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VARIATIONAL METHODS FOR CONTROLLED THERMONUCLEAR REACTOR BLANKET STUDIES*

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ABSTRACT. Variational procedures for neutron and photon transport calculations in CTR blanket studies are considered. The procedures can be readily implemented to yield accurate results in an efficient manner and will be especially useful in performing sensitivity studies and survey calculations. They are applicable to multidimensional problems and to the time-dependent, as well as the steady-state, case. The use of the dual-space approach for certain sensitivity studies is also considered. Numerical results are presented to demonstrate the efficacy of the procedures discussed.

1. INTRODUCTION

The analysis of CTR (Controlled Thermonuclear Reactor) blanket systems via neutron and photon-transport calculations has become a widespread endeavour in the past few years [1-5]. At this early stage, one is frequently interested in performing survey calculations and sensitivity studies. In general, one is most often interested in reaction rates, examples of which include the tritium production, atom displacement, and helium and hydrogen production rates, the neutron and gamma heating rates, certain transmutation rates, and so on. It is useful to determine the sensitivity of these quantities to material composition and/or material substitution in the blanket, and to the nuclear data (the cross-sections and transfer matrices). Up to this time, the procedure has been to make those changes in blanket design of interest and perform a new neutron-photon transport calculation. A separate calculation is, therefore, required for each successive change that might be considered. Clearly, it is desirable to have a procedure that allows the evaluation of the effect of these changes without the need to carry out a new transport calculation each time. Such a scheme would have a considerable advantage when performing survey and sensitivity studies and can bring about a considerable reduction in computing time and effort, especially for multidimensional problems.

Such a procedure can be formulated for CTR blanket and shield studies by using variational principles. The theory is applicable to steady-state and time-dependent problems and to coupled neutron-photon transport calculational procedures. The functionals employed are the bilinear and fractional forms of the Schwinger variational principle [6,7], suitably generalized for the needs of this work [8,9]. The Schwinger functionals, which traditionally require trial functions, and generalizations thereof, which employ trial matrices [7,10], have proven effective tools in the study of collision phenomena [11]. In reactor physics, basic studies using the variational method have been carried out by Francis et al. [8] and Selengut [12] and a generalization to allow the estimation of arbitrary linear functionals of the solution to an inhomogeneous equation has been given by Pomraning [9]. Stacey has extended the formalism to derive variational estimates and a generalized perturbation theory for ratios of linear and bilinear functionals [13], and has applied these developments to several problems in nuclear reactor physics [14].

In this paper, the variational theory is formulated and specialized for studies of CTR blanket systems. A generalized perturbation theory is presented for evaluating the sensitivity of integral quantities, such as reaction rates and ratios thereof, to changes that are made in the nuclear data or in the material composition or structure of the blanket. A new flux calculation is not required to obtain the quantities of interest. The use of variational methods to study the changes in the calculational model, such as the order of a discrete ordinates calculation, the order of scattering anisotropy, or the form of the source, is also

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discussed. Numerical results for some model calculations are given which serve to demonstrate how the theory is implemented, the types of estimates and alterations in system parameters which can be examined, and the effectiveness of the theory.

2. THEORY

A. BASIC EQUATIONS

Consider the problem of estimating a functional, $G[\phi]$, of the neutron-photon flux, $\phi(x)$, when the latter satisfies the linear equation

$$L\phi = S \tag{1}$$

L is the Boltzmann transport operator [15], S is an external source, and the variable x represents a collection of independent variables (e.g. space, time, energy). The functional $G[\phi]$ most commonly encountered is a reaction rate or a ratio of reaction rates.

Assume that we have carried out the solution to Eq.(1) in some reference configuration, obtaining a reference solution ϕ_{ref}. We are interested in changing the blanket configuration, nuclear data, or source and obtaining an estimate of $G[\phi_{pert}]$, the value of the functional when evaluated with the flux solution in the changed system, without actually solving Eq.(1) for the perturbed flux, ϕ_{pert}. We can estimate $G[\phi_{pert}]$ in a perturbed system directly from $G[\phi_{ref}]$. This is just the first-order perturbation theory result and the error made is first-order in $\delta\phi \equiv \phi_{pert} - \phi_{ref}$.

A variational estimate yields results accurate through second order in $\delta\phi$. Consider the functional, [9],

$$I_B[\Gamma^*, \phi] = G[\phi] + \langle \Gamma^*, (S_{pert} - L_{pert}\phi) \rangle \tag{2}$$

with $\langle , \rangle$ denoting a sum over discrete independent variables and an integral over continuous independent variables. This functional is stationary about functions ϕ_s and Γ_s^* for which

$$\delta I_B \equiv \frac{\delta I_B}{\delta \phi}\,\delta\phi + \frac{\delta I_B}{\delta \Gamma^*}\,\delta\Gamma^* = 0$$

for arbitrary $\delta\phi$ and $\delta\Gamma^*$. This requires that ϕ_s and Γ_s^* satisfy

$$L_{pert}\phi_s = S_{pert}$$

and

$$L^*_{pert}\,\Gamma_s^* = G'[\phi_{pert}] \tag{3}$$

where L^* is the adjoint transport operator [15] and $G'[\phi]$ is the functional derivative of G with respect to ϕ. Obviously, $\phi_s \equiv \phi_{pert}$.

The advantage of using the functional I_B of Eq.(2) to estimate the quantity of interest accrues from the observation that if arbitrary trial functions Γ^* and ϕ are used to evaluate I_B the resulting estimate of $G[\phi_{pert}]$ is accurate to second order, i.e.

$$I_B[\Gamma^*, \phi] = G[\phi] + \langle \Gamma^*, (S_{pert} - L_{pert}\phi) \rangle$$

$$\equiv G[\phi_{pert}] - \langle \delta\Gamma^*, L_{pert}\,\delta\phi \rangle$$

where $\delta\Gamma \equiv \Gamma^* - \Gamma_s$ and $\delta\phi \equiv \phi - \phi_s \equiv \phi - \phi_{pert}$. In particular, an estimate of $G[\phi_{pert}]$, accurate through second order in the errors $\delta\Gamma^*$ and $\delta\phi$, is

$$I_B[\Gamma^*_{ref}, \phi_{ref}] = G[\phi_{ref}] + \langle \Gamma^*_{ref}, (S^{pert} - L^{pert}\phi_{ref}) \rangle \tag{4}$$

where Γ^*_{ref} and ϕ_{ref} are obtained from a reference system calculation. Most conveniently, writing $L^{pert} = L^{ref} + \Delta L$ and $S^{pert} = S^{ref} + \Delta S$, the functional in Eq.(4) becomes (using $L^{ref}\phi_{ref} = S^{ref}$)

$$I_B[\Gamma^*_{ref}, \phi_{ref}] = G[\phi_{ref}] + \langle \Gamma^*_{ref}, (\Delta S - \Delta L\phi_{ref}) \rangle \tag{5}$$

In the important case of bounded linear functionals, for which $G[\phi]$ can be written as $\langle W\phi \rangle$, (and $G'[\phi] = W$) the fractional, or Schwinger, form of the variational principle is useful [8]. Using $\phi = c\phi$, $\Gamma^* = c^*\Gamma^*$ and rendering $I_B[c^*\Gamma^*, c\phi]$ stationary with respect to arbitrary variations in the constants c and c^* yields

$$I_F[\Gamma^*_{ref}, \phi_{ref}] = \frac{\langle S^{pert}\Gamma^*_{ref} \rangle \langle W^{pert}\phi_{ref} \rangle}{\langle \Gamma^*_{ref}, L^{pert}\phi_{ref} \rangle} \tag{6}$$

This fractional form is independent of the normalization of Γ^* and ϕ.

When the functional $G[\phi]$ can be written as the ratio of linear functionals, i.e.

$$G[\phi] = \langle W_1\phi \rangle / \langle W_2\phi \rangle$$

the equation to be solved for Γ^*_{ref} is, from Eq.(3),

$$L^*_{ref}\,\Gamma^*_{ref} = G'[\phi_{ref}] = \frac{W_1}{\langle W_2\phi_{ref} \rangle} - G[\phi_{ref}]\,\frac{W_2}{\langle W_2\phi_{ref} \rangle}$$

(The mathematical properties of this equation are discussed elsewhere [9, 13].) In this case, since the magnitude of Γ^*_{ref} depends inversely upon the normalization of ϕ_{ref}, I_B is independent of the normalization of Γ^*_{ref} and ϕ_{ref}.

As in synthesis methods [16], it can sometimes be advantageous to use trial functions that are linear combinations of several reference flux and

reference adjoint calculations. Write the trial functions, $\phi_t(x)$ and $\Gamma_t^*(x)$, as

$$\phi_t(x) = \sum_{i=1}^{N} a_i \phi_{ref}^i(x) = \vec{a}^T \cdot \vec{\phi}_{ref} \quad (7)$$

$$\Gamma_t^*(x) = \sum_{i=1}^{N} b_i \Gamma_{ref}^{*,i}(x) = \vec{b}^T \cdot \vec{\Gamma}_{ref}^* \quad (8)$$

where N is the number of reference functions used, T denotes transposes, and $\vec{a}$, $\vec{\phi}_{ref}$, and $\vec{\Gamma}_{ref}$ are the vectors

$$\vec{a} = \{a_i\} \quad (9)$$

$$\vec{\phi} = \{\phi_{ref}^i\} \quad (10)$$

$$\vec{\Gamma}^* = \{\Gamma_{ref}^{*,i}\} \quad (11)$$

The index i labels the i-th component of these vectors. Rendering the functional $I_B[\vec{\Gamma}^*, \vec{\phi}]$ stationary with respect to the $\{a_i\}$ and $\{b_i\}$ leads to the compact expression,

$$I[\vec{\Gamma}^*, \vec{\phi}] = \langle \vec{\Gamma}_{ref}^* \; S^{pert} \rangle^T \cdot M^{-1} \cdot \langle W \vec{\phi}_{ref} \rangle \quad (12)$$

where M^{-1} is the inverse of a matrix, M, whose elements M_{ij} are given by

$$M_{ij} = \langle \Gamma_{ref}^{*,i}, L^{pert} \phi_{ref}^j \rangle \quad (13)$$

This procedure can be important for generating accurate estimates even if a poor set of trial functions is used. It has been found, for example, that even when the partial sums, $\sum_{i=1}^{N} \phi_{ref}^i(x)$, are diverging, the procedure leading to the functional, Eq.(12), yields accurate results [10].

B. ESTIMATION OF CTR BLANKET AND SHIELD PARAMETERS

Since most CTR blanket and shield calculations are carried out using some computer code to solve the neutron-photon transport problem, it is useful to examine the discrete ordinates form of the functionals in Eqs (4) and (6). Consider the steady-state case and slab geometry. Formulations in other geometries and several dimensions, as well as inclusion of the time variable, is straightforward (the time-variable may be important for pulsed CTR systems [17]). The multigroup discrete-ordinates form of the transport operator, L, and its adjoint, L*, are

$$L_{DO} = \mu_j \frac{\partial}{\partial x} + \Sigma_T^g(x) - \sum_{g' \le g}^{G} \sum_{j'=1}^{N} \sum_{\ell=0}^{L} \left(\frac{2\ell+1}{2}\right) \times \Sigma_{s,\ell}^{g' \to g}(x) P_\ell(\mu_j) W_{j'} P_\ell(\mu_{j'}) \quad (14)$$

$$L_{DO}^* = -\mu_j \frac{\partial}{\partial x} + \Sigma_T^g(x) - \sum_{g' \ge g}^{G} \sum_{j'=1}^{N} \sum_{\ell=0}^{L} \left(\frac{2\ell+1}{2}\right) \times \Sigma_{s,\ell}^{g \to g'}(x) P_\ell(\mu_j) W_{j'} P_\ell(\mu_{j'}) \quad (15)$$

The index j labels the angular ordinate, g labels the group, W_j is the angular weight, $P_\ell(\mu)$ is the ℓ-th Legendre polynomial, N is the order of the discrete-ordinates approximation, G is the total number of energy groups, L is the order of scattering anisotropy, and the remaining notation is standard [15]. When the order of the discrete-ordinates calculation is unchanged from the reference to the altered or perturbed system, the variational expression (5) becomes

$$I_B[\Gamma_{ref}^*, \phi_{ref}] = G[\phi_{ref}] + \int dx \sum_{g=1}^{G} \sum_{j=1}^{N} \times W_j \Gamma_{ref,j}^{*,g}(x) \Delta S_j^g(x) \phi_{ref,j}^g(x) - \int dx \sum_{g=1}^{G} \sum_{j=1}^{N} \times W_j \Gamma_{ref,j}^{*,g}(x) \left[\Delta \Sigma_T^g(x) \phi_{ref,j}^g(x) - \sum_{g' \le g}^{G} \sum_{j'=1}^{N} \sum_{\ell=0}^{\max[L',L]} \times \left(\frac{2\ell+1}{2}\right) \Delta \Sigma_{s,\ell}^{g' \to g}(x) \phi_{ref,j'}^{g'}(x) P_{jj'}^\ell \right] \quad (16)$$

where

$$\Delta S_j^g(x) = \bar{S}_j^g(x) - S_j^g(x) \quad (17a)$$

$$\Delta \Sigma_T^g(x) = \bar{\Sigma}_T^g(x) - \Sigma_T^g(x) \quad (17b)$$

$$\Delta \Sigma_{s,\ell}^{g' \to g} = \bar{\Sigma}_{s,\ell}^{g' \to g}(x) - \Sigma_{s,\ell}^{g' \to g}(x) \quad (17c)$$

$$P_{jj'}^\ell = W_{j'} P_\ell(\mu_j) P_\ell(\mu_{j'}) \quad (17d)$$

The parameters with bars overhead are evaluated in the altered system whereas those without bars refer to the reference system. An analogous expression can be obtained for the fractional form, $I_F[\Gamma^*, \phi]$, in Eq.(6). Recall that in Eq.(6), Γ^* and ϕ refer to a reference system, whereas the functions S(x) and W(x) and the operator L_{DO} are evaluated in the altered system.

The variational estimation of quantities important for CTR blanket studies requires as trial functions the reference flux and one reference adjoint for each physical quantity, $G[\phi]$. The sensitivity of $G[\phi]$ to changes in the blanket configuration, nuclear data, and source can then be evaluated without the necessity of performing additional transport calculations. The integrals

which must be evaluated are similar to those which arise in first-order perturbation theory. Thus, existing codes can be used to evaluate the variational estimates.

Some specific examples of functionals, $G[\phi]$, of interest in CTR studies are:

(a) Reaction rate of i-th type

$$G[\phi] = \langle \Sigma_i \phi \rangle \tag{18}$$

$$G'[\phi] = \epsilon_i \Sigma_i(x), \quad \epsilon_i = \begin{cases} 1 & \text{spatial zone of interest} \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

(b) Ratios of i-th reaction rate to j-th reaction rate

$$G[\phi] = \frac{\langle \Sigma_i \phi \rangle}{\langle \Sigma_j \phi \rangle} \tag{20}$$

$$G'[\phi] = \frac{\Sigma_i - G[\phi]\,\Sigma_i}{\langle \Sigma_j \phi \rangle} \tag{21}$$

Regionwise estimates again are possible.

(c) Ratio of fluxes at two points in blanket or shield

$$G[\phi] = \frac{\langle \delta(x - x_0)\phi \rangle}{\langle \delta(x - x_1)\phi \rangle} \tag{22}$$

$$G'[\phi] = \frac{\delta(x - x_0) - G[\phi]\,\delta(x - x_1)}{\langle \delta(x - x_1)\phi \rangle} \tag{23}$$

(d) Neutron and gamma heating rates

$$G[\phi] = \int_{\Delta} dx\, H(x) \tag{24}$$

where Δ denotes the width of the zone in which the heating rate is required and $H(x)$ is defined as

$$H(x) = \sum_i N_i \sum_{g=1}^{G} (K^g_{n,i}\epsilon_n + K^g_{\gamma,i}\epsilon_\gamma)\phi^g(x) \tag{25}$$

The subscript i labels the i-th isotope, N_i is the atomic density of the i-th isotope, $K^g_{n,i}$ and $K^g_{\gamma,i}$ are the neutron and gamma Kerma factors [18], and

$$\epsilon_n = \begin{cases} 1 & g \in \text{(neutron groups)} \\ 0 & \text{otherwise} \end{cases}$$

$$\epsilon_\gamma = \begin{cases} 1 & g \in \text{(gamma groups)} \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

The functional derivative of $G[\phi]$ is therefore

$$G'[\phi] = \epsilon_r \sum_i N_i \sum_{g=1}^{G} (K^g_{n,i}\epsilon_n + K^g_{\gamma,i}\epsilon_\gamma) \tag{27}$$

$$\epsilon_r = \begin{cases} 1 & x \in \Delta \\ 0 & \text{otherwise} \end{cases}$$

Other quantities, such as the ratio of neutron and/or gamma currents, can similarly be formulated.

It is also clear by inspection of Eq.(16) that one can examine the following types of changes in the blanket system or calculational model: (1) changes in nuclear data (microscopic cross-sections and transfer matrices); (2) changes in material composition; (3) changes in the order of anisotropic scattering in the problem; and (4) changes in the type of source. Further, the effects of entire material replacements can be examined. The advantage of not requiring a new flux calculation each time a set of changes is made can be important when performing survey calculations and sensitivity studies, especially when studying multi-dimensional problems.

One can also examine variationally the effects on $G[\phi]$ of a change in the order of discrete ordinates from N to $\bar{N} > N$ without performing higher-order S_N calculations. Considering Eqs (4) or (6), one requires a method to generate trial functions compatible with the order $\bar{N}$ form of L^{pert}_{DO} using trial functions known only at the N quadrature points of the reference case. Linear interpolation and extrapolation can be used for this purpose [19]. Further, standard methods in approximation theory [20] can be utilized. For example, consider developing an interpolation formula for a function $\psi(\mu)$, defined on [-1, 1], given values of ψ at N distinct points, $\{\mu_j\}$, with corresponding quadrature weights $\{W_j\}$. Expanding $\psi(\mu)$ in polynomials, $F_n(\mu)$, orthogonal on [-1, 1] with weight function $\alpha_n(\mu)$ yields

$$\psi(\mu) = \sum_{n=0}^{\infty} a_n \alpha_n(\mu) F_n(\mu) \tag{28}$$

$$a_n = \int_{-1}^{1} F_n(\mu)\psi(\mu)\, d\mu \tag{29}$$

For $\psi(\mu)$ known only at the N abscissa $\{\mu_j\}$ with corresponding weights $\{W_j\}$, the approximation for a_n is

$$a_n \simeq \sum_{j=1}^{N} W_j F_n(\mu_j)\psi(\mu_j) \tag{30}$$

The most common choice for $F_n(\mu)$ is the Legendre polynomial, though Chebyshev polynomials can

also be used. Both choices will guarantee conservation of total flux and current. For the present need of developing adequate trial functions, either choice should suffice.

We have indicated how the sensitivity of integral parameters to changes in blanket configuration can be evaluated. In this development, the generalized adjoint function, Γ^*, has played a major role. In fact, Γ^* is something of a sensitivity or importance function relative to the integral parameter $G[\phi]$. When $G[\phi] = \langle W\phi \rangle$, examination of Γ^* itself provides useful insight. To understand this, consider the form taken by I_B in this case:

$$I_B[\Gamma^*, \phi] = \langle W\phi \rangle + \langle \Gamma^*, (S - L\phi) \rangle \qquad (31)$$

When ϕ_s satisfies $L\phi_s = S$, $I_B[\Gamma^*, \phi_s] = \langle W\phi_s \rangle$. On the other hand, when Γ_s^* satisfies $L^*\Gamma_s^* = W$, then $I_B[\Gamma^*_s, \phi] = \langle \Gamma^*_s S \rangle$. Thus, the stationary value of I_B is

$$I_B[\Gamma^*_s, \phi_s] = \langle W\phi_s \rangle = \langle \Gamma_s^* S \rangle \qquad (32)$$

Consequently, if one is interested in the effect of a change in the source distribution on $\langle W\phi \rangle$, Γ^* provides this information. This introduces interesting possibilities. Most obvious is the assessment of different representations of the plasma neutron source. However, in addition, a reference transport calculation may be performed to generate an incident source of neutrons and/or gamma rays from the interior at any annular surface in the blanket. Then, if a local change is to be made to the blanket, a local transport calculation may be performed to determine how the incident source is altered and Eq.(32) may be used to determine the effect upon $\langle W\phi \rangle$.

3. DEMONSTRATIVE EXAMPLES

Calculations were performed using the variational method to estimate the tritium production rate from ^{7}Li and the helium production rate. The former is important for breeding tritium while the latter is typical of quantities important for materials studies. These calculations are intended as illustrative and other quantities of interest can be estimated in an analogous manner. By choosing to estimate the tritium production rate in ^{7}Li and the helium production rate, one can simplify the calculational model. In particular, as noted by Maynard and Abdou [5, 22], these effects can be adequately studied using a six-group cross-section set that extends from 8.187 MeV to 14.918 MeV. The transport calculations were thus performed using six-group, P_1 cross-sections, slab geometry, and an S_4 discrete-ordinate approximation. (The ANISN code [23] was employed.) The above model is adequate for the present purposes.

A single reference neutron flux, ϕ_{ref}, plus the reference generalized adjoints, Γ^*_{ref}, for each quantity to be estimated are required to implement the variational method. The results follow from Eqs (4) or (6). Exact answers in the altered systems, required for comparison, were obtained by solving the altered problem using the transport code. The reference flux and adjoints

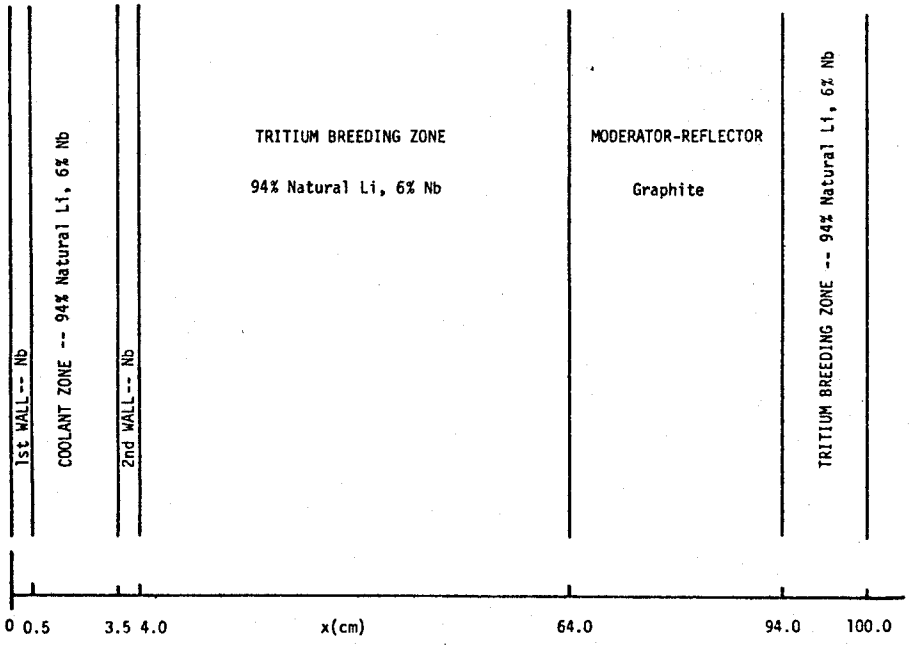

FIG. 1. Reference system.

TABLE I. EFFECT OF INCREASING NIOBIUM ABSORPTION CROSS-SECTION BY 1 BARN IN FIRST AND SECOND WALLS

Calculation	System	Total tritium production rate from ^{7}Li, at/cm^3-s	Difference from exact result, %
Direct	Reference	0.4136	13.3
Variational	Altered	0.3648	--
Direct	Altered (exact)	0.3649	--
		Helium production rate in first and second walls, at/cm^3-s x 10^{-4}	
Direct	Reference	10.685	9.00
Variational	Altered	9.718	0.93
Direct	Altered (exact)	9.809	--

TABLE II. EFFECT OF INCREASING ISOTROPIC DOWNSCATTERING CROSS-SECTIONS OF NIOBIUM BY 2 BARNS IN FIRST AND SECOND WALLS. Σ_T HELD CONSTANT.

Calculation	System	Total tritium production rate from ^{7}Li, at/cm^3-s	Difference from exact result, %
Direct	Reference	0.4136	11.4
Variational	Altered	0.4616	1.1
Direct	Altered (exact)	0.4665	--
		Helium production rate in first and second walls, at/cm^3-s x 10^{-4}	
Direct	Reference	10.685	8.8
Variational	Altered	11.676	0.2
Direct	Altered (exact)	11.697	--

were obtained from ANISN calculations on a reference system taken to be the blanket illustrated in Fig.1 with an isotropic source in the highest energy group located in the first spatial interval of the first wall.

The first alterations were changes in the nuclear data. In Table I are results for a case where the absorption and total cross-sections for niobium are increased by 1 barn in the first and second walls. This is a 25% change in the total cross-section and is analogous to increasing the optical thickness of these two zones by that percentage. The variational estimate is in excellent agreement with the exact calculation. In Table II are results for a case where the scattering transfer matrices are altered. Here, the P_0 component of the transfer cross-section to the next lower group, i.e. $\sigma_{s,0}^{g-1\to g}$ of niobium, is increased by 2 barns in both the first and second walls. This enhances the downscatter and increases $\sigma_{s,0}^{g-1\to g}$ by a factor of 20. The total cross-section in each group was not altered which has the effect of decreasing the absorption. Both this and the previous case correspond to substantial changes and yet the variational results are quite accurate. Note that, for these first two cases, first-order perturbation theory would predict no change from the reference result.

replacements, the niobium first and second walls were replaced by vanadium. The results are given in Table III. We have included the first-order perturbation theory result since it now differs from the reference result. The variational values are again very accurate.

The procedure to examine the effects of changing the source, based on the dual-space relation, $\langle S\phi^* \rangle = \langle W\phi \rangle$, is simply illustrated in Table IV. The results all agree to less than 1/2%, or to roughly within the convergence criterion employed. Of course, to use $\langle S\phi^* \rangle$ requires only an adjoint calculation to determine ϕ^* for the quantity being estimated. The results labelled $\langle W\phi \rangle$ were obtained from different flux calculations for each source considered.

TABLE III. EFFECT OF REPLACING NIOBIUM BY VANADIUM IN FIRST AND SECOND WALLS

Calculation	System	Total tritium production rate from ^{7}Li, at/cm^3-s	Difference from exact result, %
Direct	Reference	0.4136	7.3
First-order perturbation theory	Altered	0.4136	7.3
Variational	Altered	0.4402	1.2
Direct	Altered (exact)	0.4459	--
		Helium production rate in first and second walls, at/cm^3-s x 10^{-4}	
Direct	Reference	10.685	67.00
First-order perturbation theory	Altered	31.17	4.4
Variational	Altered	32.20	1.2
Direct	Altered (exact)	32.61	--

TABLE IV. SENSITIVITY TO NEUTRON SOURCE REPRESENTATION

Source [a]	Total tritium production rate from ^{7}Li, at/cm^3-s		Helium production rate in first and second walls, at/cm^3-s $\times 10^{-4}$	
	$\langle W\phi \rangle$	$\langle \Gamma^* S \rangle$	$\langle W\phi \rangle$	$\langle \Gamma^* \phi \rangle$
No. 1	0.4136	0.4137	10.685	10.698
No. 2	0.4156	0.4155	10.528	10.558
No. 3	0.3380	0.3379	6.3850	6.421

[a] No. 1: isotropic source, highest group; first spatial interval of first wall.
No. 2: isotropic source, highest group; uniformly distributed over first wall.
No. 3: isotropic source, equal strength in each group; first spatial interval of first wall.

4. CONCLUSIONS

Variational methods provide efficient procedures for performing sensitivity studies and survey calculations on CTR blanket systems. For multi-dimensional blanket calculations, one can expect further substantial savings in computing time. The integrals that must be evaluated in the variational method are of the same form as those which occur in perturbation theory. As such, existing segments of computer programs that perform perturbation-type integrals can be readily adapted for use with the variational method.

The numerical examples presented herein, on some representative CTR blanket problems, establish the promise and feasibility of the variational method in this field. Further numerical studies are required to establish the full, useful range of the variational method for these applications.

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