \sim 0.34$$

$$I_{02} \sim 0.97$$

with $\eta \sim 0.25$.

We assume that $I_{22}$ and $I_{02}$ do not change significantly when PLT parameters in TABLE 5-1 are used. In deriving Eqs. (5-51-53) Swanson neglects electrons and nonMaxwellian effects. Using Eqs. (5-51-53,55) we find that $|C|$ and $|R|$ are small. Most of the energy is either transmitted or absorbed in the resonance zone

$$E_{\text{absorb}} = 1 - |R|^2 - |T|^2 - \rho |C|^2$$

with

$$\rho = (1 - e^{-2n})^{-1}$$

We now examine ICRF heating in PLT.
5.E. Numerical Results - PLT

The PLT parameters are listed in Table 5-1. Parabolic densities (Figs. (5-4-6)) and parabolic squared temperatures (Figs. (5-7-8)) are the initial profiles. Oxygen is assumed to be the dominant impurity with $Z_{\text{eff}} \sim 2$. At the edge $T_i = T_e = 5$ eV.

In Fig. (5-9), the cold plasma $k_{1cp}^2(r)$ (dotted line) is close to $Re k_1^2$ (solid line) except at the resonance layer, where the cold plasma root $\to \infty$. T2 equals 1.0 indicates the fast wave is transmitted through the resonance zone, there being no mode conversion. In Fig. (5-8), HTEMPC equals 1.5 indicates that $T_H = 1.5 \times T_i$ in the RF calculations. If $T_H = T_i$, mode conversion would occur.

In Fig. (5-9) $Re k_1^2(r) < 0$ over a small region near $r = 123$ cm, indicating a possible evanescent zone. Since this region is small, no reflection or absorption is assumed to occur over this region. This evanescent zone lies in the region where both weak damping and WKB theory break down (see Appendix D). The steep rise in $Im k_1^2(r)$ near this region is a result of the presence of the ion-ion hybrid resonance.\(^{(32)}\)

\[
\omega^2 = \frac{X_D f_H + X_H f_D}{X_D f_D + X_H f_H} \omega_{CH} \omega_{CD}
\]

\[
X_D = \frac{n_D}{n_e}, \quad f_D = \frac{m_e}{m_D}
\]

\[
X_H = \frac{n_H}{n_e}, \quad f_H = \frac{m_e}{m_H}
\]

\[(5-58)\]
which lies near the $\omega = \omega_{\text{CH}} = 2\omega_{\text{CD}}$ resonance layer when $n_H \to 0$. As $n_H \to 0$, the evanescent zone and peaking of $\text{Im}k_{1f}^2$ disappear, resulting in a smoothing of the RF spatial depositions.

The fast wave electric field components are shown in Figs. (5-10-12). Solid lines indicate real parts while dashed lines correspond to the imaginary parts. The peaking of $\text{Im}k_{1f}^2$ and the evanescent zone produces a corresponding peaking of the electric field. The number of oscillations in the electric field indicates approximately four wavelengths fit within the device. Since the WKB approximation assumes the plasma density and temperature do not vary over a wavelength, this assumption is not as appropriate in PLT as in a reactor. However, WKB theory seems to provide reasonably accurate results when compared to the experiment.

Figures (5-10-12) show a drop in $|E^f|$ from $\approx 100$ V/cm at the low field edge to $\approx 20$ V/cm at the high field side. This drop implies the fast wave is absorbed within one pass across the cross section. No eigenmodes are observed in PLT, indicating this is correct. Both STICR-I and STICR-II neglect possible eigenmode effects. The electric field profile is obtained neglecting edge effects, a good approximation when no eigenmodes are established.

Over 93% of the RF power is deposited near the center (Figs. (5-13-14)), again indicating the RF is absorbed within one pass across the minor radius. The peaking in $\text{Im}k_{1f}^2$ and $E^f$ result in a sharp peaking of the hydrogen and deuterium RF power profiles. The evanescent zone causes the sharp decrease in the deuterium and
electron absorption profiles.

Figure (5-13) shows the fundamental minority heating of hydrogen deposition along the entire plasma cross section. Upon averaging over magnetic flux surfaces, Fig. (5-14) is obtained. This drop in power at $r = 0$ occurs because there is only one point in the flux surface. Every other region has contributions from both $+r$ and $-r$.

The deuterons, through second harmonic damping, absorb only 3% of the total RF power (Fig. (5-15)). The remaining 6.5% is deposited in the electrons over a relatively broad heating zone.

As time increases, the impurity density drops, shown in Fig. (5-16). Figures (5-17-18) show the central deuterium and hydrogen densities to increase while the edge decreases. The proton density increase at the center is faster than the deuterium increase. After 60 ms, $\frac{n_H}{n_e} < 11\%$. This increase in $n_H$ at the center, together with the decrease in electron density, lowers the mode conversion threshold. Over the last 40 ms of the RF pulse, STICR-I finds mode conversion occurring. Since the impurity level actually increases in the experiment, the mode conversion threshold should not decrease with time.

In any case, Figs. (5-19-20) show that temperature profiles are unchanged when mode conversion appears. The ion and electron temperature profiles reach steady state 40-60 ms after the RF is on, in agreement with the experiment. Figures (5-21-22) show that the electron and ion RF heating profiles remain unaffected as well.
At the end of the RF pulse, the density profiles are still parabolic. The average electron density has dropped from $3.0 \times 10^{13} \text{ cm}^{-3}$ to $2.6 \times 10^{13} \text{ cm}^{-3}$. The temperature profiles are shown in Figs. (5-23-24). The electrons are not heated while the average ion temperature increases 225 eV with $\Delta T(0) \approx 600 \text{ eV}$ in agreement with the experiment (6).

After 60 ms, mode conversion appears with the mode conversion layer between the 78th and 79th grid points ($120.0 \text{ cm} \leq r \leq 120.5 \text{ cm}$). There is still an evanescent zone which lies between the launching structure and the mode conversion layer. Between $r = 121 \text{ cm}$ and $135 \text{ cm}$, $\text{Im}k_{\perp}^2 < 0$ indicating that most of the RF wave energy may be damped before the mode conversion layer is reached. Swanson's mode conversion coefficients indicate over 58% of the wave energy which reaches the mode conversion layer is absorbed. Over 37% of the energy is reflected. Only 4.7% of the wave energy is transmitted through the layer and none of the energy is converted into the Ion Bernstein branch.

Figures (5-25-26) show the wave energy is largely damped before the wave reaches the mode conversion layer at $r = 120.5 \text{ cm}$. Over 90% of the RF power is still deposited in the protons over the same width as occurs when mode conversion is absent. Second harmonic damping increases slightly from 3% to 6% while the electron absorption decreases slightly to 4% of the total RF power.

The cyclotron resonance is between the mode conversion layer and the low field edge. Therefore, the fast wave undergoes
appreciable damping before the mode conversion layer is reached. Over 37% of the RF energy is reflected back towards the low field side. This wave must again pass through the resonance layer. This further reduces the amount of reflected power. There is some interference between the incident and reflected fast waves. A standing wave develops over half the plasma cross section. This produces oscillations in the broad electron RF heating term (Fig. (5-22)). However, electron absorption is small and the ion heating terms do not change significantly. We therefore conclude that there is very little power in the reflected wave. Mode conversion plays a small role since most of the energy is deposited in the plasma before the mode conversion layer is reached. Approximately 58% of the available mode conversion energy is deposited in the mode conversion layer, lying close to the resonance zone. Without mode conversion, the RF heating profiles would not change.

Figure (5-19) shows the electron temperature profile remaining fairly constant during the RF pulse, except for an initial drop at \( t = 0 \) ms. The peak temperature drops from 1300 eV to 950 eV at \( t = 100 \) ms while the average value drops from 440 eV to 395 eV with STICR-I. In agreement with the experiment, it showed no significant heating of the electrons. The initial drop in \( T_e \) indicates that the equilibrium Ohmic heating electron temperature profile predicted by STICR-I differs from the experimental value. With all the uncertainties in edge effects, radiation modeling, and
transport, this is to be expected. The electron temperature drops initially because the loss mechanisms (predominantly transport and radiation) are too large. The drop in $T_e$ is relatively small, indicating at least reasonable agreement in the electron temperature calculation with PLT.

The ion temperature is better known experimentally and is much less sensitive to impurity levels and radiation models. Although the ion RF heating profile $P_{iRF}(r) = P_{iRF}^D(r) + P_{iRF}^O(r) + P_{iRF}^H(r)$ is sharply peaked, the ion temperature is broader. Radial transport and charge exchange keep the ion temperature from becoming as peaked. The RF power is peaked enough that $T_i(0)$ rises from 600 eV to 1200 eV in good agreement with the experiment. However, the ion temperature is broad enough that the average ion temperature increases from 206 eV to 426 eV, also in agreement with the experiment. Having obtained agreement with PLT experiments we now use STICR-I to examine the mode conversion threshold.

5.5. Mode Conversion in PLT

With $\frac{T_H}{T_i} = 1.5$, mode conversion does not occur during the first 60 ms of the RF pulse. As time increases, $\frac{n_H}{n_e}$ increases to 11% while $n_e$ and $n_o$ decrease. These effects appear to lower the mode conversion threshold. With mode conversion occurring over the final 40 ms, neither density, temperature, nor RF heating profiles are greatly altered.
Initially $\frac{T_H}{T_1} = 1$ in PLT. Setting $C_H = 1.0$, STICRF-I finds mode conversion occurring during the entire RF pulse. Figure (5-27) shows the dispersion relation at $t = 0$ ms. There is little change in the RF dispersion relation during the RF pulse. At $t = 0$ ms, 58% of the RF energy reaching the mode conversion layer is absorbed, 37% reflected, and 4.7% transmitted. At $t = 100$ ms, these numbers change to 59%, 27% and 14%. No wave energy is mode converted. The mode conversion layer shifts slightly from 122.5 cm $< r < 123.0$ cm to 120.0 cm $< r < 121.0$ cm.

This shift in the mode conversion layer has little effect on the RF power deposition. Most of the wave energy is absorbed before the mode conversion layer is reached. Over 90% of the RF power is still deposited in the protons in the same heating zone as before. Neither the equilibrium densities nor temperatures change. Equilibrium is still obtained during the RF pulse.

When $C_H = 2.0$, mode conversion never occurs. Figures (5-28-29) show the evanescent zone disappearing as the ion temperature increases. As the evanescence shrinks, the dip in both the deuterium and electron heating profiles lessens. The dominant heating mechanism remains minority heating. Figures (5-30-31) show little change in the hydrogen heating profile. When $C_H = 2.0$, there is a slight decrease in the amount of hydrogen heating, from 89.6% to 88%. The heating zone increases slightly with temperature from 12-14 cm to 20 cm. As the ion resonant heating zone widens, second harmonic damping increases from 3% to 7% at the expense of
electron heating, dropping from 7% to 4.6% of the RF power.

Again we find little change in the equilibrium densities and temperatures with equilibrium being reached between 40-60 ms.

From these three cases, $C_H = 1.0, 1.5, 2.0$, the following conclusions can be made. After the Ohmic heating phase, $T_H = T_1$ so the PLT RF heating phase begins in the mode conversion regime. Although mode conversion occurs, fundamental minority heating of protons is the dominant ion heating mechanism. Chapter 4 showed $C_H$ becoming larger than one as time increases. As $C_H$ increases, the mode conversion threshold rises until mode conversion disappears. As $T_1$ increases, the ion resonant heating width increases from 12-14 cm to 20 cm.

However, the presence of mode conversion or the increase of the cyclotron resonance layer does not alter the density and temperature profiles. Mode conversion may be difficult to find in PLT since these profiles are not changed in its presence. Whether mode conversion occurs or not, STICR-I finds good agreement with the experiment.

To investigate the influence of $k_\parallel$ on mode conversion, we change $k_\parallel$ from 0.1 cm$^{-1}$ to 0.2 cm$^{-1}$ with $C_H = 1.5$. The equilibrium plot of $k_{r2}^2(r)$ (Fig. (5-32)) shows that mode conversion is eliminated. The resonance zone is now very broad and $\text{Im} k_{r2}^2$ no longer has a sharp peak. There are now wide evanescent regions on the plasma edges. Although these regions may result in surface heating$^{(24,27)}$, we assume that the fast wave can penetrate through the low field
evanescent zone without losing energy.

The proton heating width is quite broad, increasing from 25 cm at $t = 0$ ms to 35 cm at $t = 100$ ms. This widening produces an increase in fundamental heating from 233 kW at $t = 0$ to 302 kW at 100 ms (Fig. (5-33)). Initially, when $k_n$ is increased electron Landau and transit time damping is much larger than in the smaller $k_n$ case, 91 kW compared to 23 kW. As the ion temperature increases, the amount of electron heating drops, reaching 40 kW at 100 ms (Fig. (5-35)). Figure (5-34) shows the deuterium RF profile.

The broadening of the ion heating profiles and the increase in electron heating do not affect the density or electron temperature profiles. However, the ion temperature profile (Fig. (5-36)) is changed. While $\Delta T_i \sim 220$ eV is close to our previous results, $\Delta T_i(0) \sim 500$ eV is too low. If the ion heating zone is substantially larger than 20 cm, the RF power density at the center must drop. This drop in power density results in a smaller ion temperature increase in the center. Whereas the occurrence of mode conversion does not affect our results, a broadening of the heating zone does.

Although the WKB slab model breaks down in the plasma center, the RF heating width appears to agree with the experiment. The RF power deposition within this zone may vary (perhaps a smoothing of the power deposition) when a more complete treatment is used. However, changes in temperatures and densities should be minimal.
We conclude that our RF heating model simulates accurately the density and temperature profiles in PLT.

In a reactor, Appendix D shows the WKB slab model to be valid over a wider region than in PLT. The WKB model assumes that the densities and temperatures do not change over distances of a wavelength. In a reactor, the characteristic scale lengths over which the plasma density and temperature vary are larger than in PLT. In addition, the wavelengths, shown in Figs. (5-50-52), become shorter. These two effects tend to improve the WKB approximations.

In simulating a reactor startup it is necessary to compute, as accurately as possible, the probable density and temperature profiles. If NUWMAK follows "PLT empirical" scaling we conclude that STIRC-II, using the 1-D WKB slab model of ICRF heating, should be more accurate in a reactor and should predict the density, temperature and RF heating profiles.

5.G. Fundamental Minority Heating of Deuterium in NUWMAK

In this section we examine fundamental heating of deuterium in the NUWMAK conceptual tokamak reactor\(^{(14,18)}\). Table 5-2 lists the plasma parameters. In this scenario, \(\omega = \omega_\text{CD}(r=0)\) or \(f = 46 \text{ MHz}\). To avoid the possible mode conversion complications and to reduce the influence of the ion-ion hybrid resonance, the maximum concentration of deuterium in a tritium plasma is found to be 10%. When the concentration of deuterium is increased, mode
conversion at the ion-ion hybrid resonance begins to dominate over fundamental heating. Although heating at this resonance has been observed in TFR (31) and has been examined theoretically (27-28, 33), the physics is not clearly understood. Therefore, we examine RF heating scenarios which minimize the influence of this resonance.

The initial parabolic densities and temperatures, obtained after the Ohmic heating phase, are shown in Figs. (5-37-41). Fully stripped carbon with an initial parabolic profile of average density, \( n_c = 5.5 \times 10^{10} \text{ cm}^{-3} \), is assumed to be the dominant impurity. To avoid initial numerical oscillations at the edge, \( n_e \) (edge) = \( 3 \times 10^{13} \text{ cm}^{-3} \). As time increases, \( n_e \) (edge) drops to \( 1.0 \times 10^{12} \text{ cm}^{-3} \). STICR-II finds that approximately 125 MW of RF power for 2.0 sec provides enough heating to ignite the plasma. The temperature profiles at the time of RF shutoff are shown in Figs. (5-42-43). The average plasma temperature has increased from 1.5 keV to 6.3 keV. Since there is no source of impurities, the carbon density remains parabolic but decreases to a spatially averaged value of \( 3.35 \times 10^{10} \text{ cm}^{-3} \).

The fast wave and cold plasma \( k_{cp}^2(r) \) are shown in Fig. (5-44). \( T_2 = 1.0 \) indicates that no mode conversion occurs. The fast wave is transmitted through the mode conversion layer. Figure (5-44) shows a broad region, 50-100 cm wide, where \( \text{Im} k_{f}^{2} > 0 \). This broad resonance surface lies between the antenna and the mode conversion surface. As the plasma heats, this layer becomes wider.
(see Fig. (5-45)). The resonance layer produces substantial damping of the incoming wave through electron Landau and transit time damping, before the wave energy can reach the mode conversion layer. This damping greatly diminishes any mode conversion effect on the heating profiles.

The initial electron RF power deposition is shown in Figs. (5-46-47). Figure (5-46) shows that electron heating occurs before mode conversion layer is reached, which Fig. (5-47) shows to be over 62% of the total RF power.

The remaining RF power is deposited in the deuterons as shown in Figs. (5-48-49). Since the mode conversion layer lies near \( r = 490 \) cm, we see that most of the deuterium heating also occurs before this layer is reached.

The components of the RF electric field are shown in Figs. (5-50-52). The wave damping is very strong in the cyclotron resonance zone. The electric field strength is greatly reduced by the time the wave reaches the mode conversion layer. From these figures and the RF power deposition, we can conclude that even if mode conversion did occur, as in PLT, the RF power deposition would not change. The wave is too strongly damped before the mode conversion layer is reached.

As the plasma heats and the resonance zone widens, the RF power deposition shifts towards the low field side, away from the mode conversion layer. At \( t = 1.5 \) sec, the peak electron heating has shifted \( \approx 28 \) cm and been reduced from 79 MW to 75 MW (Figs.
(5-53-54)). Figures (5-55-56) show the deuterium heating also shifting towards the low field side. The broadening of the cyclotron resonance layer produces a stronger damping of the incoming fast wave and an increase in fundamental minority heating.

Figure (5-57) shows the alpha heating remains center-peaked. At \( t = 2.0 \text{ sec} \), 233 MW of power goes to the electrons while 68 MW goes to the ions. Since the alphas are preferentially slowing down on the electrons \( T_e > T_i \). Keeping the alpha power peaked near the center can reduce the amount of auxiliary heating needed to reach ignition\(^{34}\). Ignition can be achieved in the center before global ignition is reached. An outwardly propagating burn wave can then ignite the remaining plasma.

The feedback control keeps the deuterium and tritium densities constant with time, while the alpha density increases with time. However, at the RF shut off point, the alpha density is only \( 1.3 \times 10^{12} \text{ cm}^{-3} \) (Fig. (5-58)), still too low to absorb any RF power. The plasma temperature runs away under "PLT empirical" scaling. Both \( T_e(0) \) and \( T_i(0) > 36 \text{ keV} \) at \( t = 3.0 \text{ sec} \). There is enough alpha power being produced by the time the RF is turned off to keep the plasma ignited.

A 50\%D-50\%T plasma would be optimal from the standpoint of alpha production. Therefore, we now return to the heating scenario examined in Chapter 2.
5.1. Second Harmonic Heating of Deuterium in NUWMAK

Table 5-3 lists the plasma parameters for this second harmonic heating scenario. The increased concentration of deuterium (50%D-50%T) produces the greatest number of alpha fusions for the same plasma temperature. Approximately 100 MW of RF power \( f = 92 \text{ MHz} \) for 1.5 sec must be provided for the plasma to reach ignition. The initial electron, deuterium and tritium density profiles are again parabolic with \( n_D = n_T = 1.5 \times 10^{14} \text{ cm}^{-3} \). The initial alpha, carbon and temperature profiles are the same as before (Figs. (5.40-41)).

With the increased concentration of deuterium, Fig. (5.59) shows that mode conversion now occurs. The mode conversion layer is between the 65th and 66th grid points \( (503.5 \text{ cm} \leq r \leq 504.5 \text{ cm}) \). As the temperature increases, the cyclotron resonance layer again widens. At \( t = 1.5 \text{ sec} \), the resonance layer increases up to 50 cm in width towards the low field side. Meanwhile, the mode conversion layer shifts towards the high field side while also increasing in width. At \( t = 1.5 \text{ sec} \), the mode conversion layer lies between the 71st and 75th mesh points \( (404.5 \text{ cm} \leq r \leq 492.5 \text{ cm}) \).

Using the mode conversion coefficients in Eqs. (5.49-50), we find that the fast wave is completely reflected at the mode conversion layer. However, as in the previous scenario, the incoming fast wave is so strongly damped before the mode conversion layer is reached that mode conversion effects are minimal.
Second harmonic heating of deuterium with third harmonic heating of tritium \((\omega = 2\omega_{CD} = 3\omega_{CT})\) was first proposed as a heating scheme because direct ion heating was thought to occur\(^{(13-14, 35-36)}\). However, the higher densities and temperatures which will occur in the reactor regime lead to substantial electron Landau and transit time damping of the fast wave before the center is reached. Figures \((5-60, 62)\) show that initially, over \(84\%\) of the RF power is damped on the electrons before the mode conversion layer is reached. The remaining power is also deposited in the deuterons before the mode conversion layer is reached \((\text{Figs. (61, 63)})\). No power is deposited directly in the tritons.

As the plasma temperature increases, the RF heating again shifts away from the mode conversion layer and towards the low field side. As the averaged electron temperature increases, more power is deposited in the electrons. At \(t = 1.5\) sec, over \(96\%\) of the RF power is absorbed by the electrons. The shift in the peak RF heating is small enough to keep both the densities and temperatures peaked at the center. The alpha heating profiles are still centrally peaked at \(t = 1.5\) sec \((\text{Fig. (5-64)})\). When the RF is turned off, there is almost 200 MW of alpha power being deposited in the plasma.

The electron and ion RF time-dependent heating profiles are shown in Figs. \((5-62-63)\). The deuterium and tritium densities again remain constant. The carbon density, although decreasing, also remains parabolic. Figures \((5-65-66)\) show that the carbon
and alpha densities are too low during the RF pulse to absorb any power. Figures (5-67-68) again show the central ion and electron temperatures running away under PLT scaling.

When $k_n = 0.2 \, \text{cm}^{-1}$, we again conclude that electron Landau and transit time damping, away from the mode conversion zone, are the dominant RF heating processes. Mode conversion appears to play a minor role in this case.

When $k_n$ is decreased from $0.2 \, \text{cm}^{-1}$ to $0.1 \, \text{cm}^{-1}$, the mode conversion layer increases. For the same initial conditions as before, the cyclotron resonance layer decreases while the mode conversion layer increases in width ($502.5 \, \text{cm.} \leq r \leq 504.5 \, \text{cm}$), as shown in Fig. (5-69). In this case, both deuterium ($\sim 59 \, \text{MW}$) and direct tritium absorption ($\sim 0.6 \, \text{MW}$) increase while electron heating decreases ($\sim 1 \, \text{MW}$). Although the ion heating is more peaked in this case, neither the density nor temperature profiles are affected. The same amount of RF power is needed to reach ignition.

When $k_n$ is changed to $0.3 \, \text{cm}^{-1}$, mode conversion is completely eliminated (Fig. (5-70)). Electron Landau and transit time damping is again the dominant heating process absorbing over 84% of the RF power at $t = 0$ sec. The remaining power is deposited in the deuterons. Figure (5-71) shows that the electron heating profile does not differ from Fig. (5-62) where mode conversion occurred. Increasing $k_n$ increases the deuterium heating width (Fig. (5-72)). However, the density and temperature profiles are unaffected. The same amount of power is needed as in the previous example.
From these three cases, \( k_n = 0.2 \text{ cm}^{-1}, 0.1 \text{ cm}^{-1}, 0.3 \text{ cm}^{-1} \), we find that as \( k_n \) increases, the mode conversion threshold rises. Mode conversion appears to be eliminated for \( k_n \geq 0.3 \text{ cm}^{-1} \). As \( k_n \) increases, the cyclotron resonance zone increases in width. Using McVey's mode conversion coefficients, we find that mode conversion does not affect the temperature and density profiles or the amount of RF power needed to reach ignition. The energy which reaches the mode conversion layer is reflected and passes through the resonance layer again on the way back to the low field side. If a standing wave developed, it would occur over half of the minor cross section and not contain much energy. In our model, it appears that the fast wave is damped within one pass across the minor cross section and no eigenmode should develop.

In both fundamental and second harmonic heating, electron heating is the dominant RF process. Both the Fokker-Planck and space-time studies show that \( T_i \sim T_e \) in a reactor as long as the auxiliary heating is near the center. The amount of auxiliary power needed is independent of which species, ions or electrons, is heated. The requirement is dependent on the relative concentration of deuterium to tritium. Since electron heating appears to dominate, we investigate higher ion harmonic heating.

5.1. Third Harmonic Heating of Deuterium in NUWMAK

We now examine a 50%D-50%T plasma with \( \omega = 3\omega_{CD} \) (\( f = 138 \text{ MHz} \)). As the RF frequency increases, waveguides decrease in size.
If waveguides are used for the launching structure, a three meter wide waveguide for $\omega = \omega_{CD}$ becomes one meter wide in this case. A waveguide one meter wide should be smaller than neutral beam ports in a reactor. At RF frequencies of 100–200 MHz, efficient high power RF sources are still available. Therefore, 3rd or 4th ion harmonic heating could be an attractive heating scheme from a technological viewpoint.

For $\omega = 3\omega_{CD}$, the same initial profiles as in the $2\omega_{CD}$ cases are used. As the plasma temperature increases, the cyclotron resonance surface again widens (Figs. (5-73-74)). Although mode conversion appears within 500 ms after the RF is turned on, there is again no affect on the temperature and density profiles. Figure (5-75) shows 97% of the RF power being deposited in the electrons before the mode conversion layer is reached. The remaining power is deposited in the deuterons. The ion heating profile is shown in Fig. (5-76). The electron heating again shifts towards the low field side as the electron temperature increases.

As the electron heating shifts, the temperature profiles begin to flatten in the center (Figs. (5-77-78)). The density profiles remain parabolic, causing the alpha power production to remain centrally peaked (Fig. (5-79)). The amount of RF heating needed to reach ignition equals that required in second harmonic heating.
We conclude that \( \omega = 3\omega_{CD} \) appears to be an attractive heating regime. There is substantial electron heating, close to the center but away from the mode conversion layer. Mode conversion effects on densities and temperatures should be minimal. In the next chapter, we compare the results obtained by the three different RF models which have been used.
### Table 5-1

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<p>| Initial $\bar{n}_0$ (cm$^{-3}$) | $\sim 5.4 \times 10^{11}$ |
| Initial $n_D$ (edge) (cm$^{-3}$) | $5.8 \times 10^{12}$ |
| Initial $n_H$ (edge) (cm$^{-3}$) | $5.8 \times 10^{11}$ |
| $T_i$ (edge) (eV)                  | 5 |
| $T_e$ (edge) (eV)                  | 5 |
| $T_o$ (eV)                        | 5 |
| Initial $T_i$ (o) (eV)             | 600 |
| Initial $T_e$ (o) (eV)             | 1300 |
| Initial $T_i$ (eV)                 | $\sim 206$ |
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At RF Shutoff

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MODE CONVERSION

No (for $\frac{N_D}{N_e} \leq 10\%$)
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Possibly for $t \geq 500$ ms
Table 5-3 (cont.)
At RF Shutoff

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Table 5-3 (cont.)
LIST OF FIGURES

(5-1) Computational mesh spacing for RF calculations and transport calculations; poloidal symmetry is assumed and in the transport calculations the plasma is symmetric about r = 0.

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(5-3) Complex reflection and mode conversion coefficients in PLT for waves incident from the high field side (subscript 1) and the low field side (subscript 2) (reproduced from Ref(5-28)).

(5-4) Electron density in PLT at t = 0 ms.

(5-5) Deuterium density in PLT at t = 0 ms.

(5-6) Hydrogen density in PLT at t = 0 ms.

(5-7) Electron temperature in PLT at t = 0 ms.

(5-8) Ion temperature in PLT at t = 0 ms.

(5-9) Fast wave Re$k_1^2$ (solid line), Im$k_1^2$ (dashed line) and cold plasma $k_1^2$ (dotted line) in PLT, t = 0 ms; $|T|^2 = 1.0$, $|R|^2 = |C|^2 = 0.0$.

(5-10) Re$E_+$ (solid line) and Im$E_+$ (dashed line) in PLT at t = 0 ms.

(5-11) Re$E_-$ (solid line) and Im$E_-$ (dashed line) in PLT at t = 0 ms.

(5-12) Re$E_\|_\|$ (solid line) and Im$E_\|_\|$ (dashed line) in PLT at t = 0 ms.

(5-13) Hydrogen RF power deposition along the entire cross section in PLT at t = 0 ms.
(5-14) Hydrogen RF power deposition summed over flux surfaces in PLT at \( t = 0 \) ms.

(5-15) Deuterium RF power deposition summed over flux surfaces in PLT at \( t = 0 \) ms.

(5-16) Oxygen density profile in PLT.

(5-17) Deuterium density profile in PLT.

(5-18) Hydrogen density profile in PLT.

(5-19) Electron temperature profile in PLT.

(5-20) Ion temperature profile in PLT.

(5-21) Ion RF power deposition in PLT (\( P^i_{RF} = P^D_{RF} + P^H_{RF} + P^O_{RF} \)); \( P_{RF} = 350 \) kW.

(5-22) Electron RF power deposition in PLT; \( P_{RF} = 350 \) kW.

(5-23) Electron temperature in PLT at RF shutoff, \( t = 100 \) ms.

(5-24) Ion temperature in PLT at \( t = 100 \) ms.

(5-25) Hydrogen RF power deposition along the entire cross section in PLT at \( t = 100 \) ms.

(5-26) Hydrogen RF power deposition summed over flux surfaces in PLT at \( t = 100 \) ms.

(5-27) \( R_{ekf}^2 \) and \( R_{e i.b}^2 \) (solid lines) and \( Imk_{ekf}^2 \) and \( Imk_{e i.b}^2 \) (dashed lines) for PLT, \( k_u = 0.1 \) cm\(^{-1}\), \( T_H / T_D = C_H = 1.0 \) at \( t = 0 \) ms. Mode conversion occurs between grid points 73 and 74 (122.5 cm \( \leq r \leq 123.0 \) cm).

(5-28) \( R_{ekf}^2 \) and \( R_{e i.b}^2 \) (solid lines) and \( Imk_{ekf}^2 \) and \( Imk_{e i.b}^2 \) (dashed lines) for PLT, \( k_u = 0.1 \) cm\(^{-1}\), \( C_H = 2.0 \) at \( t = 0 \) ms. No mode conversion occurs.

(5-29) \( R_{ekf}^2 \) and \( R_{e i.b}^2 \) (solid lines) and \( Imk_{ekf}^2 \) and \( Imk_{e i.b}^2 \) (dashed lines) for PLT \( k_u = 0.1 \) cm\(^{-1}\), \( C_H = 2.0 \), \( t = 100 \) ms. Evanescent zone in center disappears.

(5-30) Hydrogen RF power deposition summed over flux surfaces in PLT, \( C_H = 2.0 \), \( t = 0 \) ms.
(5-31) Hydrogen RF power deposition summed over flux surfaces in PLT, $C_H = 2.0$, $t = 100$ ms.

(5-32) $Re k^2_f$ and $Re k^2_{I.B.}$ (solid lines) and $Im k^2_f$ and $Im k^2_{I.B.}$ (dashed lines) for PLT, $k_n = 0.2$ cm$^{-1}$, $C_H = 1.5$, $t = 100$ ms.

(5-33) Hydrogen RF power deposition summed over flux surfaces in PLT, $k_n = 0.2$ cm$^{-1}$, $C_H = 1.5$, $t = 100$ ms.

(5-34) Deuterium RF power deposition summed over flux surfaces in PLT, $k_n = 0.2$ cm$^{-1}$, $C_H = 1.5$, $t = 100$ ms.

(5-35) Hydrogen RF power deposition summed over flux surfaces in PLT, $k_n = 0.2$ cm$^{-1}$, $C_H = 1.5$, $t = 100$ ms.

(5-36) Ion temperature in PLT, $k_n = 0.2$ cm$^{-1}$, $C_H = 1.5$, $t = 100$ ms.

(5-37) Electron density in NUWMAX, $f = f_{CD}$, $t = 0$ sec.

(5-38) Deuterium density in NUWMAX, $f = f_{CD}$, $t = 0$ sec.

(5-39) Tritium density in NUWMAX, $f = f_{CD}$, $t = 0$ sec.

(5-40) Electron temperature in NUWMAX, $f = f_{CD}$, $t = 0$ sec.

(5-41) Ion temperature in NUWMAX, $f = f_{CD}$, $t = 0$ sec.

(5-42) Electron temperature in NUWMAX, $f = f_{CD}$ at RF shutoff, $t = 2.0$ sec.

(5-43) Ion temperature in NUWMAX, $f = f_{CD}$, $t = 2.0$ sec.

(5-44) $Re k^2_f$ (solid line), $Im k^2_f$ (dashed line) and cold plasma $k^2_\perp$ (dotted line) for NUWMAX, $f = f_{CD}$, at $t = 0$ sec. No mode conversion, $|T|^2 = 1.0$, $|R|^2 = |C|^2 = 0.0$.

(5-45) $Re k^2_f$ (solid line), $Im k^2_f$ (dashed line) and cold plasma $k^2_\perp$ (dotted line) for NUWMAX, $f = f_{CD}$, at $t = 1.5$ sec. No mode conversion, $|T|^2 = 1.0$, $|R|^2 = |C|^2 = 0.0$.

(5-46) Electron RF power deposition along the entire cross section in NUWMAX, $f = f_{CD}$, $t = 0$ sec.

(5-47) Electron RF power deposition summed over flux surfaces in NUWMAX, $f = f_{CD}$, $t = 0$ sec.
Deuterium RF power deposition along the entire cross section in NUWMAK, \( f = f_{CD}, \ t = 0 \) sec.

Deuterium RF power deposition summed over flux surfaces in NUWMAK, \( f = f_{CD}, \ t = 0 \) sec.

Re\( E_+ \) (solid line) and Im\( E_+ \) (dashed line) in NUWMAK, \( f = f_{CD}, \ t = 0 \) sec.

Re\( E_- \) (solid line) and Im\( E_- \) (dashed line) in NUWMAK, \( f = f_{CD}, \ t = 0 \) sec.

Re\( E_{II} \) (solid line) and Im\( E_{II} \) (dashed line) in NUWMAK, \( f = f_{CD}, \ t = 0 \) sec.

Electron RF power deposition along the entire cross section in NUWMAK, \( f = f_{CD}, \ t = 1.5 \) sec.

Electron RF power deposition summed over flux surfaces in NUWMAK, \( f = f_{CD}, \ t = 1.5 \) sec.

Deuterium RF power deposition along the entire cross section in NUWMAK, \( f = f_{CD}, \ t = 1.5 \) sec.

Deuterium RF power deposition summed over flux surfaces in NUWMAK, \( f = f_{CD}, \ t = 1.5 \) sec.

Alpha power deposited in the electrons and ions in NUWMAK, \( f = f_{CD}, \ t = 2.0 \) sec.

Alpha density in NUWMAK, \( f = f_{CD}, \ t = 2.0 \) sec.

Re\( k_{lf}^2 \) and Re\( k_{lI.B.}^2 \) (solid lines) and Im\( k_{lf}^2 \) and Im\( k_{lI.B.}^2 \) (dashed lines) in NUWMAK, \( f = 2f_{CD}, \ t = 0 \) sec. Mode conversion occurs with the mode conversion layer between the 65th and 66th grid points (503.5 cm \( \leq r \leq 504.5 \) cm).

Electron RF power deposition along the entire cross section in NUWMAK, \( f = 2f_{CD}, \ t = 0 \) sec.

Deuterium RF power deposition along the entire cross section in NUWMAK, \( f = 2f_{CD}, \ t = 0 \) sec.

Electron RF power profile in NUWMAK, \( f = 2f_{CD} \).
(5-63) Ion RF power profile in NUWMAK, $f = 2f_{CD}$, $P^i_{RF} = P^D_{RF} + P^T_{RF} + P^C_{RF} + P^α_{RF}$.

(5-64) Alpha power deposited in the electrons and ions in NUWMAK, $f = 2f_{CD}$ at RF shutoff, $t = 1.5$ sec.

(5-65) Carbon density in NUWMAK, $f = 2f_{CD}$, $t = 1.5$ sec.

(5-66) Alpha density in NUWMAK, $f = 2f_{CD}$, $t = 1.5$ sec.

(5-67) Ion temperature in NUWMAK, $f = 2f_{CD}$, $t = 1.5$ sec.

(5-68) Electron temperature in NUWMAK, $f = 2f_{CD}$, $t = 1.5$ sec.

(5-69) $\text{Re}k^2_{\phi}$ and $\text{Re}k^2_{\Omega B}$ (solid lines) and $\text{Im}k^2_{\phi}$ and $\text{Im}k^2_{\Omega B}$ (dashed lines) in NUWMAK, $f = 2f_{CD}$, $k_\| = 0.1 \text{ cm}^{-1}$, $t = 0$ sec. Mode conversion occurs with the mode conversion layer between the 65th and 67th grid points (502.5 cm $\leq r \leq 504.5$ cm).

(5-70) $\text{Re}k^2_{\phi}$ and $\text{Re}k^2_{\Omega B}$ (solid lines) and $\text{Im}k^2_{\phi}$ and $\text{Im}k^2_{\Omega B}$ (dashed lines) in NUWMAK, $f = 2f_{CD}$, $k_\| = 0.3 \text{ cm}^{-1}$, $t = 0$ sec. No mode conversion occurs.

(5-71) Electron RF power profile in NUWMAK, $f = 2f_{CD}$, $k_\| = 0.3 \text{ cm}^{-1}$.

(5-72) Deuterium RF power profile in NUWMAK, $f = 2f_{CD}$, $k_\| = 0.3 \text{ cm}^{-1}$.

(5-73) $\text{Re}k^2_{\phi}$ (solid line), $\text{Im}k^2_{\phi}$ (dashed line) and cold plasma $k^2_\perp$ (dotted line) in NUWMAK, $f = 3f_{CD}$, $t = 0$ sec.

(5-74) $\text{Re}k^2_{\phi}$ (solid line), $\text{Im}k^2_{\phi}$ (dashed line) and cold plasma $k^2_\perp$ (dotted line) in NUWMAK, $f = 3f_{CD}$, $t = 1.5$ sec.

(5-75) Electron RF power profile in NUWMAK, $f = 3f_{CD}$.

(5-76) Ion RF power profile in NUWMAK, $f = 3f_{CD}$, $P^i_{RF} = P^D_{RF} + P^T_{RF} + P^C_{RF} + P^α_{RF}$.

(5-77) Electron temperature in NUWMAK, $f = 3f_{CD}$ at RF shutoff, $t = 1.5$ sec.

(5-78) Ion temperature in NUWMAK, $f = 3f_{CD}$, $t = 1.5$ sec.

(5-79) Alpha power deposited in the electrons and ions in NUWMAK, $f = 3f_{CD}$, $t = 1.5$ sec.
TRANSPORT CALCULATIONS

Fig. (5-1)

NUWMAK

Grid Points, \( n \geq 1 \)

TRANSMITTED FAST WAVE, \( E_a T \)

ION BERNSTEIN WAVE, \( E_a C_E \)

REFLECTED FAST WAVE, \( E_a R \)

INCIDENT FAST WAVE, \( E^i \)

SCHEMATIC OF THE MODE CONVERSION PROCESS

Fig. (5-2)
Reflection and conversion coefficients as a function of $k_{ll}$ (upper scale) or $n$ (lower scale) for PLT parameters: $n_a = 6.2 \times 10^{13} m^3$, $R_o = 1.3 m$, $B_o = 3.6 W/m^2 (f = 55 MHz)$, $S_o = 0.01$, $n_f = 0$. Dashed lines show variation without absorption. Subscripts $r$ and $i$ denote real and imaginary parts of the conversion coefficients and, although some parts change sign, the magnitudes of $C_1$ and $C_2$ decrease monotonically with increasing $n$.

Fig. (5-3)
Fig. (5-4) AVERAGE ELECTRON DENSITY (CH-3) = 2.9773 x 10^9

Fig. (5-5) AVERAGE DEUTERIUM DENSITY (CH-3) = 2.3483 x 10^9

Fig. (5-6) AVERAGE HYDROGEN DENSITY (CH-3) = 2.3443 x 10^9
ELECTRON TEMPERATURE

ION TEMPERATURE

Fig.(5-7)

Fig.(5-8)

Fig.(5-9)
Fig. (5-21)

Fig. (5-22)
Fig. (5-23)

Fig. (5-24)
RF POWER TO HYDROGEN

TIME (MSEC) = 100.0000

Fig.(5-25)

RF POWER TO HYDROGEN

TIME (MSEC) = 100.0000

TOTAL POWER (WATTS) = 3.1593 x 10^5
% RF POWER = 90.239

Fig.(5-26)
Fig. (5-33)

Fig. (5-34)

Fig. (5-35)
ELECTRON TEMPERATURE

Fig. (5-42)

ION TEMPERATURE

Fig. (5-43)
FIG. (5-44)

FIG. (5-45)
RF POWER TO ELECTRONS

Fig. (5-46)

RF POWER TO ELECTRONS

TOTAL POWER (WATTS) = 7.8959 x 10^7 x RF POWER = 62.625

Fig. (5-47)
RF POWER TO DEUTERIUM

Fig.(5-48)

RF POWER TO DEUTERIUM

TOTAL POWER IN coded - 1.0479×10^7 2 RF POWER - 37.051

Fig.(5-49)
RF POWER TO ELECTRONS

TIME (MSEC) = 1500.000

Fig.(5-53)

RF POWER TO ELECTRONS

TIME (MSEC) = 1500.0000  R (CM)

TOTAL POWER (WATTS) = 7.8072 x 10^2  Z RF POWER = 39.439

Fig.(5-54)
Fig. (5-55)

RF POWER TO DEUTERIUM

Time (msec) = 1500.0000

Fig. (5-56)

RF POWER TO DEUTERIUM

Time (msec) = 1500.0000

Total Power (Watts) = 5.0347x10^7
RF Power = 39.863
Fig. (5-57)

Fig. (5-58)
ION TEMPERATURE

Fig. (5-67)

ELECTRON TEMPERATURE

Fig. (5-68)
Fig. (5-77)

Fig. (5-78)

Fig. (5-79)
CHAPTER 5 REFERENCES


CHAPTER 6

COMPARISON OF NUMERICAL RESULTS

6.A. PLT

In this section, the results for PLT obtained from the Fokker-Planck code, FORCE, and the space-time code, STICR-I are compared. The Fokker-Planck calculations revealed that only the proton distribution becomes non-Maxwellian in PLT. The protons become hotter than the deuterons, reaching an effective temperature of $1.7 \times T_D$ at the time of the RF shutoff. Therefore, two ion energy balance equations should be solved when using a space-time transport code. STICR-I, however, uses only one.

Although the ratio of $T_H$ to $T_D$ is important in determining the mode conversion threshold, mode conversion itself leaves the density and temperature profiles unchanged. STICR-I neglects non-Maxwellian effects but does allow $T_H \neq T_D$ in the RF calculations. However, FORCE assumes that all of the wave energy is absorbed in the cyclotron layer near the plasma center. While these two codes examine the same experiment, they include different aspects. If the results obtained are different, it is necessary to understand why.

Figure (6-1) shows the change in the spatially averaged electron and ion temperatures with time. It appears that the
initial temperatures do not agree between the two codes. This is because the average temperature is defined differently. In the space time code

$$\bar{T}_{i,e} = \frac{\int_0^a T_{i,e}(r) r \, dr}{\int_0^a r \, dr}.$$  \hfill (6-1)

In FORCE, the average ion temperature is a density weighted equivalent Maxwellian temperature

$$\bar{T}_i = \frac{\frac{2}{3} \sum \int \frac{1}{2} m v^2 f_\beta(v) \, d^3v}{\sum f_\beta(v) \, d^3v}.$$  \hfill (6-2)

while the electron temperature is

$$\bar{T}_e = \frac{\frac{2}{3} \int \frac{1}{2} m_e v^2 f_e(v) \, d^3v}{\int f_e(v) \, d^3v}.$$  \hfill (6-3)

In STICR-I, this would be equivalent to defining the average temperature as

$$\bar{T} = \frac{\int_0^a n(r) T(r) \, r \, dr}{\int_0^a n(r) \, r \, dr}.$$  \hfill (6-4)
If the density-weighted temperature was used in STICR-I, the two codes would agree. From the Fokker-Planck code, we find

\[ \Delta T_1 \sim 150 \text{ eV} \]  \hspace{1cm} (6-5)

while from the space-time code

\[ \Delta T_1 \sim 220 \text{ eV}. \]  \hspace{1cm} (6-6)

These results are comparable. Both codes show the average ion temperature doubling, in good agreement with the experiment. This results from the good agreement between charge exchange, radial losses and RF heating terms shown in Fig. (6-2). The slight difference between the two ion temperature increases may be a result of the more accurate treatment of Coulomb collisions in the Fokker-Planck calculations.

Figure (6-1) shows that \( T_e \) initially drops in the space-time code. This drop results from a difference in the value of the Ohmically heated \( T_e \) predicted by STICR-I, and the one which actually occurs. Upon using the experimental value, \( T_e \) drops. Figure (6-4) shows the mechanism which causes this decrease. Initially, the \( \sim 20 \text{ kW} \) of RF heating is more than offset by the \( \sim 80 \text{ kW} \) of power lost through radiation. The difference between the Ohmic heating (\( \sim 500 \text{ kW} \)) and the radial loss (\( \sim 350 \text{ kW} \)) together with the radiation results in \( \sim 100 \text{ kW} \) of net heating power. Since \( T_e > T_i \), the electrons initially lose \( \sim 600 \text{ kW} \) to the ions. The
net result is a loss of ~500 kW of power from the electrons. Consequently, $T_e$ drops. As $T_e$ drops, both the radiation and re-thermalization losses decrease. After 10 ms, $T_i > T_e$ in STICR-I. The electron-ion equilibration terms now become an energy source for the electrons. The electron temperature then begins to rise. The electron temperature in STICR-I reaches an equilibrium value near $t = 60$ ms, when the RF, Ohmic, and electron-ion equilibration heating terms equal the radial and radiation loss terms. After the initial drop in the electron temperature, $\Delta T_e \approx 50$ eV, in agreement with both the experiment and FORCE.

On comparing Fig. (6-4) to Fig. (6-3), one can see the reason behind the initial discrepancy in $\Delta T_e$ between the two codes. The radiation and RF heating values calculated by FORCE agree with the STICR-I results. However, the Ohmic heating term is larger while the radial loss term is smaller. These results, coupled with the more accurate Coulomb collisions calculations ($T_H \neq T_D$) enables the electron temperature to initially rise. Unlike STICR-I, if no RF was used, $T_e$ would remain constant rather than fall. Unlike STICR-I, during the RF pulse, $T_e > T_D, T_o$ during the RF pulse. There is always a net loss of energy from the electrons due to collisions, even though $T_H > T_e$ during most of the pulse. The electrons lose approximately 100 kW through collisions. This balances the net heating from Ohmic heating minus radial loss term. Therefore, the electron temperature increases less than 50 eV.
We find that STICR-I and FORCE agree with each other as well as with the actual PLT ICRF heating experiment. In PLT, the spatial effects of ICRF heating are small. Most of the heating occurs in the center, as assumed in FORCE. Mode conversion does not appear to affect either the RF power deposition or the density and temperature profiles. In addition, since \( n_D > n_H \), the non-Maxwellian aspects of ICRF heating do not affect the overall ion temperature. The majority of the ions remain Maxwellian, as assumed in STICR-I. Even though \( T_H \sim 1.5 \times T_D \), since \( \frac{n_H}{n_D} = 0.1 \), this effect is small on the ion energy balance. One can assume that \( T_H = T_D = T_i \). In comparison, NUWMAK shows a great disagreement between codes.

6.B. NUWMAK

Chapter 4 showed that in both the fundamental and second harmonic heating cases, the deuterons and tritons were Maxwellian with \( T_D \approx T_n \), even though most of the RF power was deposited in the deuterons. The only species found to be non-Maxwellian, excepting the alphas, was the carbon impurity in the fundamental heating case. Whereas FORCE assumes that all of the RF power is deposited at the center, STICR-II shows that this is not the case. Depending upon the ion harmonic and \( k_n \), from 40%–95% of the RF power is absorbed by the electrons before the plasma center is reached. We therefore expect the carbon impurity to remain Maxwellian when the cyclotron resonance is kept near the center.
There is not enough RF power available at the center to produce a non-Maxwellian impurity.

For this same reason, mode conversion effects should be small when the RF wave is launched from the low field side. In this case, the wave, which is largely damped before the center is reached, must pass through the resonance zone to reach the mode conversion layer, which lies on the high field side.

Contrary to both the 0-D transport code, PORT, and the Fokker-Planck code, FORCE, the dominant RF heating mechanism, as one approaches reactor-like parameters, should be electron Landau and transit time damping. Although ion heating is dominant at the center, there is little wave energy reaching this region.

Both PORT and FORCE predict dominant ion heating in the second harmonic heating case. STICR-I does not. Therefore, we conclude that the predominance of electron heating is caused by spatial effects. The plasma should remain Maxwellian.

Comparing PORT and STICR-II for the same case, $\omega = 2\omega_\text{CP}$, we see that even though the plasma is larger and denser, STICR-II predicts that less auxiliary heating energy must be supplied. The 225 MJ calculated by PORT may actually be a lower bound since there are no impurities, $Z_{\text{eff}} = 1$. PORT neglects the profile effects which allow ignition to be achieved at the center before global ignition occurs. This can reduce the amount of heating power needed. In addition, PORT uses trapped particle scaling.
This scaling transports more energy radially than does "empirical PLT" scaling. Although the RF heating profiles do not change with scaling, the amount of auxiliary heating required to reach ignition is very sensitive to the scaling law. If STICR-II used trapped particle scaling, as much as 100 MJ of additional RF heating would be needed.

FORCE, which neglects charge exchange, radial, and radiation losses, and Ohmic heating, obtains $T_e \approx T_i$. Because of the high densities and temperatures in a reactor, the electron-ion equilibration time is quite short. Coupling this with the space-time results, we conclude that the requirement for auxiliary heating should be the same when either the ions or the electrons are heated, as long as the power is deposited near the plasma center. ICRF produced enhancement of the alpha power production should be minimal. Since electron heating is dominant higher ion harmonic heating, which heats electrons, may become more attractive from a technological standpoint. There appears to be no significant heating benefit in using fundamental or second harmonic heating over higher harmonics.
LIST OF FIGURES

(6-1) \[ T_e \text{ and } T_i \text{ in PLT}, P_{RF} = 350 \text{ kW}. \]

(6-2) \[ \text{Spatially-averaged ion RF heating, radial and charge exchange losses in PLT.} \]

(6-3) \[ \text{Spatially-averaged electron Ohmic heating, radial and radiation losses in PLT.} \]

(6-4) \[ \text{Spatially-averaged RF and Ohmic heating, radial and radiation losses, and the electron-ion equilibration heating (solid line) and cooling (dashed line) term.} \]
CHAPTER 7

SUMMARY AND CONCLUSIONS

In Chapter 1 we saw that ICRF was quite efficient in heating DIVA, TFR and PLT. The main body ion temperature doubled in several experiments and the overall heating efficiency was often greater than 80%, approaching that obtained by neutral beams.

The 0-D transport code with trapped particle scaling, discussed in Chapter 2, predicted that 175 MW for 1 second with $\omega = 2\omega_{cD}$ was enough power to ignite the NUWMAK conceptual reactor. In this scenario, the dominant heating mechanism was second harmonic damping by finite Larmour radius effects. The 0-D code neglects spatial effects which, as shown in Chapter 5, are very important.

The complete 3x3 hot plasma dispersion tensor, accurate for any temperature for ion harmonics up to five, was derived in Chapter 3. We have included impurities and, in the reactor cases, alphas. The complex value of $k_\perp$ obtained for a given real value of $k_{\parallel}$ and $\omega$ agrees well with the cold plasma value. The results compare quite favorably with those obtained by Stix in his reactor case.

In Chapter 4 we used a 0-D in real space, 2-D in velocity space, time-dependent, multi-species Fokker-Planck code to examine PLT and NUWMAK. In this chapter, as in Chapter 2, we have neglected spatial and any possible mode conversion effects. In PLT over 80% of the RF power was absorbed, by the presence of protons (~10%),
through fundamental minority damping. The protons became non-Maxwellian with $T_H/T_D \sim 1.7$ and a high energy tail of 18 keV. The deuterium, electrons and oxygen species remained Maxwellian while the spatially-averaged deuterium temperature doubled. The electron temperature, as in the experiment, did not increase significantly. We were able to simulate the PLT experimental results when charge exchange, radial and radiation losses, and Ohmic heating were included.

The dispersion relation calculations assumed that each distribution could be represented by an "equivalent Maxwellian" distribution. As a possible improvement, one could represent the high energy proton tail in PLT as a sum of Maxwellian distributions.

In NUWMAK, the dominant heating was fundamental minority heating of deuterium, when $\omega = \omega_{cD}$, and second harmonic damping when $\omega = 2\omega_{cD}$. In either case, deuterium, tritium and electrons all remained Maxwellian with $T_D \approx T_T \approx T_e$. Little, if any, ICRF produced fusion reactor Q enhancement was found. If the electrons were heated directly by the RF, the amount of auxiliary power needed to reach ignition would remain the same.

In Chapter 5 we examined ICRF heating in PLT and NUWMAK using a WKB slab model coupled to a 1-D multi-species, time-dependent transport code. Using the slab model, we found that in PLT over 90% of the RF power is deposited in the protons at the plasma center over a width of 12-14 cm. We were able to numerically simulate both the ion temperature rise of 600 eV at the center, and the
average ion temperature doubling using "PLT empirical" scaling.
Again, the electron temperature did not rise.

The mode conversion threshold was calculated numerically. The incident fast wave could mode convert to an Ion Bernstein wave if the two dispersion relation roots coalesced over a finite grid mesh. This criterion appears to agree with the analytical expression obtained by Jacquinot. We found that PLT starts in the mode conversion regime. As the plasma heats, the mode conversion threshold rises so that over the final 40 ms of the RF pulse, no mode conversion occurs.

In NUWMAK, due to the higher densities and temperatures, the dominant heating mechanisms were electron Landau and transit time damping. In both NUWMAK and PLT, most of the RF power is absorbed by the plasma before the mode conversion layer is reached. The occurrence of mode conversion does not appear to change either the density or temperature profiles.

In both studies, we assumed that all ions had the same temperature. Although Chapter 4 showed this to be the case for NUWMAK, this does not hold for PLT. In our RF calculations, we found that the ratio $T_H/T_D$ affects the mode conversion threshold. The threshold becomes higher when this ratio is increased. For a more accurate calculation for PLT, two energy balance equations should be used. In addition, we have neglected the possible effects of having a high energy proton tail on the dispersion properties. In PLT calculations, one should couple Fokker-Planck and transport calculations.

For PLT calculations, we used reflection, transmission
and mode conversion coefficients for a slab model obtained by Swanson. For the NUWMAK cases these coefficients were obtained from work performed by McVey. In either case, electron effects were neglected and both formulisms break down at high temperatures \((\lambda + 1)\). In addition, since \(I_{02}\) and \(I_{22}\) represent complicated integral expressions which must be solved numerically, we have assumed that these values do not change significantly when PLT parameters change. Improvements in calculating the various reflection, transmission and mode conversion coefficients should also lead to more accurate results.

In addition, we have neglected any possible 2D and 3D effects. Although poloidal effects may be small, there appears to be a focusing effect when waves are launched from a coil with a finite length in the poloidal direction. We have also neglected all boundary effects and the possibility of eigenmode formation. The WKB slab model is more accurate when the wave is damped within one pass across the minor cross section. In PLT, this appears to be the case; we obtain good agreement with the experiment.

In Chapter 6, we find good agreement between the Fokker-Planck and 1-D transport codes in the PLT simulations. In the reactor cases, spatial effects are very important. These effects produce dominant electron absorption of the RF power which neither the 0-D transport nor the Fokker-Planck codes predict.

In both PLT and NUWMAK, we have neglected the ion-ion hybrid resonance layer. When the minority concentration becomes large (>10%), this layer can dominate the heating process when \(\omega = \omega_{ci}\).
This has been observed in recent TFR experiments. We have examined cases where the minority concentration is low enough or the RF frequency is at an ion harmonic which minimizes this effect. Both the WKB and weak damping formulas break down in the vicinity of this resonance layer. When the ion-ion hybrid resonance layer is in the plasma and not in close proximity to the cyclotron resonance layer, different reflection, transmission and mode conversion coefficients must be used. Although we have not examined these cases, the formulism has been set up to do this. One needs only to determine the position of the ion-ion hybrid resonance layer and add the appropriate wave coefficients.

The following could improve the accuracy of our ICRF heating description:

1. Add a second energy balance equation to the 1-D transport code;
2. Couple the Fokker-Planck and transport codes in performing PLT calculations;
3. Generalize the 3x3 dispersion tensor to include anisotropy and high energy tails;
4. Add coefficients to examine ion-ion hybrid resonance effects;
5. Include boundary effects to examine possible eigenmodes;
6. Improve calculations of reflection, transmission and mode conversion coefficients; and
7. Use ray tracing to examine 2D and 3D effects.
The above seven suggestions are nontrivial and will all involve much work. The codes which have been used are written in such a way that they can be easily updated.

There remains a large amount of work which needs to be done before we can say that we truly understand ICRF heating. We have provided a good start; other theses shall surely follow.
FLOW CHART OF RF DISPERSION RELATION USED IN THE FOKKER - PLANCK CODE.
subroutine rfield

* Material: B. McVee, Univ. of Wisconsin-Madison Report PLP 716 (Feb., 1977)

These subroutines and functions are used in the Fokker-Planck code force
the RF dispersion relation routines in the space-time codes STICK-I and
stick-II are slightly different.

Sets up the calculation of complex KERP and the RF electric field

The RF electric field is the fast wave root.
The next largest root is the ion cyclotron or ion Bernstein root

\( \omega \) = RF frequency (rad/sec)

\( \Delta r \) = position along minor radius (+ in Fokker-Planck code)

\( \Delta n \) = number of species

1 = 1 for electrons

\( \omega \) = cyclotron frequency (rad/sec)

\( \Delta n \) = density (cm\(^{-3}\))

\( \Delta E_{\perp} \) = perpendicular energy (keV)

\( \Delta E_{||} \) = parallel energy (keV)

\( \Delta v_{\perp} \) = perpendicular thermal velocity (cm/sec)

\( \Delta v_{||} \) = parallel thermal velocity (cm/sec)

\( \Delta \omega \) = plasma frequency (rad/sec)

\( \Delta n_{2} \) = \( \Delta n \)

\( m_{\text{Harm}} \) = harmonic number

\( \theta_{0} \) = argument of fried-con te integral

\( z_{1} \) = fried-con te integral

\( f_{10} \) = derivative of fried-con te integral

\( \Phi_{0} \) = \( \theta_{0} \) (see T. K. Pu, Ph.D. Thesis, 1977)

\( \Phi_{10} \) = \( \Phi_{10} \) (see Brian McVee, PLP 716 Feb., 1977)

\( z(1) \) = fast wave KERP

\( z(2) \) = ion Bernstein KERP

x(k) = KERP

rfield calculates the RF electric field components for unbounded plasma

*external det

complex det

complex x(3), z(1,5), dz(1,5)

common/p, pb,pagen, pio2, c, chara, lima, dir, din(15), nsin(15), nesin, nsin, nesin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin, nsin,
121 IF(1.eq.1) ZM1=0.
122 UZ2(1)=4.*QFREDEN(1)*(CHARAC*NUMB(1))/EMASS(1)
123 UP(1)=GART(UZ2(1))
124 DO 4 N=1,NNH
125 AN=N*NH
126 ZETA(AN,1)=(U-AN*UW(1))/XPARN/VTIZ(1)
127 GO TO 105
128 105 ZETA(AN,1)=(U-AN*UW(1))/XPARN/VTIZ(1)
129 DBA(33)=ZETA(AN,1)
130 DBA(34)=ZETA(AN,1)
131 CALL FCREAL(ZETA(AN,1),ZR,ZL,DZR,DZL,NTM,REM,1.0,E7)
132 ZLN(1)=CMLOX(ZR,0.0)+CMLOX(0.1,E7)
133 GO TO 105
134 DZL(AN,1)=CMLOX(DZR,0.0)+CMLOX(DZL,0.1,E8)
135 CALL FCREAL(ZETA(AN,1),ZR,ZL,DZR,DZL,NTM,REM,1.0,E7)
136 ZLN(1)=CMLOX(ZR,0.0)+CMLOX(0.1,E7)
137 DZL(AN,1)=CMLOX(DZR,0.0)+CMLOX(DZL,0.1,E8)
138 FHTH(AN,1)=2.*ZLN(1)*ZLN(1)/VTIZ(1)-XPARN*ZLN(1)*DZL(AN,1)/W
139 FHTH(AN,1)=2.*ZLN(1)*ZLN(1)/VTIZ(1)-XPARN*ZLN(1)*DZL(AN,1)/W
140 FPHI(AN,1)=(1.-AN*UW(1))/ZLN(1)/W
141 GO TO 107
142 107 FPHI(AN,1)=(1.-AN*UW(1))/ZLN(1)/W
143 DBA(35)=REAL(FHTH(AN,1))
144 DBA(36)=AIMAG(FHTH(AN,1))
145 DBA(37)=REAL(FPHI(AN,1))
146 DBA(38)=AIMAG(FPHI(AN,1))
147 DBA(39)=REAL(FPHI(AN,1))
148 DBA(40)=AIMAG(FPHI(AN,1))
149 DBA(41)=REAL(FPHI(AN,1))
150 DBA(42)=AIMAG(FPHI(AN,1))
151 4 CONTINUE
152 GO TO 108
153 108 ZETA(11,1)=W*XPARN/VTIZ(1)
154 DBA(43)=ZETA(11,1)
155 CALL FCREAL(ZETA(11,1),ZR,ZL,DZR,DZL,NTM,REM,1.0,E7)
156 ZLN(1,1)=CMLOX(ZR,0.0)+CMLOX(0.1,E7)
157 DZL(1,1)=CMLOX(DZR,0.0)+CMLOX(DZL,0.1,E8)
158 FHTH(11,1)=2.*ZLN(1,1)/VTIZ(1)-XPARN*ZLN(1,1)*DZL(1,1)/W
159 FHTH(11,1)=2.*ZLN(1,1)/VTIZ(1)-XPARN*ZLN(1,1)*DZL(1,1)/W
160 DBA(44)=REAL(FHTH(11,1))
161 DBA(45)=AIMAG(FHTH(11,1))
162 DBA(46)=REAL(FPHI(11,1))
163 DBA(47)=AIMAG(FPHI(11,1))
164 GO TO 109
165 109 XK(1)=.5*UP2(1)/ZI(1)/W*XPARN
166 CONTINUE calculation cold plasma dispersion relation (KXPAR)**2
167 CONTINUE calculation cold plasma dispersion relation (KXPAR)**2
168 IF(1.eq.1) GO TO 1
169 R=*H2(1)/UC(1)=(*UC(1))
170 Z=*H2(1)/UC(1)=(*UC(1))
171 D1=1.-2.*W2(1)/UC(1)=(*W(UC))
172 1 CONTINUE
173 R1=R*W/S
174 XK=KXPAR**D4/S
175 OK=(*UC(1))**K2
176 XRCR=(*KX-2.)*KXPAR*KXPAR+KX**K2)/(*OK-KXPAR*OK)/S
177 GO TO 110
178 110 Z1=CMLOX(XRCR,8.)
179 X1=10.*XRCR
180 Z2=CMLOX(X1,10.*XRCR)
call zanluf(den,1.e-7,6.0,1.1,z,200,infor,ier)
132 do 9 nn=1,2
133 xip2(nn)=z(nn)
134 ao to 111
135 111 xpass(nn)=csqrt(z(nn))
136 dba(-48)=real(xpass(nn))
137 ao to 112
138 112 dba(+9)=aimag(xpass(nn))
139 5 continue
140 dba(65)=real(z(1))
141 dba(66)=aimag(z(1))
142 dba(67)=real(xpass(1))
143 dba(68)=aimag(xpass(1))
144 call rfld
145 999 call rfcstuf
146 return
147 end
* subroutine fcreal(or,zr,zi,dr,z1,n,rem,lo1)
* WRITTEN BY BRIAN NOVEY, Univ. of Wisconsin-Madison
201 comment CALCULATES FRIED-COTTE INTEGRALS FOR REAL ARGUMENTS
203 comment B. Fried, Conte, THE PLASMA DISPERSION FUNCTION (1961)
204 comment or IS THE REAL ARGUMENT
205 comment zr IS THE REAL PART OF THE INTEGRAL
206 comment zi IS THE IMAGINARY PART OF THE INTEGRAL
207 comment dr IS THE REAL PART OF THE DERIVATIVE OF THE INTEGRAL
208 comment dz1 IS THE IMAGINARY PART OF THE DERIVATIVE OF THE INTEGRAL
209 comment n IS THE NUMBER OF ITERATIONS USED TO CALCULATE THE INTEGRAL
210 comment rem IS THE REMINDER
211 comment lo1 IS THE TOLERANCE SPECIFIED TO CALCULATE THE INTEGRAL
212 * n=8
213 sarpi=1.772454
214 * abs(or)
215 if(x,g.e.1.5) ao to 11
216 if(x,g.e.4.) ao to 12
217 14 call asur(x,zr,dr,zi,n,rem,lo1)
218 1n=1
219 31 ao=--=2
220 if(asag.lt.-8.e1) ao to 81
221 z1=sarpi*exp(aos)
222 if(in=0.1) ao to 92
223 dzi=2.9xzi
224 ao to 20
225 92 dr=2.9x(1+x*zi)
226 dzi=2.9xzi
227 ao to 20
228 91 zi=0.
229 dzi=dzi
230 20 if(or.g.t.8.) return
231 zr=zi
232 dzi=dzi
233 11 call por(x,zr,dr,zi,n,rem,lo1)
12 call intr(x,zr,n,rem,toi)
13 ao to 31
14 12 call intr(x,zr,n,rem,toi)
15 ao to 31
16 end subroutine asur(x,zr,dzr,n,rem,toi)
17 *
18 * WRITTEN BY BRIAN McCoy, Univ. of Wisconsin-Madison
19 cmt cmt CALCULATES FRIED-CONTE INTEGRAL USING ASYMPTOTIC EXPANSION FOR LARGE ARGUMENTS
20 cmt cmt ZETA .at. 4.
21 *
22 *
23 reml=1.
24 s=1.
25 dzr=0.1
26 b=1./x
27 do 10 n=1,40
28 x=real(n)
29 p=p+2e(2,x*(1-1.))*.5
30 if(n.eq.1) go to 11
31 10 pl=pl+b
32 ao to 12
33 11 pl=pl+b
34 12 rem=abs(pl)/abs(s)
35 if(rem.1 or 10) ao to 13
36 13 if(rem.1 or 10) ao to 13
37 rem=rem
38 pl=pl+b
39 dzr=dzr+pl
40 do 10 n=1,40
41 dzr=2.*dzr+pl
42 return
43 end subroutine intr(x,zr,n,rem,toi)
44 *
45 *
46 cmt cmt WRITTEN BY BRIAN McCoy, Univ. of Wisconsin-Madison
47 cmt cmt CALCULATES FRIED-CONTE INTEGRAL FOR 1.5 at. ZETA .ie. 4.
48 *
49 *
50 dimension (f(10))
51 11=0
52 13 1=1.5
53 35 n=8
54 13 f(2)=2.*x(f(1)+x(f(1))
55 10 x=x+1.7
56 11=x=f(1)
57 10 f(x+2)=2.*x(f(x)+2.*x(f(x))
58 n+4.835e3=1000*exp(2))*.125
59 19 b+b+n
60 11=1
61 19 b+b+n
62 n=1.
63 12 i=2.8
64 x=x+1
**Written by Bryan McVey, Univ. of Wisconsin-Madison**

**Calculates Poisson Series Expansion of Fried-Conde Integral for Small Arguments**

```fortran
301 np=2n+1
302 12 fx(i)=fx(i)+p%f(i)
303 303 n=n+1
304 if(i+1.eq.0) go to 13
305 14 zr=f(i)
306 rem=1.0/float(n)+rem
307 return
308 end
309 subroutine pour(x,zr,n,rem,ret)
310 *
311 *
312 writing function
313 *
314 writing function
315 *
316 *
317 z=1.
318 do 10 n=1,40
319 xi=1.0/float(n)
320 p=1.0-zr/2.0*(xi+1.0)
321 if(n.ea.1) go to 11
322 p2=p/2.0
323 if(n.ge.12) go to 11
324 11 p2=pi
325 12 rem=rem*p2/abs(s)
326 if(rem.lt.tol) go to 13
327 10 s=s*p2
328 13 zr=-2.0*wx
329 return
330 end
331 complex function kxx(x)
332 *
333 *
334 this function is the kxx element of the hot plasma dielectric tensor
335 complex kxx
336 common/meant,lu,jk,x1,neq,mmeq,mm,monch,12,nsor,ndata,nnhch,
337 cmw1,cmw2,cmw3,cmw4,cmw5,cmw6,cmw7,cmw8,cmw9,cw1,cw2,
338 cmw10,cmw11,cmw12,cmw13,cmw14,cmw15,cmw16,cmw17,cmw18,
339 cmw19,cmw20,cmw21,cmw22,cmw23,cmw24,cmw25,cmw26,cmw27,
340 cmw28,cmw29,cmw30,cmw31,cmw32,cmw33,cmw34,cmw35,cmw36,
341 cmw37,cmw38,cmw39,cmw40,cmw41,cmw42,cmw43,cmw44,cmw45,
342 cmw46,cmw47,cmw48,cmw49,cmw50,cmw51,cmw52,cmw53,cmw54,
343 cmw55,cmw56,cmw57,cmw58,cmw59,cmw60,cmw61,cmw62,cmw63,
344 cmw64,cmw65,cmw66,cmw67,cmw68,cmw69,cmw70,cmw71,cmw72,
345 cmw73,cmw74,cmw75,cmw76,cmw77,cmw78,cmw79,cmw80,cmw81,
346 cmw82,cmw83,cmw84,cmw85,cmw86,cmw87,cmw88,cmw89,cmw90,
347 cmw91,cmw92,cmw93,cmw94,cmw95,cmw96,cmw97,cmw98,cmw99,
348 cmw100,cmw101,cmw102,cmw103,cmw104,cmw105,cmw106,cmw107,
349 cmw108,cmw109,cmw110,cmw111,cmw112,cmw113,cmw114,cmw115,
350 cmw116,cmw117,cmw118,cmw119,cmw120,cmw121,cmw122,cmw123,
351 cmw124,cmw125,cmw126,cmw127,cmw128,cmw129,cmw130,cmw131,
352 cmw132,cmw133,cmw134,cmw135,cmw136,cmw137,cmw138,cmw139,
353 cmw140,cmw141,cmw142,cmw143,cmw144,cmw145,cmw146,cmw147,
354 cmw148,cmw149,cmw150,cmw151,cmw152,cmw153,cmw154,cmw155,
355 cmw156,cmw157,cmw158,cmw159,cmw160,cmw161,cmw162,cmw163,
356 cmw164,cmw165,cmw166,cmw167,cmw168,cmw169,cmw170,cmw171,
357 cmw172,cmw173,cmw174,cmw175,cmw176,cmw177,cmw178,cmw179,
360 kxx=complx(0.0,0.0)
361 du 200 i=1,nea
362 unfr(i,1,1)=0.
363 200 continue
364 du 1=1,nea
365 if(zam(i).eq.cmplx(0.0,0.0)) go to 222
366 do 2=n,nea
367 km=kam(i)
368 nx=km(i)
369 go to 101
370 101 kxx=kxx+ck((i)*cmexp(-zlam(i))*m*m*bost
```
1(m,1)=fthet(1,m,1)+fthet(m,1))/zlam(i)

dba(29)=real(kxx)

ao = 102

dba(30)=aimaq(kxx)

2 continue

ao = 103

222 kxx=kxx+.5*ck(i)*(fthet(1,1)+fthet(6,1))

103 kxx=kxx+aimaq(1,1)/neq(8,1)

dba(31)=real(kxx)

ao = 104

184 dba(32)=aimaq(kxx)

aprf(1,1)=aimaq(kxx)-aprf(1,1)-aprf(1,2)-aprf(1,3)-aprf(1,4)

1 continue

return

end

complex function kxx(kxx)

*

* THIS FUNCTION IS THE Kxx ELEMENT OF THE HOT PLASMA DIELECTRIC TENSOR

* 1

complex xx

common/cont/lu, ix, jx1, neq, mmaq, mxx, nonch, lz, nsor, nste, niah,

ccnana, kquad, luda, madd, lpx, mx, lu, llin, luha1, luha2, mxw3,m,

comp4, mxx, neq, mxx, nxmg, nnnug, nxm0, lpx, lpx, lpx,

comxx, mxx, mxx, mxx, mxx, mxx, mxx, mxx, mxx, mxx,

complex fthet, fphi, besd, besl, xkero, zlam, kxx2

common/rffm, u6, v6, t6, z6, f6, fthet(1,6), fphi(11,6), frea,

xbeta7, zeta(1,6), zeta(6,1), besl(6,1), co2(6), xkxx2(2,6), v6, irr

x, xkero2, besd(6,6), ck(6), inter(2), bset(7), zlam(6,6), rrr, pourx

complex rfd, rfd, rfd, rfd, rfd, rfd, rfd, rfd, rfd, rfd, rfd,

comxx/6, xnor, xnor, xnor, xnor, dka(39)

1, aprf(6,6), aprf(6,6), power, rffm, rffm, rffm, rffm(3), baxias

c, rxx, s, rxx, rxx, rxx, rxx, rxx, rxx, rxx

do 200 n1=1, neq

aprf(2,1)=6.

200 continue

do 1 n1=1, neq

do 2 m1=n1, niah

continue

ao = 101

101 kxx=kmxx(6,6,1)+ck(i)+cexp(zlam(i))==(besd(m,1)-

1besl(m,1))*(fthet(m,1)-fthet(m,1))

2 continue

do(27)=real(kxx)

ao = 102

102 dba(28)=aimaq(kxx)

aprf(2,1)=real(kxx)-aprf(2,1)-aprf(2,2)-aprf(2,3)-aprf(2,4)

1 continue

return

end

complex function kxz(kxx)

*

* THIS FUNCTION IS THE Kxz ELEMENT OF THE HOT PLASMA DIELECTRIC TENSOR

* 1

complex xx

common/cont/lu, ix, jx1, neq, mmaq, mxx, nonch, lz, nsor, nste, niah,
A page of code from a scientific or technical document. The text contains complex mathematical expressions and functions, with some annotations like comments and function calls. The content appears to be related to computational tasks, possibly involving physics or engineering.
do 2 m=1,ninh
sominh
ao to 103
183 Kuz*Kuz*cck(1)*cexp(-zlam(1))*cexp(*bas1
(1,m,1)/zlam(1)-2*zlam(1)*casd(m,1)-cas1(m,1))*cflth(m,1)
1+cflth(m,1))
2 continue
103 ao to 230
231 Kuz*Kuz+cmix(1,ne,0.0)+.5*cck(1)*cflth(1,0)+cflth(6,1)
230 dba(23)=real(kuz)
ao to 104
104 dba(24)=imaq(kuz)
opr(3,3)-imaq(kuz)-opr(3,1)-opr(3,2)-opr(3,3)-opr(3,4)
1 continue
return
end
complex function kuz(xx)
THIS FUNCTION IS THE KUZ ELEMENT OF THE HOT PLASMA DIELECTRIC TENSOR
common

complex xx
common/contlu,jx,jx1,neq,nmaa,mmx,mann,iz,nsor,nstp,nninh,
ennonea,ksspm,luam,mdm,lu1mx,lu1mx,lu1li,lu1li,lu1li,lu1li,lu1li,
lu1mx,lu1li,lu1li,lu1li,lu1li,lu1li,lu1li,lu1li,lu1li,lu1li,
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complex xx

common/ncom1,v1,wp1,vwp1,wp2,change,lima,dir,dirin(5),nsein(5),
     x15,nswb,v1,nswb,v1,nswb,v1,nswb,v1,nswb,v1,nswb,v1,
     lum2,lim3,xim1,lim2,xim3,xim1,lim2,xim3,uratkov,potent,dpot,
     xmix,xxmix,kbund,kaund,kxund,kxund,kxund,kxund,kxund,kxund,
     kbeta,mmix,kbeta,kbeta,kbeta,kbeta,kbeta,kbeta,kbeta,kbeta,
     kxn,xxmix,xxmix,kbxund,xxmix,kbxund,xxmix,kbxund,xxmix,kbxund,
     xkxund,xxmix,xxmix,kbxund,xxmix,kbxund,xxmix,kbxund,xxmix,kbxund,
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     xkxund,xxmix,xxmix,kbxund,xxmix,kbxund,xxmix,kbxund,xxmix,kbxund,
     xkxund,xxmix,xxmix,kbxund,xxmix,kbxund,xxmix,kbxund,xxmix,kbxund,
ulations elements for fast or ion Bernstein wave for given

Calculates dielectric elements for fast ion Bernstein wave for given

* Calculates dielectric elements for fast ion Bernstein wave for given
CALCULATES THE UNNORMALIZED RF ELECTRIC FIELD COMPONENTS FOR THE FAST
AND/OR IDW CYCLOTRON WAVES FOR AN INFINITE UNIFORM PLASMA USING WAVE EQUATION.

w_i = \frac{\partial E}{\partial t} + V \cdot \nabla E

\nabla \cdot E = 0

\nabla \times \mathbf{E} = 0

\nabla \cdot \mathbf{B} = 0

\nabla \times \mathbf{B} = \mu_0 \mathbf{J} - \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}

\text{with } \mathbf{E} = E_i \hat{i} + E_j \hat{j} + E_k \hat{k}

\text{and } \mathbf{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}

The RF power deposited in species 'i' is given by the weak damping formula:

\text{power}_i = \frac{1}{2} n_i \frac{\partial \mathbf{E}^2}{\partial t}

where \( n_i \) is the number density of species 'i'.
APPENDIX B

Kennel and Engelmann have assumed that a spectrum of randomly phased modes exist in order to justify the concept of particle diffusion caused by several random velocity displacements. A monochromatic, rather than a spectrum, of waves can be used if a plasma ion, having passed through the resonant heating zone, encounters many collisions so as to forget the phase of the cyclotron heating field before it passes through the heating zone again. The time between passes through this zone is

\( \tau_p \sim \frac{\pi R q}{v_n} \) \hspace{1cm} (B-1)

and the variation in \( \tau_p \) is given by

\( \Delta \tau_p \sim \frac{\pi R q \Delta v_n}{v_n^2} \) \hspace{1cm} (B-2)

After half a poloidal rotation, Coulomb scattering causes an RMS deviation, \( \Delta v_n \), on the order of \( \left( \langle \Delta v_n \rangle^2 \right)^{1/2} \), where \( \langle \Delta v_n \rangle^2 \) is the diffusion coefficient \( (4-16,33) \). Ions "forget" the cyclotron phase between transists when \( \omega_{ci} \Delta \tau_p \gg 1 \) or

\( \frac{\langle \Delta v_n \rangle^2}{v_n^2} \gg \frac{1}{\left( \omega_{ci} \tau_p \right)^2} \frac{1}{\tau_p} \) \hspace{1cm} (B-3)
where the diffusion coefficient is \((13, 16)\)

\[
\langle (\Delta v_n)^2 \rangle = \sum_f \frac{C_f}{v} G(\xi_f v)
\]

\[
C_f = \frac{8\ln n_\sigma^2 \sigma_f^2 e^{\frac{1}{2} \ln \Lambda}}{m_f^2}, \quad \xi_f^2 = \frac{m_f}{2kT_f}
\]

(B-4)

\[
G(x) = \frac{\phi(x) - x\phi'(x)}{2x^2}, \quad \phi(x) = \frac{2}{\pi^{\frac{3}{2}}} \int_0^x \exp(-y^2)dy.
\]

From Table B-1, we see that Eq. (B-3) is valid for both PLT and NUWMAK.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>PLT</th>
<th>NUWMAK</th>
</tr>
</thead>
<tbody>
<tr>
<td>R (cm)</td>
<td>130</td>
<td>512.5</td>
</tr>
<tr>
<td>q</td>
<td>~ 3</td>
<td>~ 3</td>
</tr>
<tr>
<td>$T_f$ (keV)</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>$V_n \approx v_{thi}$ (cm/s)</td>
<td>$2.2 \times 10^7$</td>
<td>$6.2 \times 10^7$</td>
</tr>
<tr>
<td>$\tau_p$ (ms)</td>
<td>0.056</td>
<td>0.078</td>
</tr>
<tr>
<td>$\omega_{ci}$ (r/s)</td>
<td>$1.57 \times 10^8$</td>
<td>$2.9 \times 10^8$</td>
</tr>
<tr>
<td>$n_f$ (cm$^{-3}$)</td>
<td>$3 \times 10^{13}$</td>
<td>$3 \times 10^{14}$</td>
</tr>
<tr>
<td>$m_f/m_p$</td>
<td>2.0</td>
<td>2.5</td>
</tr>
<tr>
<td>$\varepsilon n\Omega$</td>
<td>~ 10</td>
<td>~ 10</td>
</tr>
<tr>
<td>$\frac{&lt;(\Delta V_n)^2&gt;}{V_n^2}$ (sec$^{-1}$)</td>
<td>340</td>
<td>96.5</td>
</tr>
<tr>
<td>$\frac{1}{(\omega_{ci}\tau_p)} = \frac{1}{\tau_p}$ (sec$^{-1}$)</td>
<td>$2.3 \times 10^{-4}$</td>
<td>$2.5 \times 10^{-5}$</td>
</tr>
</tbody>
</table>
APPENDIX C

From Eq. (4-16), one term of C may be written as

\[ C = \frac{|E_+|^2}{32Z^2e_n^2n_o^2} \delta(n\omega_{c_B} - \omega + k_n\nu_n)j_{n-1}^2 \]  \hspace{1cm} (C-1)

where \( J_n \equiv J_n(k_n\nu_n/\omega_{c_B}) \).

With

\[ k_n = (m - 1)q/qR \] \hspace{1cm} (C-2)

and with \( m = 0, l = 100 \) and \( R \approx R_0 \approx 500 \text{ cm} \) for NUWMAK or

\( m = 0, l = 13, R \approx 130 \text{ cm} \) for PLT.

\[ k_n = -1 \text{ cm}^{-1} \equiv -k_n \text{ for PLT and} \]

\[ k_n = -2 \text{ cm}^{-1} \equiv -k_n \text{ for NUWMAK} \] \hspace{1cm} (C-3)

so that

\[ C = \frac{|E_+|^2}{32Z^2e_n^2n_o^2} \delta(n\omega_{c_B} - \omega - k_n\nu_n)j_{n-1}^2 \] \hspace{1cm} (C-4)

On averaging over \( \theta \)

\[ \langle C \rangle_{\theta} \approx \frac{|E_+|^2}{32\pi Z^2e_n^2n_o^2} \int_0^\pi \int_{n-1} \delta(n\omega_{c_B} - \omega - k_n\nu_n) d\theta. \] \hspace{1cm} (C-5)

Letting

\[ x \equiv \frac{n\omega_{c_B}R_o}{R_o + \sin\theta} \] \hspace{1cm} (C-6)
then

\[ dx = - \frac{n_{\omega c_{\beta o} R}}{(R_o + as_{\theta})^2} \, d\theta = - \frac{ac_{\theta}x^2}{R_o n_{\omega c_{\beta o}}} \, d\theta. \]  

\[ (C-7) \]

Therefore

\[ d\theta = - \frac{n_{\omega c_{\beta o} R}}{ax^2 \sin \theta} \, dx. \]  

\[ (C-8) \]

Since \( \sin \theta = (R/a)(n_{\omega c_{\beta o}}/x - 1) \)

\[ \cos \theta = (1 - (R/a)^2(n_{\omega c_{\beta o}}/x - 1)^2)^{1/2} = \xi(x). \]  

\[ (C-9) \]

Therefore

\[ d\theta = - \frac{n_{\omega c_{\beta o} R}}{ax^2 \xi(x)} \, dx \]  

\[ (C-10) \]

and

\[ \langle C \rangle_{\theta} = \frac{|E_+|^2 J_{n-1}^2 R_{\omega c_{\beta o}}}{32 \pi^2 e^2 n_a} \int_0^\pi \frac{\delta(x - k_{n} v_{n} - \omega)}{x^2 \xi(x)} \, dx \]  

\[ (C-11) \]

\[ = \frac{|E_+|^2 J_{n-1}^2 R_{\omega c_{\beta o}}}{32 \pi^2 e^2 n_a (\omega + k_{n} v_{n})^2 \xi(x_a)} \]  

\[ (C-12) \]

where

\[ \xi(x_a) = \left\{ 1 - \frac{(R/a)^2(n_{\omega c_{\beta o}}/\omega + k_{n} v_{n})^2 - 1)^{1/2}} \right\} \]  

\[ (C-13) \]
Since \( \omega + k_n v_n = n \omega_{cB0} \),

\[
\langle C \rangle_{\theta} = \frac{|E_+|^2 J_{n-1}^2 R_{n-1,0} \omega_{cB0}}{32 Z^2 e^2 n_o a n w_{cB}^2 \xi(x_o)} \sim \frac{|E_+|^2 J_{n-1}^2 R_{n-1,0}}{32 Z^2 e^2 n_o a n w_{cB0}^2 \xi(x_o)} \tag{C-14}
\]

with \( \omega_{cB} \sim \omega_{cB0} \).

A similar procedure may be used to derive the other two terms of the expression for \( C \) given in Eq. (4-17).
APPENDIX D

The first approximation used in the ICRF model is the weak damping formula

\[ P = \frac{\omega}{2} \vec{E} \cdot \vec{K} \cdot \vec{E} \quad (D-1) \]

which determines the RF power deposition. Equation (D-1) holds when

\[ \frac{\text{Im} k_L}{\text{Re} k_L} << 1. \quad (D-2) \]

Since there is little energy in the Ion Bernstein branch, the first approximation used is

\[ \varepsilon = \frac{\text{Im} k_{Lf}}{\text{Re} k_{Lf}} << 1. \quad (D-3) \]

The remaining approximations are in the WKB method used to find the RF electric field. Again, since there is little energy in the Ion Bernstein mode, we calculate epsilons for the fast wave only. The WKB approximations should be better for the Ion Bernstein wave (5-27). To obtain the fast wave electric field for Eq. (D-1), we use the following WKB type equation

\[ \frac{d^2 E_f}{dr^2} + k_f^2(r) E_f = 0. \quad (D-4) \]
There are several approximations used in obtaining the solution for Eq. (D-4)

\[ E \propto \frac{1}{k_{\perp f}^2} e^{i \int k_{\perp f} \cdot dr}. \]  \hspace{1cm} (D-5)

To find the solution we assume that

\[ E \propto e^{i \phi}. \]  \hspace{1cm} (D-6)

Equation (D-4) becomes

\[- (\phi')^2 + i \phi'' + k_{\perp f}^2 = 0.\]  \hspace{1cm} (D-7)

If we assume that

\[ |\phi''| \ll k_{\perp f}^2 \] \hspace{1cm} (D-8)

then we have

\[ \phi' = \pm k_{\perp f} \] \hspace{1cm} (D-9)

or

\[ \phi = \pm \int k_{\perp f} \cdot dr. \] \hspace{1cm} (D-10)

For \( |\phi''| \) to be small

\[ \frac{|\phi''|}{|k_{\perp f}^2|} \sim \frac{k_{\perp f}'}{k_{\perp f}^2} \equiv \epsilon_1 \ll 1. \] \hspace{1cm} (D-11)
For a higher order approximation to Eq. (D-4), from Eq. (D-9)

\[ \phi'' + k_{\perp f} \left( \phi' \right)' = 0 \]  \hspace{1cm} (D-12)

Therefore

\[ -\left( \phi' \right)^2 + i k_{\perp f}' + k_{\perp f}^2 = 0 \]  \hspace{1cm} (D-13)

\[ \left( \phi' \right)^2 \sim k_{\perp f}^2 + ik_{\perp f}' \]  \hspace{1cm} (D-14)

\[ \phi \sim + \int k_{\perp f} \cdot dr + \frac{i}{4} \ln k_{\perp f}^2 \]  \hspace{1cm} (D-15)

which yields the solution

\[ E_f^e(r) \sim \frac{C_+}{k_{\perp f}^{\frac{3}{2}}} e^{\int k_{\perp f} \cdot dr} + \frac{C_-}{k_{\perp f}^{\frac{3}{2}}} e^{-i \int k_{\perp f} \cdot dr} \]  \hspace{1cm} (D-16)

For a wave traveling from the low field side \( C_- = 0 \) so

\[ E_f^e(r) = \frac{C_+}{k_{\perp f}^{\frac{3}{2}}} e^{i \int k_{\perp f} \cdot dr} \]  \hspace{1cm} (D-17)

For Eq. (D-16) to hold

\[ \left| \frac{\phi''}{k_{\perp f}^2} \right| \ll 1 \]  \hspace{1cm} (D-18)
From Eq. (D-14)

\[ \phi' \propto k_{lf} + i \frac{k_{lf}'}{2k_{lf}} \]  

(D-19)

therefore

\[ \phi'' \propto k_{lf} + i \frac{k_{lf}''}{2k_{lf}} - i \frac{(k_{lf}')^2}{2k_{lf}^3} \]  

(D-20)

so that

\[ \varepsilon_2 \equiv \left| \frac{\frac{k_{lf}'}{2k_{lf}^2} + i \frac{k_{lf}''}{2k_{lf}^3}}{1 - i \frac{(k_{lf}')^2}{2k_{lf}^3}} \right| \ll 1. \]  

(D-21)

By inserting Eq. (D-17) into Eq. (D-14) a similar approximation is obtained

\[ \varepsilon_3 \equiv \left| \frac{3}{4} \frac{(k_{lf}')^2}{k_{lf}^2} - \frac{k_{lf}''}{2k_{lf}} \right| \ll 1. \]  

(D-22)

Since \( k_{lf} \) is a complicated function of \( r \), we have fitted both \( \text{Re} k_{lf}(r) \) and \( \text{Im} k_{lf}(r) \) to polynomials. Both \( \text{Re} k_{lf}(r) \) and \( \text{Im} k_{lf}(r) \) are divided into 12 intervals, 11 having 10 points while the region on the inside plasma edge has 7 points. In each interval \( \text{Re} k_{lf} \) and \( \text{Im} k_{lf} \) is approximated by a fifth order polynomial so that

\[ \left| \sum_{i=0}^{5} A_i r^i \text{Re} k_{lf} \right| \quad \text{and} \quad \left| \sum_{i=0}^{5} B_i r^i \text{Im} k_{lf} \right| \]  

are minimized in each of
the 12 regions. By using a polynomial fit in each zone, $\varepsilon_1$, $\varepsilon_2$, and $\varepsilon_3$ can be calculated. In the Re$k_{1f}$ and the Im$k_{1f}$ plots, the solid curve is the actual value while the dashed line represents the polynomial approximation. In the epsilon plots, $\varepsilon_2$ is the solid curve, the dashed line is $\varepsilon$, the dotted line is $\varepsilon_1$, while the chaindot line plots $\varepsilon_3$. In all the epsilon figures $\varepsilon_1 \gtrsim \varepsilon_2 \gtrsim \varepsilon_3$.

Figures (D-1-2) show the real and imaginary parts of $k_{1f}$ for PLT at $t = 0$ with $C_H = 1.5$. The mode conversion layer near $r = 125$ cm can be seen. The polynomial fit is accurate except near the mode conversion layer. Im$k_{1f}$ is nonzero at the inner edge where the wave does not penetrate and in the cyclotron layer. We see that Im$k_{1f}$ is large in a region between the mode conversion layer and the launching structure at $r = 175$ cm. The sharp rise in Im$k_{1f}$ due to the presence of the ion-ion hybrid resonance can be seen. In Fig. (D-3) we see there are three regions where both the weak damping and WKB formulations breakdown. The region at the outer edge does not affect the calculations. This region is an evanescence zone and also appears in the NUWMAK calculations. In PLT, little power is absorbed in this region (34) so we assume that the wave is launched inside of this zone. The inner edge also is an evanescence region. In both PLT and NUWMAK, little, if any energy reaches the inner surface. Therefore, this inner zone does not affect the calculations.
The important region is near the center 120 cm ≤ r ≤ 130 cm where mode conversion may occur. In this thin layer, most of the RF is deposited. Both the WKB and weak damping formulas break down in this region. In comparison to the experiment, the RF heating width predicted by STICR-I seems to be correct. The actual deposition within this 10 cm needs a more accurate description. The effect of a more complete calculation on either the density or temperature profiles should be minimal.

When \( C_H = 1.0 \) or 2.0, the areas where both the WKB and weak damping formulations are valid remain the same. Mode conversion does not appear to affect the results, the RF power deposition should be unaffected by the presence of mode conversion in our model.

In PLT, where \( k_w = 0.2 \text{ cm}^{-1} \), the sharp change in Re\( k_{\perp f} \) and Im\( k_{\perp f} \) over the plasma center is eliminated as shown in Figs. (D-4-5). There are wide evanescence regions on both sides of the plasma. Since the wave is assumed to penetrate the outside evanescence region without absorption and does not reach the inner edge with much energy, these regions can be neglected in Fig. (D-6). Both the WKB and weak damping formulations are valid over the entire resonance layer at the center. There is a region on the high field side (98 cm ≤ r ≤ 115 cm) where the WKB theory is not valid. However, the RF wave is damped before this region is reached. In addition, the polynomial approximation of Im\( k_{\perp f} \) is
not too accurate in this zone and may contribute to making $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ too large. The exact expression, becomes large over a very small width in this interval, when $\text{Re}L_{1f} \to 0$.

We next proceed to examine the WKB and weak damping approximations in the conceptual reactor NUWMAK. In the fundamental heating case, we find that $\text{Re}L_{1f}$ and $\text{Im}L_{1f}$ are more accurately approximated by polynomials (Fig. (D-7-8)). In Fig. (D-9), we see there are again edge regions where both formulations break down. Again, by assuming that the incoming wave can propagate through the outer evanescence zone, this region can be ignored. The wave is damped before the inner edge is reached so this region can also be neglected. Unlike FLT, both formulations are valid over the plasma center. As the plasma temperature increases, $\text{Re}L_{1f}$ becomes smoother (Fig. (D-10)). The damping zone spreads and $\text{Im}L_{1f}$ becomes smoother at the plasma center (Fig. (D-11)). Figure (D-12) shows the WKB and weak damping formulas improving as the temperature increases.

Figures (D-13-21) pertain to second harmonic heating in NUWMAK. On comparing Figs. (D-13-15) where mode conversion occurs to Figs. (16-21) where mode conversion is absent, we find little change. In second harmonic heating the zones of validity are unaffected by temperature. Both the WKB and weak damping formulas are valid over a wider region in comparison to fundamental heating.

Figures (D-22-24) deal with third harmonic heating in NUWMAK. $\text{Re}L_{1f}$ and $\text{Im}L_{1f}$ are very smooth over the resonance zone
and are accurately approximated by polynomials. Both formulas hold well over the central regions and break down near the plasma edges. As the plasma temperature increases, the epsilons increase, although both formulations appear to remain valid.

In conclusion, for PLT, mode conversion does not appear to influence the areas of validity. There are regions at both edges where the formulas break down. Little power is deposited in these regions so their effects on the RF depositions should be minimal. Near the plasma center, where most of the power is deposited, both formulas break down. However, the heating width which is predicted agrees well with the experiment. The ratio \( \frac{T_H}{T_D} \) does not affect the widths of the regions of validity.

For NUWMMAK, both approximations become good over the resonance zone. There is still an inner and outer edge region where the approximations break down. However, when mode conversion is present, the inner edge cannot be reached, the wave being reflected at the mode conversion layer. When mode conversion is absent, the wave is damped before the inner edge is reached. The assumption of no power being absorbed in the outer evanescence zone allows us to ignore this region as well. Mode conversion does not appear to affect the validity of either formulation.

As the RF frequency increases, both WKB and weak damping approximations become better. As the plasma temperature increases, the approximations used in the calculations of fundamental heating
improve. For second and third harmonic heating, the epsilon values increase slightly. On the whole, both approximations appear to hold over the entire region of NUWMAK where heating, either ion or electron, occur. Our conclusions should be more accurate in NUWMAK than in PLT. The slab model should be more accurate in large devices such as a reactor.
LIST OF FIGURES

(D-1) \( \text{Re}_{k_{\perp}} \) (solid line) and the fifth order polynomial fit (dashed line) for PLT, \( C_H = 1.5, k_n = 0.1 \text{ cm}^{-1}, t = 0 \text{ ms} \).

(D-2) \( \text{Im}k_{\perp} \) (solid line) and the fifth order polynomial fit (dashed line) for PLT, \( C_H = 1.5, k_n = 0.1 \text{ cm}^{-1}, t = 0 \text{ ms} \).

(D-3) \( \varepsilon \) (dashed line), \( \varepsilon_{1} \) (dotted line), \( \varepsilon_{2} \) (solid line), \( \varepsilon_{3} \) (chaindot line) for PLT, \( C_H = 1.5, k_n = 0.1 \text{ cm}^{-1} \).

(D-4) \( \text{Re}_{k_{\perp}} \) (solid line) and the fifth order polynomial fit (dashed line) for PLT, \( C_H = 1.5, k_n = 0.2 \text{ cm}^{-1}, t = 0 \text{ ms} \).

(D-5) \( \text{Im}k_{\perp} \) (solid line) and the fifth order polynomial fit (dashed line) for PLT, \( C_H = 1.5, k_n = 0.2 \text{ cm}^{-1}, t = 0 \text{ ms} \).

(D-6) \( \varepsilon \) (dashed line), \( \varepsilon_{1} \) (dotted line), \( \varepsilon_{2} \) (solid line), \( \varepsilon_{3} \) (chaindot line) for PLT, \( C_H = 1.5, k_n = 0.2 \text{ cm}^{-1} \).

(D-7) \( \text{Re}_{k_{\perp}} \) (solid line) and the fifth order polynomial fit (dashed line) for NUWMAK, \( f = f_{cD}, t = 0 \text{ sec} \).

(D-8) \( \text{Im}k_{\perp} \) (solid line) and the fifth order polynomial fit (dashed line) for NUWMAK, \( f = f_{cD}, t = 0 \text{ sec} \).

(D-9) \( \varepsilon \) (dashed line), \( \varepsilon_{1} \) (dotted line), \( \varepsilon_{2} \) (solid line), \( \varepsilon_{3} \) (chaindot line) for NUWMAK, \( f = f_{cD}, t = 0 \text{ sec} \).

(D-10) \( \text{Re}_{k_{\perp}} \) (solid line) and the fifth order polynomial fit (dashed line) for NUWMAK, \( f = f_{cD}, t = 1.5 \text{ sec} \).

(D-11) \( \text{Im}k_{\perp} \) (solid line) and the fifth order polynomial fit (dashed line) for NUWMAK, \( f = f_{cD}, t = 1.5 \text{ sec} \).

(D-12) \( \varepsilon \) (dashed line), \( \varepsilon_{1} \) (dotted line), \( \varepsilon_{2} \) (solid line), \( \varepsilon_{3} \) for NUWMAK, \( f = f_{cD}, t = 1.5 \text{ sec} \).
(D-13) \( \text{Re}k_{\perp} \) (solid line) and the fifth order polynomial fit (dashed line) for NUWMAK, \( f = 2f_{CD} \), \( k_{\parallel} = 0.1 \text{ cm}^{-1} \), \( t = 0 \text{ sec} \).

(D-14) \( \text{Im}k_{\perp} \) (solid line) and the fifth order polynomial fit (dashed line) for NUWMAK, \( f = 2f_{CD} \), \( k_{\parallel} = 0.1 \text{ cm}^{-1} \), \( t = 0 \text{ sec} \).

(D-15) \( \epsilon \) (dashed line), \( \epsilon_{\perp} \) (dotted line), \( \epsilon_{2} \) (solid line), \( \epsilon_{3} \) (chaindot line) for NUWMAK, \( f = 2f_{CD} \), \( k_{\parallel} = 0.1 \text{ cm}^{-1} \), \( t = 0 \text{ sec} \).

(D-16) \( \text{Re}k_{\perp} \) (solid line) and the fifth order polynomial fit (dashed line) for NUWMAK, \( f = 2f_{CD} \), \( k_{\parallel} = 0.2 \text{ cm}^{-1} \), \( t = 0 \text{ sec} \).

(D-17) \( \text{Im}k_{\perp} \) (solid line) and the fifth order polynomial fit (dashed line) for NUWMAK, \( f = 2f_{CD} \), \( k_{\parallel} = 0.2 \text{ cm}^{-1} \), \( t = 0 \text{ sec} \).

(D-18) \( \epsilon \) (dashed line), \( \epsilon_{\perp} \) (dotted line), \( \epsilon_{2} \) (solid line), \( \epsilon_{3} \) (chaindot line) for NUWMAK, \( f = 2f_{CD} \), \( k_{\parallel} = 0.2 \text{ cm}^{-1} \), \( t = 0 \text{ sec} \).

(D-19) \( \text{Re}k_{\perp} \) (solid line) and the fifth order polynomial fit (dashed line) for NUWMAK, \( f = 2f_{CD} \), \( k_{\parallel} = 0.3 \text{ cm}^{-1} \), \( t = 0 \text{ sec} \).

(D-20) \( \text{Im}k_{\perp} \) (solid line) and the fifth order polynomial fit (dashed line) for NUWMAK, \( f = 2f_{CD} \), \( k_{\parallel} = 0.3 \text{ cm}^{-1} \), \( t = 0 \text{ sec} \).

(D-21) \( \epsilon \) (dashed line), \( \epsilon_{\perp} \) (dotted line), \( \epsilon_{2} \) (solid line), \( \epsilon_{3} \) (chaindot line) for NUWMAK, \( f = 2f_{CD} \), \( k_{\parallel} = 0.3 \text{ cm}^{-1} \), \( t = 0 \text{ sec} \).

(D-22) \( \text{Re}k_{\perp} \) (solid line) and the fifth order polynomial fit (dashed line) for NUWMAK, \( f = 3f_{CD} \), \( k_{\parallel} = 0.2 \text{ cm}^{-1} \), \( t = 0 \text{ sec} \).
(D-23) \( \text{Im} k_L \) (solid line) and the fifth order polynomial fit (dashed line) for NUWMAK, \( f = 3 f_{CD} \), \( k_H = 0.2 \text{ cm}^{-1} \), \( t = 0 \text{ sec} \).

(D-24) \( \epsilon \) (dashed line), \( \epsilon_1 \) (dotted line), \( \epsilon_2 \) (solid line), \( \epsilon_3 \) (chaindot line) for NUWMAK, \( f = 3 f_{CD} \), \( k_H = 0.2 \text{ cm}^{-1} \), \( t = 0 \text{ sec} \).
Fig. (D-1) Real Fast K Perp

Fig. (D-2) Imag Fast K Perp

Fig. (D-3) Epsilon
Fig. (D-10) Real Fast K Perp

Fig. (D-11) Imag Fast K Perp

Fig. (D-12) Epsilon
Fig. (D-13)

REAL FAST K PERP

Fig. (D-14)

IMAG FAST K PERP

Fig. (D-15)

EPSILON
Fig.(D-16)

Fig.(D-17)

Fig.(D-18)