Effects of the Fast Wave on the Drift-Cyclotron Loss-Cone Model

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Abstract

Particle orbits in the presence of the fast wave fields are calculated. It is found that the resonant interaction of ions with the wave can be neglected at $\omega_0 = 2\omega_{ci}$, provided that the wave electric field strength is less than a critical value E_c (= $4k_{\parallel}T/e$). The dominant effect on the dispersion relation of the drift-cyclotron loss-cone mode is thus found to be due to the non-resonant effects of the wave on electrons and ions. It is found that a fast wave with $\omega_0 = 2\omega_{ci}$ destabilizes the mode. For $\omega_0 < 2\omega_{ci}$, fast waves can stabilize the mode for certain values of the plasma β, which is calculated numerically.

I. Introduction

Fast wave ion heating at $2\omega_{ci}$ in the tandem mirror plugs has been proposed to replace part of the neutral beam power and is to be tested in the Phaedrus experiment.[1] Fast waves may also be used to heat electrons in the plugs of the thermal barrier tandem mirror. In this paper, we study the effect of the fast wave fields on the drift-cyclotron loss-cone mirror instability which is believed to exist in the recent mirror experiments[2,3] and to enhance the particle scattering into the loss-cone.

The effect of the fast wave fields on the microinstabilities was studied by Ivanov and Soboleva.[4] However, they did not consider the ion drift motion across the field which is comparable to that of the electrons at $2\omega_{ci}$, and the finite β effect which is important for the drift-cyclotron loss-cone mode.[5] In this paper, we take both effects into account.

The paper is organized as follows. In Sec. II, we calculate the particle orbits in the fast wave fields and discuss various nonlinear effects on the mode. The dispersion relation of the drift-cyclotron loss-cone mode in the presence of the fast wave fields is derived in Sec. III. In Sec. IV, we discuss stabilizing and destabilizing effects for various values of the fast wave frequency $\omega_0 \lesssim 2\omega_{ci}$ and plasma β. Concluding remarks are given in Sec. V.

II. Particle Orbits

The electric field of the fast wave is assumed to have the form

$$\vec{E} = \hat{e}_x E_{xo} \cos \psi + \hat{e}_y E_{yo} \sin \psi \quad , \tag{1}$$

where $\psi = k_x x - \omega_0 t$, as shown in Fig. 1. The electric field in the z direction is small due to large electric conductivity along the magnetic field line.[6] The Doppler shift due to the finite wave vector $k_{f\parallel}$ in the z direction is neglected, and can easily be taken into account by replacing ω_0 by $\omega_0 - k_{f\parallel} v_\parallel$. In the nonresonant case, neglecting the Doppler shift is justified since $\omega_{ci} \gg k_{f\parallel} v_\parallel$. The equations of motion for the particles in the x and y direction can then be written as

$$\dot{v}_x = \frac{eE_x}{m} + \omega_c v_y + \frac{ek_x E_y}{m\omega_0} v_y \quad ,$$
$$\dot{v}_y = \frac{eE_y}{m} - \omega_c v_x - \frac{ek_x E_y}{m\omega_0} v_x \quad , \tag{2}$$

where $\omega_c = eB/mc$. The Lorentz force due to the wave magnetic field associated with the wave electric field is included in Eq. (2). The formal solutions for Eq. (2) are then

$$v_x = \omega_c y + \frac{e}{m}\int E_x \, dt + \frac{ek_x}{m\omega_0}\int v_y E_y \, dt \equiv \omega_c y + \dot{H}_y \quad ,$$
$$v_y = -\omega_c x + \frac{e}{m}\int E_y \, dt - \frac{ek_x}{m\omega_0}\int v_x E_y \, dt = -\omega_c x + \dot{H}_x \quad , \tag{3}$$

where $\dot{H}_{x(y)} \equiv dH_{x(y)}/dt$. Substituting Eq. (3) into Eq. (2), we obtain

$$\ddot{x} + \left(\omega_c^2 + \frac{ek_x \omega_c E_y}{m\omega_0}\right)x = \omega_c \dot{H}_x + \frac{eE_x}{m} \quad ,$$
$$\ddot{y} + \left(\omega_c^2 + \frac{ek_x \omega_c E_y}{m\omega_0}\right)y = -\omega_c \dot{H}_y + \frac{eE_y}{m} \quad . \tag{4}$$

Assuming $ek_xE_{x(y)}/m\omega_c^2 \ll 1$, we can solve Eq. (4) by the Bogoliubov asymptotic expansion method.[7,8] The solution to Eq. (4) is then

$$y = a \sin(\omega_c t + \phi) + y_1 \quad , \tag{5}$$

where $a = v_\perp/\omega_c$, $v_\perp$ is the perpendicular speed of the particle and

$$y_1 = \sum_{\substack{n=-\infty \\ n\omega_c + \omega_o \neq 0,\, \pm\omega_c}}^{\infty} \frac{\frac{eE_{yo}}{m}\left(\frac{n\omega_c + \omega_o}{\omega_o}\right) J_n(\alpha) + \frac{\omega_c}{n\omega_c + \omega_o}\frac{eE_{xo}}{m}\left[J_n(\alpha) + \frac{\alpha\omega_c}{\omega_o}\frac{E_{yo}}{E_{xo}} J_n'(\alpha)\right]}{(n\omega_c + \omega_o)^2 - \omega_c^2}$$

$$\times \sin\left[n\left(\omega_c t + \phi + \frac{\pi}{2}\right) + \omega_o t\right] \quad , \tag{6}$$

where $\alpha = k_x v_\perp/\omega_c$, J_n is the Bessel function of order n, and $J_n' = dJ_n/d\alpha$. We only write down the position vector in the y direction since this is the quantity needed to calculate $\mu_{e(i)}$, which is the ratio of the electron (ion) excursion distance relative to the wavelength of the drift-cyclotron loss-cone mode. For the resonant case, $n\omega_c + \omega_o = \pm\omega_c$, the Larmor radius a and the gyrophase ϕ are no longer constants. Both a and ϕ are changing with time at the rates

$$\dot{a} = \frac{eE_{yo}}{m\omega_c}\left[J'_{-\ell}(\alpha) + \frac{E_{xo}}{E_{yo}}\frac{\ell}{\alpha} J_{-\ell}(\alpha)\right] \cos\left[\ell\left(\phi + \frac{\pi}{2}\right)\right] \quad , \tag{7a}$$

$$\dot{\phi} = -\frac{eE_{yo}}{m\omega_c}\left[\left(\frac{\ell}{\alpha} + \frac{\alpha}{\ell}\right) J_{-\ell} - \frac{E_{xo}}{E_{yo}} J_\ell'(\alpha)\right] \sin\left[\ell\left(\phi + \frac{\pi}{2}\right)\right] \quad , \tag{7b}$$

where $\ell \equiv \omega_o/\omega_c$ is an integer. The calculations to obtain Eqs. (6) and (7) are shown in Appendix A.

Two nonlinear processes are described by Eqs. (6) and (7). The largest term in Eq. (6), the n = 0 term, is the nonresonant, coherent motion of the particles in the presence of the external wave fields. This is responsible for the wave coupling effects. The resonant motion is described in Eq. (7). The latter is responsible for particle trapping and heating. Which one is more important?

The usual assumption is to neglect the frequency spread due to the non-uniformity of the magnetic field and the Doppler shift. This is true if the following ordering holds[9]

$$\delta\omega \equiv |\omega - n\omega_{ci}| \gg |n\omega'_{ci}v_{\parallel}|^{1/2} \gg k_{\parallel}v_{\parallel} \quad , \tag{8}$$

where n is an integer, $\omega'_{ci} = d\omega_{ci}/ds$, $k_{\parallel} = (d\phi/ds)/\phi$, and s is the distance along the magnetic field line. The terms $|n\omega'_{ci}v_{\parallel}|^{1/2}$ and $k_{\parallel}v_{\parallel}$ in Eq. (8) are the frequency spread due to the magnetic field nonuniformity and Doppler shift, respectively. In the presence of the fast wave fields, we must consider the frequency spread due to $\dot{\phi}$ and $k\dot{a}$, where k is the wave vector of the drift-cyclotron loss-cone mode. Since $\dot{\phi}$ and $k\dot{a}$ are proportional to the electric field strength, the following ordering will hold

$$\delta\omega \gg |n\omega'_{ci}v_{\parallel}|^{1/2} \gtrsim k\dot{a} \gg k_{\parallel}v_{\parallel} \gtrsim \dot{\phi} \quad , \tag{9}$$

if $E_{yo} < 2k_{\parallel}T_i/e$ at $\omega_o = 2\omega_{ci}$. At $\omega_o = 2\omega_{ci}$, $E_{xo} \simeq 2E_{yo}$, thus

$E_{xo} < 4k_{\parallel}T_i/e \equiv E_c$ in order to satisfy Eq. (9). The definition of E_c is valid if $k_x a_i \lesssim 0.4$, which is the case for the Phaedrus experiment. Thus as long as $E_{xo} < E_c$, we can neglect the resonant ion motion in the fast wave fields. The effect of the possible stochastic motion due to the resonance between the gyromotion and bounce motion can be neglected since stochastic motion occurs on a slower time scale, namely, the bounce period.

III. Dispersion Relation

We have concluded that the nonresonant, coherent particle motion is important and thus we can use the dispersion relation derived in our earlier work.[10] However, we must now also consider the ion drift in the fast wave fields, since $|\mu_i|$ is comparable to $|\mu_e|$ at $\omega_o \sim 2\omega_{ci}$ where $\mu_\sigma = (e_\sigma/m_\sigma)\, kJ_o(\alpha_\sigma)\, (E_{yo} + \omega_{c\sigma} E_{xo}/\omega_o)/(\omega_o^2 - \omega_{c\sigma}^2)$ for species σ. The coupled dispersion relation including the induced ion drift can be written as

$$(E_y')_n + \sum_\sigma \sum_{p,s} x_\sigma(\omega_{s-p})\, J_p(\mu_\sigma)\, J_{n+p-s}(\mu_\sigma)(E_y')_s - \sum_{p,s} \overline{x}_e(\omega_n,\omega_{s-p})$$

$$\times\, p(n+p-s)\, J_p(\mu_e)\, J_{n+p-s}(\mu_e)\, (E_y')_s + \sum_{p,s} pJ_p(\mu_e)\, J_{n+p-s}(\mu_e)$$

$$\times\, x_e'(\omega_{s-p})(E_y')_s - \sum_{p,s} (n+p-s)\, J_p(\mu_e)\, J_{n+p-s}(\mu_e)\, x_e'(\omega_n)\, (E_y')_s = 0 \qquad (10)$$

where

$$\overline{x}_e(\omega_n,\omega_{q-p}) = \left(\frac{\omega_{pi}^2}{\omega_{ci}^2}\right)\left(\frac{\omega_{pi}^2}{k^2c^2}\right)\left(\frac{\omega_o^2}{\omega_n\omega_{q-p}}\right),$$

$$x_e'(\omega_{q-p}) = \left(\frac{\omega_{pi}^2}{\omega_{ci}^2}\right)\left(\frac{\omega_{pi}^2}{k^2c^2}\right)\left(\frac{\omega_o}{\omega_{q-p}}\right),$$

$$\chi_i(\omega_n) = -2\pi(\frac{\omega_{pi}^2}{\omega_{ci}})(\frac{\omega_n}{k^2}) \sum_\ell \int du_\perp^2 \frac{\partial f_o/\partial u_\perp^2}{\ell - \omega_n/\omega_{ci}} J_\ell^2(\alpha_i) ,$$

and

$$\omega_n = \omega + n\omega_o .$$

Again assuming $|\mu_e|$ and $|\mu_i| < 1$, we only have to consider the components with frequency ω and $\omega_0 \pm \omega$ in Eq. (10), and obtain

$$\varepsilon_d = -\frac{(\mu_e - \mu_i)^2}{4} (\frac{(\chi_i - \chi_i^+)(\chi_e - \chi_e^+)}{\varepsilon^+} + \frac{(\chi_i - \chi_i^-)(\chi_e - \chi_e^-)}{\varepsilon^-}) , \tag{11}$$

where $\varepsilon_d = 1 + \chi_e + \chi_i$, $\varepsilon^\pm = 1 + \chi_e^\pm + \chi_i^\pm$.

From Eq. (11), we see that the sideband waves are determined by the relative motion between ions and electrons. At $\omega_0 \sim 2\omega_{ci} << \omega_{ce}$, we have

$$\mu_e = -\frac{ckE_{xo}}{B_o\omega_o} , \quad \mu_i = \frac{J_o(k_x a_i)}{3} \frac{ck(E_{xo} + E_{yo})}{B_o\omega_o} . \tag{12}$$

The electron and ion electric susceptibilities at $\omega = \omega \pm \omega_0$ are

$$\chi_e^\pm = \frac{\omega_{pi}^2}{\omega_{ci}^2} (\frac{m_e}{m_i} + \frac{\omega_{pi}^2}{k^2c^2} - \frac{\varepsilon\omega_{ci}}{k(\omega \pm \omega_o)}) , \quad \chi_i^\pm = \frac{\omega_{pi}^2}{\omega_{ci}^2} \frac{D}{k^3a_i^3} (\Omega \pm f\pi) \cot (\Omega \pm f\pi) , \tag{13}$$

where $f \equiv \omega_o/\omega_{ci}$, $\Omega = \pi\omega/\omega_{ci}$, $D = (2/\sqrt{\pi})(R + 1)^{3/2}/(R + \sqrt{R})$, and R is the mirror ratio. Substituting Eq. (13) into Eq. (11), we obtain the dispersion relation of the drift-cyclotron loss-cone mode in the presence of the fast wave

$$\frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{m_e}{m_i} + \frac{\omega_{pi}^2}{k^2c^2} - \frac{\varepsilon\pi}{k\Omega}\left(1 - \frac{|\mu_e - \mu_i|^2}{4}\frac{N}{Q}\right) + \frac{D}{k^3a_i^3}\,\Omega\cot\Omega = 0 \quad , \qquad (14)$$

where

$$N = (f\pi)^2 \frac{D}{k^3a_i^3}\left\{\frac{\Omega\cot\Omega - (\Omega - f\pi)\cot(\omega - f\pi)}{\Omega^2 - (f\pi)^2}\left[\frac{\varepsilon\pi}{k\Omega} - \frac{D}{k^3a_i^3}\Omega\cot\Omega + \frac{D}{k^3a_i^3}\right.\right.$$

$$\left. \times\ (\Omega + f\pi)\cot(\Omega + f\pi)\right] - \frac{\Omega\cot\Omega - (\Omega + f\pi)\cot(\Omega + f\pi)}{\Omega^2 - (f\pi)^2}$$

$$\left.\times \left[\frac{\varepsilon\pi}{k\Omega} - \frac{D}{k^3a_i^3}\Omega\cot\Omega + \frac{D}{k^3a_i^3}(\Omega - f\pi)\cot(\Omega - f\pi)\right]\right\} \quad ,$$

and

$$Q = \left[\frac{\varepsilon\pi}{k\Omega}\frac{f\pi}{\Omega + f\pi} - \frac{D}{k^3a_i^3}\Omega\cot\Omega + \frac{D}{k^3a_i^3}(\Omega + f\pi)\cot(\Omega + f\pi)\right]\left[-\frac{\varepsilon\pi}{k\Omega}\frac{f\pi}{\Omega - f\pi}\right.$$

$$\left. - \frac{D}{k^3a_i^3}\Omega\cot\Omega + \frac{D}{k^3a_i^3}(\Omega - f\pi)\cot(\Omega - f\pi)\right] \quad .$$

IV. Stability Analysis

From Eq. (14) we see that the pondermotive force produced by the beating between sideband waves and fast waves "effectively" modifies the density gradient in the dispersion relation. The fast wave can destabilize the mode if $N/Q > 0$, and stabilize the mode if $N/Q < 0$. The factor N/Q has been calculated numerically for zeroth-order frequency Ω and the wave vector k at various values of plasma β. The results are shown in Figs. 2-4. We find that the fast wave with $\omega_0 = 2\omega_{ci}$ will destabilize the mode for all plasma β. This can be understood by noting that at $\omega_0 = 2\omega_{ci}$ (or f = 2), the factor N/Q is reduced to

$$\frac{N}{Q} = -\ 8\pi^2\left(\frac{D}{k^3a_i^3}\cot\Omega\right)^2 \frac{\omega_0^2}{\omega_0^2 - \omega^2}\left[\left(\frac{\varepsilon\pi}{k\Omega} - \frac{\varepsilon\pi}{k(\Omega + 2\pi)} + \frac{2\pi D}{k^3a_i^3}\cot\Omega\right)\right.$$

$$\left. \times \left(\frac{\varepsilon\pi}{k\Omega} - \frac{\varepsilon\pi}{k(\Omega - 2\pi)} - \frac{2\pi D}{k^3a_i^3}\cot\Omega\right)\right]^{-1} \quad . \tag{15}$$

Since the numerator of Eq. (15) is always positive, the sign of N/Q is determined by the denominator which is negative for all plasma β. For Phaedrus parameters, e.g., $\beta = 0.3$, $\omega_0 = 2\omega_{ci}$, $T_i = 1$ keV, and B = 2 kG, the critical plasma radius required for drift-cyclotron loss-cone stability will be increased by 10% if the wave electric field $E_{xo} \sim 60$ V/cm, which is less than $E_c \sim 300$ V/cm. Since $E_{xo} < E_c$, our assumption is justified. With the same plasma parameters and electric field strength, but $\omega_0 = 1.8\omega_{ci}$, the critical plasma radius is reduced by 10%. Since the field strength is within the range of possibilities for the Phaedrus ion cyclotron heating experiment, these stabilization and destabilization effects may be able to be tested in the experiment.

V. Concluding Remarks

We have shown from the particle orbit calculation, that the effect of resonant ion motion with the external wave at $\omega_0 = 2\omega_{ci}$ on the drift-cyclotron loss-cone mode can be neglected if the wave electric field strength E_{xo} is less than a critical value $E_c = 4k_\parallel T_i/e$. The effect of nonresonant ion and electron motion on the mode can stabilize or destabilize the mode depending on the wave frequency ω_0 and plasma β. At $\omega_0 = 2\omega_{ci}$, the fast wave can destabilize the mode for all plasma β. The field strength required to stabilize or destabilize the mode is about 100 V/cm and might be achievable in the Phaedrus experiment.

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Appendix A

The equation of motion for the particle in the fast wave fields can be written as

$$\ddot{x} + \left(\omega_c^2 + \frac{ek_x\omega_c E_y}{m\omega_o}\right)x = \omega_c \dot{A}_x + \frac{eE_x}{m} \quad ,$$

$$\ddot{y} + \left(\omega_c^2 + \frac{ek_x\omega_c E_y}{m\omega_o}\right)y = - \omega_c \dot{A}_y + \frac{eE_y}{m} \quad , \tag{A1}$$

where $\dot{A}_x$ and $\dot{A}_y$ are defined as

$$\dot{A}_x = \frac{e}{m} \int E_x \, dt + \frac{ek_x}{m\omega_o} \int v_y E_y \, dt \quad ,$$

$$\dot{A}_y = \frac{e}{m} \int E_y \, dt - \frac{ek_x}{m\omega_o} \int v_x E_y \, dt \quad . \tag{A2}$$

Assuming $ek_x E_{y(x)}/m\omega_c^2 \ll 1$, we obtain the zeroth-order solutions of Eq. (A1)

$$x = - a \cos (\omega_c t + \phi) \quad , \quad y = a \sin (\omega_c t + \phi) \quad , \tag{A3}$$

and

$$v_x = v_\perp \sin (\omega_c t + \phi) \quad , \quad v_y = v_\perp \cos (\omega_c t + \phi) \quad , \tag{A4}$$

where $v_\perp$ is the initial perpendicular velocity of the particle, and ϕ is the initial gyrophase. To the next order, we assume the solution in the y direction has the form

$$y = a \sin (\omega_c t + \phi) + y'(a,\phi,\omega_o t) + \ldots$$

$$\dot{a} = A'(a,\phi) \quad , \tag{A5}$$

$$\dot{\phi} = B'(a,\phi) \quad ,$$

where y', A', and B' are first order corrections to Eq. (A4). Then, first order equations in the y direction can be written as

$$\ddot{y}' + \omega_c^2 y' = - \frac{e k_x \omega_c E_y}{m\omega_o} y - \omega_c \dot{H}_y + \frac{eE_y}{m} - 2A'\omega_c \cos (\omega_c t + \phi)$$
$$- 2a\omega_c B' \sin (\omega_c t + \phi) \quad , \tag{A6}$$

where y in Eq. (A6) is the zeroth-order solution given in Eq. (A3). Fourier analyzing the terms $\dot{H}_y$, eE_y/m, and $ek_x\omega_c E_y y/m\omega_o$ on the right hand side of Eq. (A6) we obtain

$$H_y = - \sum_{\substack{n=-\infty \\ n\omega_c+\omega_o\neq 0}}^{\infty} \left[\frac{eE_{xo}}{m} J_n(\alpha) + \frac{ek_x v_\perp E_{yo}}{m\omega_o} J_n'(\alpha)\right] \frac{\cos [n(\omega_c t + \phi + \frac{\pi}{2}) + \omega_o t]}{(n\omega_c + \omega_o)^2} \quad ,$$

$$\frac{eE_y}{m} = - \frac{eE_{yo}}{m} \sum_{n=-\infty}^{\infty} J_n(\alpha) \sin [n(\omega_c t + \phi + \frac{\pi}{2}) + \omega_o t] \quad , \tag{A7}$$

$$\frac{ek_x\omega_c E_y y}{m\omega_o} = \frac{e}{m}\frac{\omega_c}{\omega_o} E_{yo} \sum_{n=\infty}^{\infty} nJ_n(\alpha) \sin [n(\omega_c t + \phi + \frac{\pi}{2}) + \omega_o t] \quad ,$$

where $\alpha = k_x v_\perp/\omega_c$, J_n is the Bessel function of order n, and $J_n(a) = dJ_n/d\alpha$. The Fourier component of H_n when $n\omega_c + \omega_o = 0$ is zero. Substituting Eq. (A7) into (A6), Fourier analyzing y' as

$$y' = \sum_{n=-\infty}^{\infty} \{Y_n^{(1)} \cos [n(\omega_c t + \phi + \frac{\pi}{2}) + \omega_o t + Y_n^{(2)} \sin [n(\omega_c t + \phi + \frac{\pi}{2}) + \omega_o t]\} \quad , \tag{A8}$$

and setting the coefficient of the sine or cosine function equal to zero, we obtain $Y_n^{(1)} = 0$, $A' = 0$, $B' = 0$, and

$$Y_n^{(2)} = \sum_{n=-\infty}^{\infty} \{\frac{eE_{yo}}{m} (\frac{n\omega_c + \omega_o}{\omega_o}) J_n(\alpha) + \frac{\omega_c}{n\omega_c + \omega_o} \frac{eE_{xo}}{m} [J_n(a) + \frac{\alpha\omega_c}{\omega_o}\frac{E_{yo}}{E_{xo}} J'_n(\alpha)]\} \tag{A9}$$

$$\times [(n\omega_c + \omega_o)^2 - \omega_c^2]^{-1} \quad ,$$

if $(n\omega_c + \omega_o)^2 \neq \omega_c^2$. If $(n\omega_c + \omega_o) = \omega_c^2$, we obtain

$$A' = \frac{eE_{yo}}{m\omega_c} [J'_{-\ell}(\alpha) + \frac{E_{xo}}{E_{yo}} \frac{\ell}{\alpha} J_{-\ell}(\alpha)] \cos [\ell(\phi + \frac{\pi}{2})] \quad ,$$

$$B' = - \frac{eE_{yo}}{ma\omega_c} [(\frac{\ell}{\alpha} + \frac{\alpha}{\ell}) J_{-\ell}(\alpha) - \frac{E_{xo}}{E_{yo}} J'_{-\ell}(a)] \sin [\ell(\phi + \frac{\pi}{2})] \quad , \tag{A10}$$

where $\ell = \omega_0/\omega_c$. Substituting Eqs. (A9) and (A10) into (A5), we obtain Eqs. (6) and (7).

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Figure Captions

Fig. 1. Configuration of the coordinates.

Fig. 2. N/Q as a function of plasma β at $\omega_0 = 2\omega_{ci}$.

Fig. 3. N/Q as a function of plasma β at $\omega_0 = 1.9\omega_{ci}$.

Fig. 4. N/Q as a function of plasma β at $\omega_0 = 1.8\omega_{ci}$.

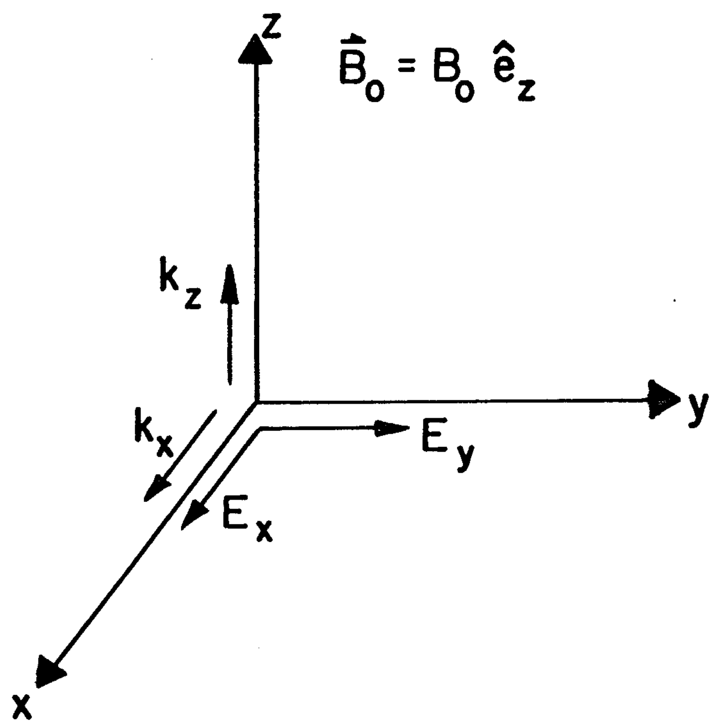

Figure 1

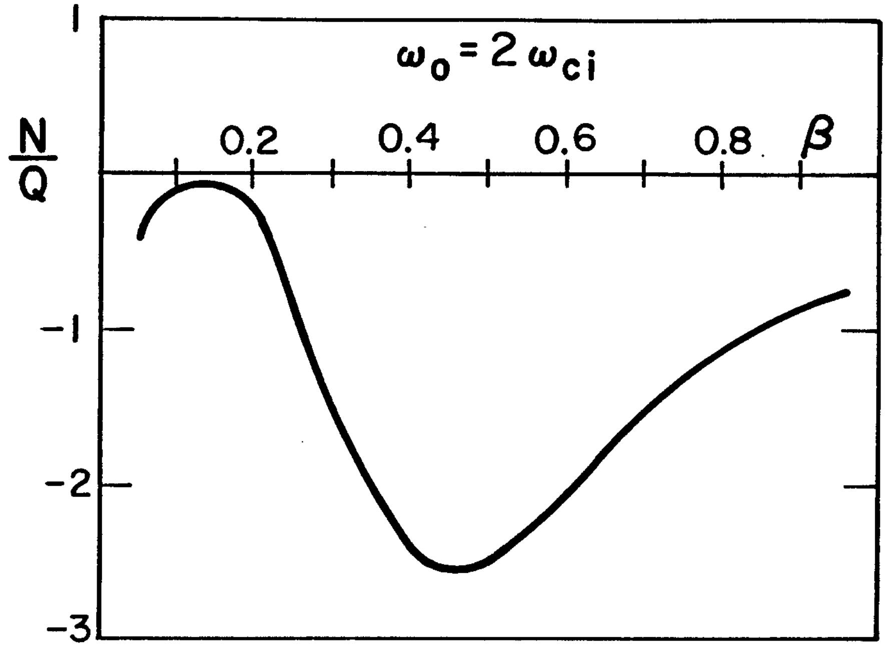

Figure 2

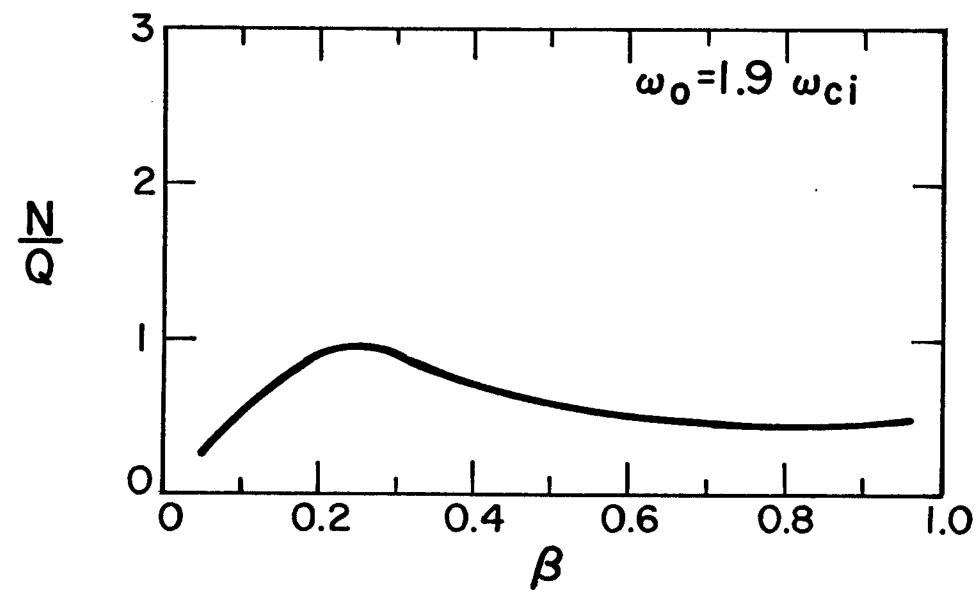

Figure 3

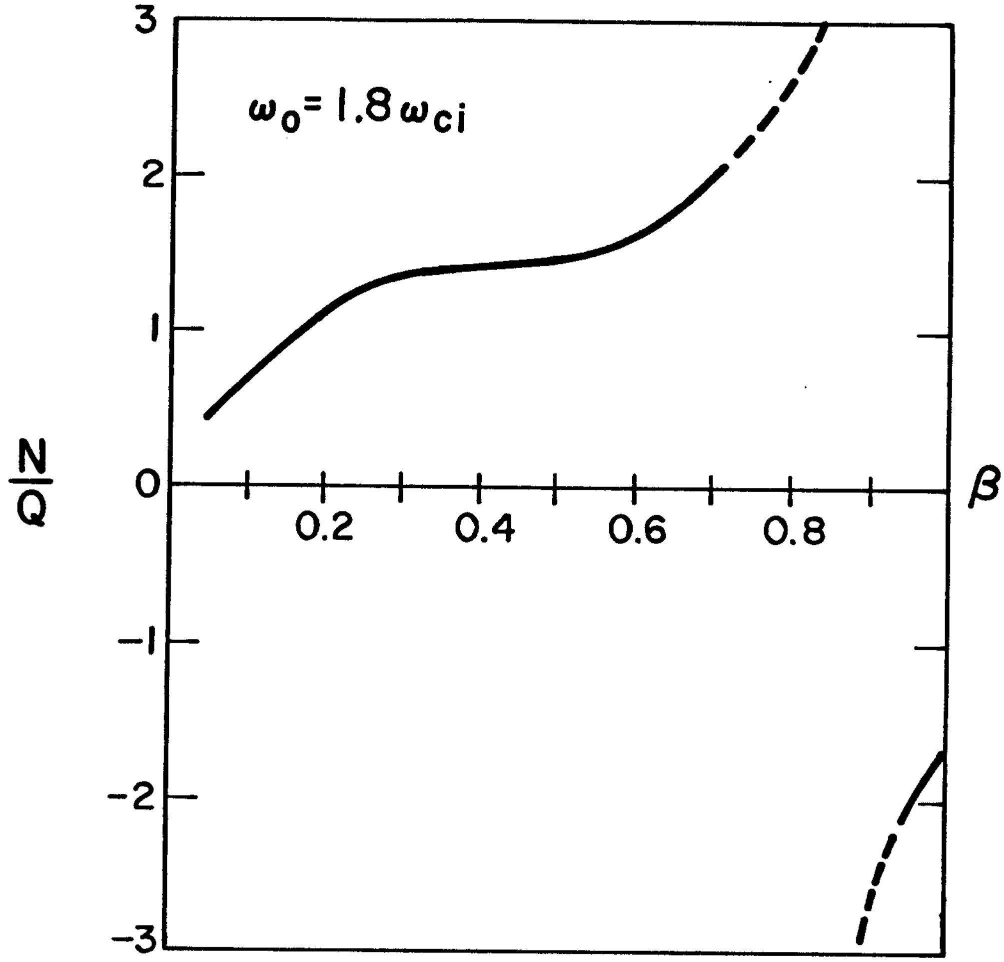

Figure 4