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Abstract

Simulation of Transport in Central Cell of a Tandem Mirror.* L.L. Lao, R.W. Conn, Univ. of Wisconsin. -- We have modified our tokamak radial transport code to include end loss terms for simulation of the central cell plasma in a tandem mirror. The plug is described using zero dimensional particle and energy balance equations with fixed radial profiles. Calculations have been done using classical transport coefficients and plasma parameters of the Wisconsin tandem device PHAEDRUS. The results show that radial ion conduction loss is not negligible compared with axial energy loss. If the transport coefficients are a few times the classical values, a beneficial effect accrues because a lower T_i leads to better axial confinement. Then, for the same injected power in the end plugs, both T_e and the plug density can be higher. Thus, ϕ_e is higher. When the transport coefficients are ten times the classical values, T_i decreases from 17 eV to 10 eV in PHAEDRUS. Radial electron conduction losses are small. A value of κ_e about 15 $\kappa_e^{classical}$ does not appreciably alter the energy balance picture. Radial particle loss is small. The equilibrium radial plasma profiles in the central cell tend to follow those of the plug. A flat ion energy profile with moderately flat density profile in the plug yields a flat ambipolar profile in the central cell.

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We present a study of radial transport in a tandem mirror. A set of fluid equations for central-cell plasma is obtained by assuming both the ions and the electrons have a Maxwellian distribution in velocity space. The axial and azimuthal dependence in configuration space are eliminated by approximating the magnetic field in central cell and end plugs as three square wells. The axial particle and energy losses are represented by an absorption term along each flux tube. The electron fluid is assumed to have a uniform temperature along each flux tube and the electron energy balance equation is averaged over the flux tube volume in the central cell and end plugs. Plug ions are assumed to have fixed radial density and radial energy profiles with average values determined by the zero dimensional particle and energy balance equations. Together with the quasi-neutral condition and conservation of magnetic flux in each flux tube, we have a set of closed equations. These are discussed in detail in (1).

Central Cell Equations

$$\frac{\partial n_c}{\partial t}(r,t) = -\frac{1}{r}\frac{\partial}{\partial r}(r\Gamma_i) - \frac{n_c^2}{2}\langle\sigma v\rangle_{DT}\,\gamma_{DT} - \frac{n_c}{\tau_{\shortparallel c}} + S_c$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}n_cT_c\right) = \frac{1}{4}n_c^2\langle\sigma v\rangle_{DT}\,E_\alpha U_\alpha^i\,\gamma_{DT} - \frac{1}{r}\frac{\partial}{\partial r}r\left(Q_i + \frac{3}{2}T_c\Gamma_i\right) + Q_{ei} + Q_{auxi} - \frac{n_c(q\phi_c + T_c)}{\tau_{\shortparallel c}} - Q_{cx}$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}\, n_c T_e + 3\, V_{pc}\, n_p T_e\right) = \frac{1}{4}\, n_c^2 <\sigma v>_{DT}\, E_\alpha U_\alpha^e\, \gamma_{DT}$$

$$- \frac{1}{r}\frac{\partial}{\partial r}\, r\, \left(Q_e + \frac{3}{2}\, T_e\, \Gamma_e\right) - Q_{ei} - Q_{rad} + Q_{auxe}$$

$$+ \frac{2V_{pc}\, n_p\, (E_p - \frac{3}{2}\, T_e)}{\tau_{drag}^p} - 2\, V_{pc}\, \frac{n_p}{\tau_{\parallel}^p}\, (e\phi_e + e\phi_c + T_e)$$

$$- (e\phi_e + T_e)\, \frac{n_c}{\tau_{\parallel}^c}$$

Plug equations

$$\frac{d\bar{n}_p}{dt} = - \frac{\bar{n}_p}{\tau_{\parallel}^p} + S_p$$

$$\frac{d}{dt}\, (\bar{n}_p \bar{E}_p) = - \frac{\bar{n}_p\, (\bar{E}_p - \frac{3}{2}\, \bar{T}_e)}{\tau_{drag}^p} - \frac{\bar{n}_p\, E_{out}}{\tau_{\parallel}^p} + P_{auxi}$$

$$+ \left(1 + \frac{\sigma_{cx}}{\sigma_i}\right)\, S_p\, E_{inj} - \frac{\sigma_{cx}}{\sigma_i}\, S_p\, \bar{E}_p$$

$$n_p\, (R) = (\alpha + 1)\, \bar{n}_p\, \left(1 - \left(\frac{R}{R_{max}}\right)^2\right)^\alpha$$

$$E_p\, (R) = \frac{\alpha + \beta + 1}{\alpha + 1}\, \bar{E}_p\, \left(1 - \left(\frac{R}{R_{max}}\right)^2\right)^\beta$$

Flux conservation

Initially, at time $t = 0$

$$\int_0^r B_c(x)\, x\, dx = \int_0^R B_p(y)\, y\, dy$$

at time t

$$\int_0^{r(t)} B_c(x)\, x\, dx = \int_0^{r(t-\Delta t)} B_c(x)\, x\, dx$$

$$\int_0^{R(t)} B_p(y)\, y\, dy = \int_0^{R(t-\Delta t)} B_p(y)\, y\, dy$$

Quasi-neutrality

$$\frac{n_{ec}^2(r)}{(n\tau_{\parallel})_c^e} + \frac{2\, V_{pc}\, n_{ep}^2\, (r)}{(n\tau_{\parallel})_p^e} + \frac{1}{r}\frac{\partial}{\partial r} r\Gamma_e$$

$$= \frac{n_{ic}^2\, (r)}{(n\tau_{\parallel})_c^i} + \frac{2\, V_{pc}\, n_{ip}^2\, (r)}{(n\tau_{\parallel})_p^i} + \frac{1}{r}\frac{\partial}{\partial r} r\Gamma_i$$

where

$U_{\alpha e}$, $U_{\alpha i}$ = fraction of alpha particle energy to electrons and ions, respectively

γ_{DT} = 1 for DT plasma, 0 otherwise

Q_{ei} = electron and central-cell ion rethermalization term

Q_{rad} = bremsstrahlung, line, recombination, and synchrotron radiation term

Q_{auxi}, Q_{auxe} = ion and electron auxiliary heating term

Q_i, Q_e = central-cell ion and electron heat flux

Γ_i, Γ_e = central-cell ion and electron particle flux

S_c = central-cell particle source

Q_{cx} = central-cell ion charge exchange loss term

E_{out} = average energy carried out by each plug ion

V_{pc} = $\dfrac{\text{flux tube volume in plug}}{\text{flux tube volume in central cell}}$

The boundary conditions for the central-cell equations are:

$$\Gamma,\ Q_e,\ Q_i \,\Big|_{r=0} = 0$$

$$(\alpha_e\ \alpha_i\ \alpha_n)\begin{pmatrix} T_e \\ T_i \\ n \end{pmatrix}_{r=r_{max}} + (\beta_e\ \beta_i\ \beta_n)\begin{pmatrix} \frac{\partial T_e}{\partial r} \\ \frac{\partial T_i}{\partial r} \\ \frac{\partial n}{\partial r} \end{pmatrix}_{r=r_{max}} = 0$$

We have modified our tokamak radial transport code[2] to solve these equations. The difference in characteristic time for central-cell and plug plasma allows their solutions without major modification of the code itself. At each time step n the plug equations are first advanced in time by subdividing the time interval into smaller time steps and using the value of T_e at time n. The central-cell equations are then advanced using the plug parameters at time $n + 1$. In the following we summarize the result of calculations using classical transport coefficients and parameters of the Wisconsin tandem device PHAEDRUS.[3] Classically,

$$\begin{pmatrix} \bar{Q}_e \\ \bar{Q}_i \\ \bar{\Gamma}_i \end{pmatrix} = \bar{\bar{D}} \begin{pmatrix} \bar{\nabla} r T_e \\ \bar{\nabla} r T_c \\ \bar{\nabla} r n_c \end{pmatrix}$$

$$D_{11} = -\ 7.567 \times 10^{-5}\ C_{11} \frac{n_c^2\ \ell n \Lambda}{T_e^{1/2}\ B^2}$$

$$D_{22} = -\ 9.796 \times 10^{-4}\ C_{22} \frac{n_c^2\ \ell n \Lambda}{T_c^{1/2}\ B^2}$$

$$D_{31} = 8.15 \times 10^{-6}\, C_{31} \frac{n_c^2 \, \ell n\Lambda}{T_e^{\frac{3}{2}} B^2}$$

$$D_{32} = -1.629 \times 10^{-5}\, C_{32} \frac{n_c^2 \, \ell n\Lambda}{T_e^{\frac{3}{2}} B^2}$$

$$D_{33} = -1.629 \times 10^{-5}\, C_{33} \frac{n_c \, \ell n\Lambda}{T_e^{\frac{3}{2}} B^2} (3T_e - T_c)$$

$$D_{12} = D_{13} = 0$$

$$D_{21} = D_{23} = 0$$

where C_{11}, C_{22}, C_{31}, C_{32}, C_{33} are flush factors, temperature is in eV, density is in cm^{-3}, and time is in seconds.

1. Comparison between space-time and point model

The average parameters obtained from the one-dimensional space-time code is compared against a point code(4) in Table 1 for PHAEDRUS and a Q = 10 tandem reactor (5). The agreement is within 10%.

2. Effects of boundary conditions

Both fixed and gradient boundary conditions are used. For fixed boundary conditions, the density and temperatures of central cell plasma at the edge are set at constant low values. For gradient boundary conditions, the extrapolation lengths are set equal to 3 cm. The equilibrium radial profiles obtained are compared in Figs. 1a - 1b. The plug is assumed to vary radially as

$$n_p = n_{pmax} \left(1 - \left(\frac{R}{R_{max}}\right)^2\right)^{\alpha}$$

$$E_p = E_{pmax} \left(1 - \left(\frac{R}{R_{max}}\right)^2\right)^{\beta}$$

$$\alpha = 1, \quad \beta = .1$$

The solutions are not sensitive to the boundary conditions.

3. Effects of transport coefficients

The dominant classical radial transport is due to ion conduction. The equilibrium profiles, when the transport coefficients are 1 and 10 times the classical values, are shown in Figs. 2a - 2b. When the transport coefficients are a few times the classical values, a beneficial effect accrues because a lower T_i leads to better axial confinement. For the same injected power in end plugs, both T_e and the plug density are higher. The relative fraction of various particle and energy losses are shown in Table 2. If the transport coefficients are 10 times the classical values, radial transport becomes dominant.

4. Effects of plug density profiles

The equilibrium profiles for the cases when the plug density is relatively flat (α = .2) and peaked (α = 1) are shown in Figs. 3a - 3b. They tend to follow the plug profiles. In particular, for the case α = .2, the ambipolar potential profile, $\phi_e(r)$, is fairly flat over most of the central-cell volume.

Acknowledgement

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Table 1

Comparison Between Space-time and Point Models

PHAEDRUS

	Point	Space-time
$n_p (cm^{-3})$	1.0×10^{13}	9.8×10^{12}
E_p(keV)	2.5	2.4
R_p(vaccum)	1.8	1.8
$n_c (cm^{-3})$	4.1×10^{12}	4.0×10^{12}
T_c(ev)	16.0	16.7
T_e(ev)	50	54
ϕ_e(ev)	295	298
ϕ_c(ev)	45	48
$\frac{V_c}{V_p}$	20	18

Tandem Reactor*

	Point	Space-time
$n_p (cm^{-3})$	3.8×10^{14}	3.9×10^{14}
E_p(keV)	1080	930
R_p(vaccum)	1.2	1.2
$n_c (cm^{-3})$	3.6×10^{13}	3.9×10^{13}
T_c(keV)	39	44
T_e(keV)	44	42
ϕ_e(keV)	--	260
ϕ_c(keV)	103	100
P_{in}(MW)	300	310
P_{th}(MW)	3000	3000
Q	10	10

*See UWFDM-267 "Parametric Studies of Tandem Mirror Reactors," K.C. Shaing, R.W. Conn, J. Kesner.

Table 2

Relative Fraction of Particle and Energy Losses for PHAEDRUS

When the Transport Coefficients are 1 and 10

Times the Classical Values

	(D_{cl}, X_{cl})	$10(D_{cl}, X_{cl})$
radial ion conduction	.050	.79
radial ion convection	.005	.02
axial loss	.945	.19
radial electron conduction	6×10^{-4}	.04
radial electron convection	.01	.23
radiation	10^{-5}	5×10^{-5}
ion drag	.19	.39
axial loss	.80	.34
axial particle loss	.98	.47
radial particle loss	.02	.53

Reference

1. L. L. Lao, University of Wisconsin, Ph.D. thesis proposal (1978).

2. W. A. Houlberg, R.W. Conn, Nucl Science Eng., 64 (1977), 141.

3. R. S. Post, J. Kesner, J. Scharer, R. W. Conn, Bull. Am. Phys. Soc., 22, 1094 (1977).

4. K. C. Shaing, private communication.

5. K. C. Shaing, R. W. Conn, J. Kesner, Bull. Am. Phys. Soc., 23, 882 (1978).

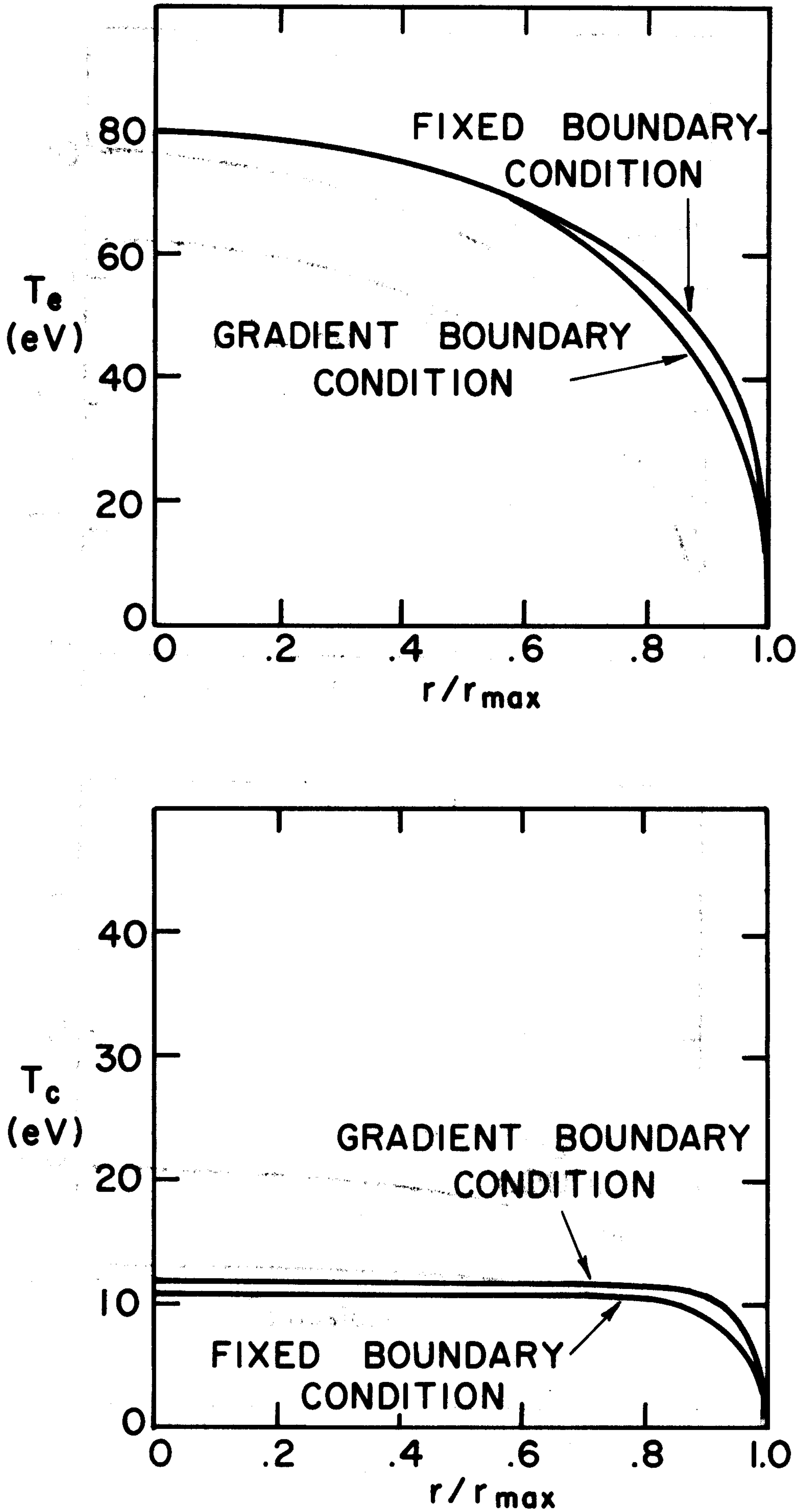
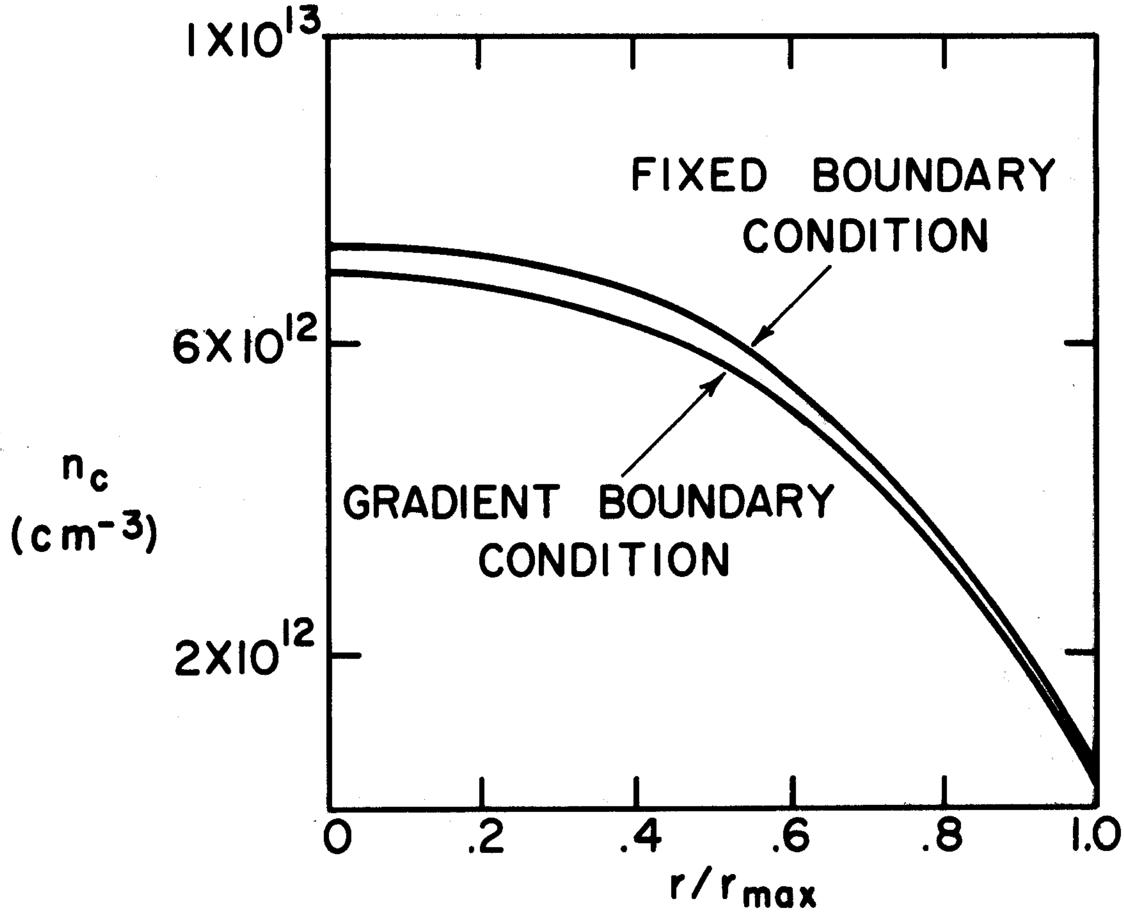


Fig.1a Equilibrium profiles for PHAEDRUS with 10 times classical transport coefficients



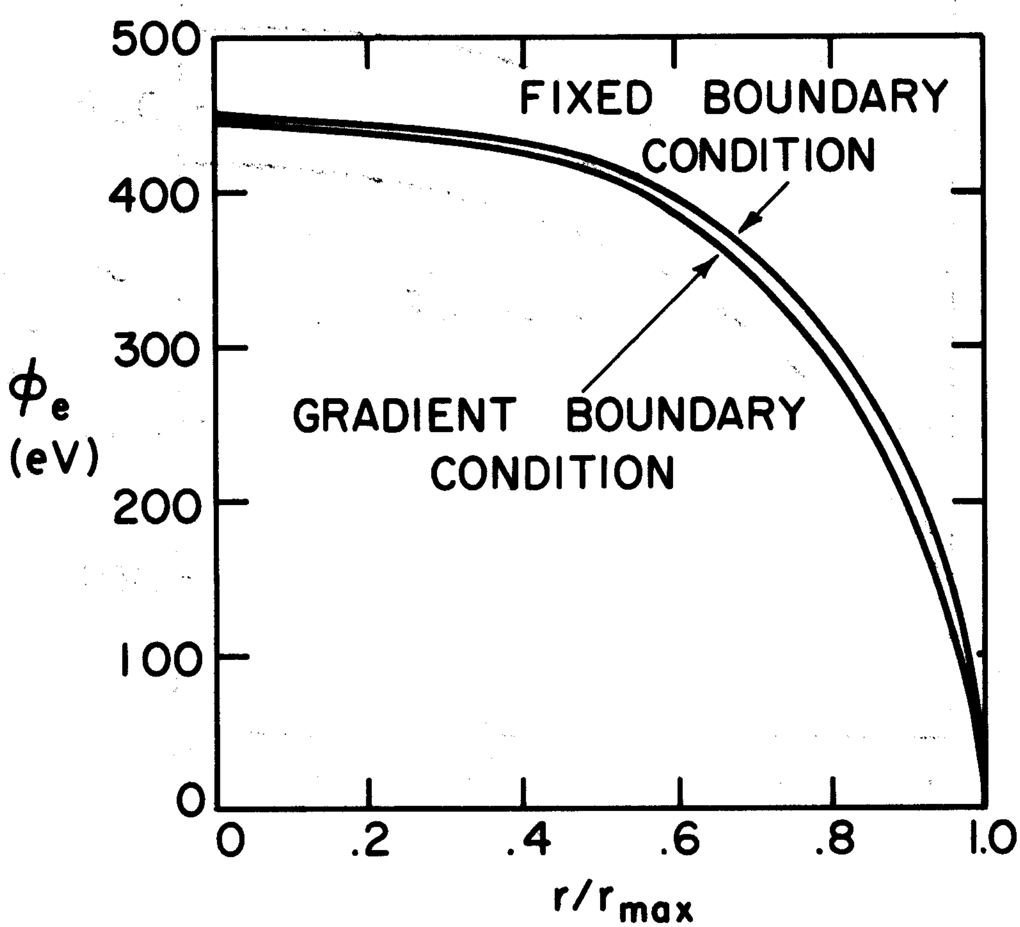


Fig. 1b Equilibrium profiles for PHAEDRUS with 10 times classical transport coefficients

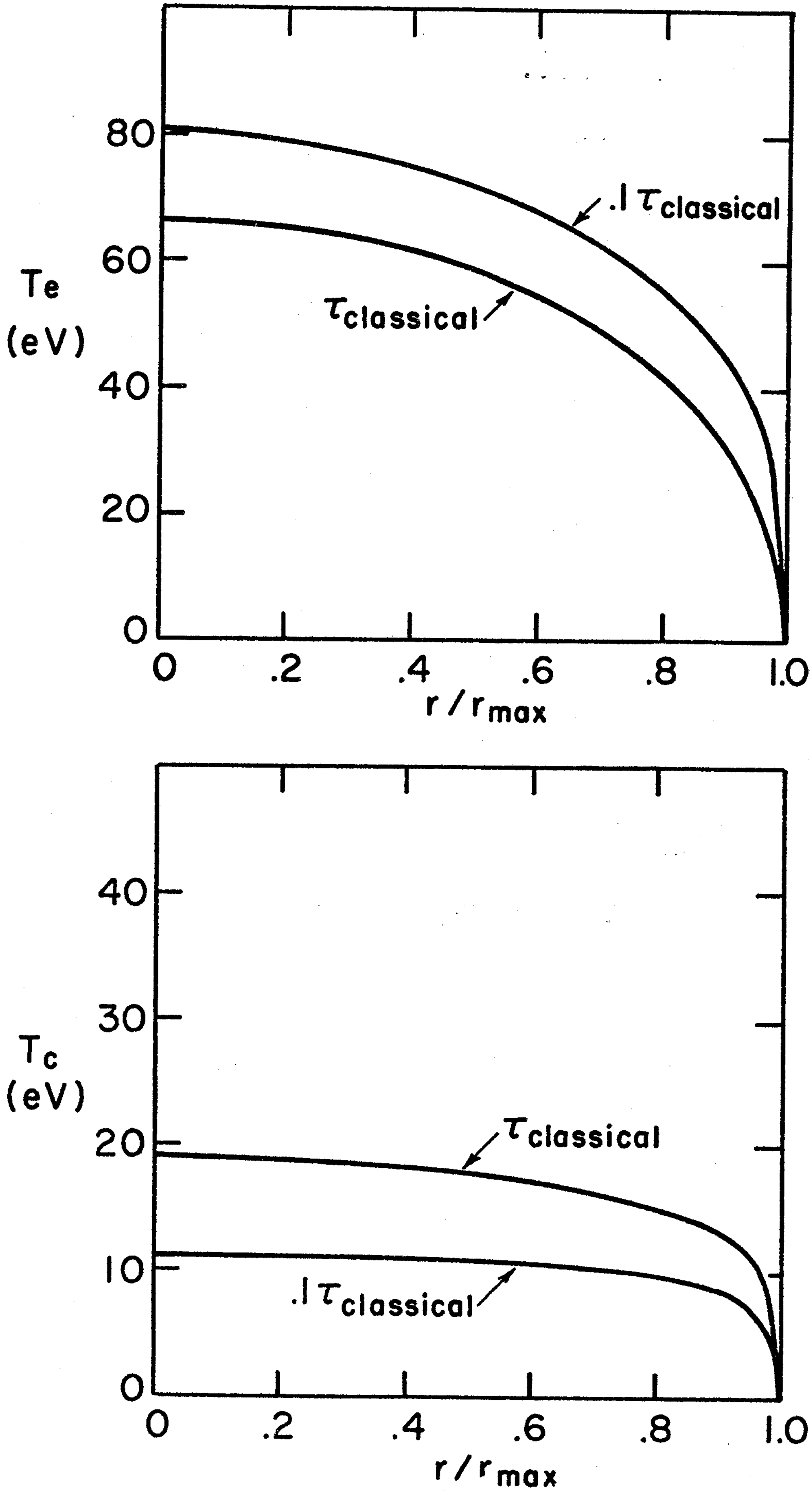


Fig.2a Equilibrium profiles for PHAEDRUS, α = 1., β = 0.1, fixed boundary conditions

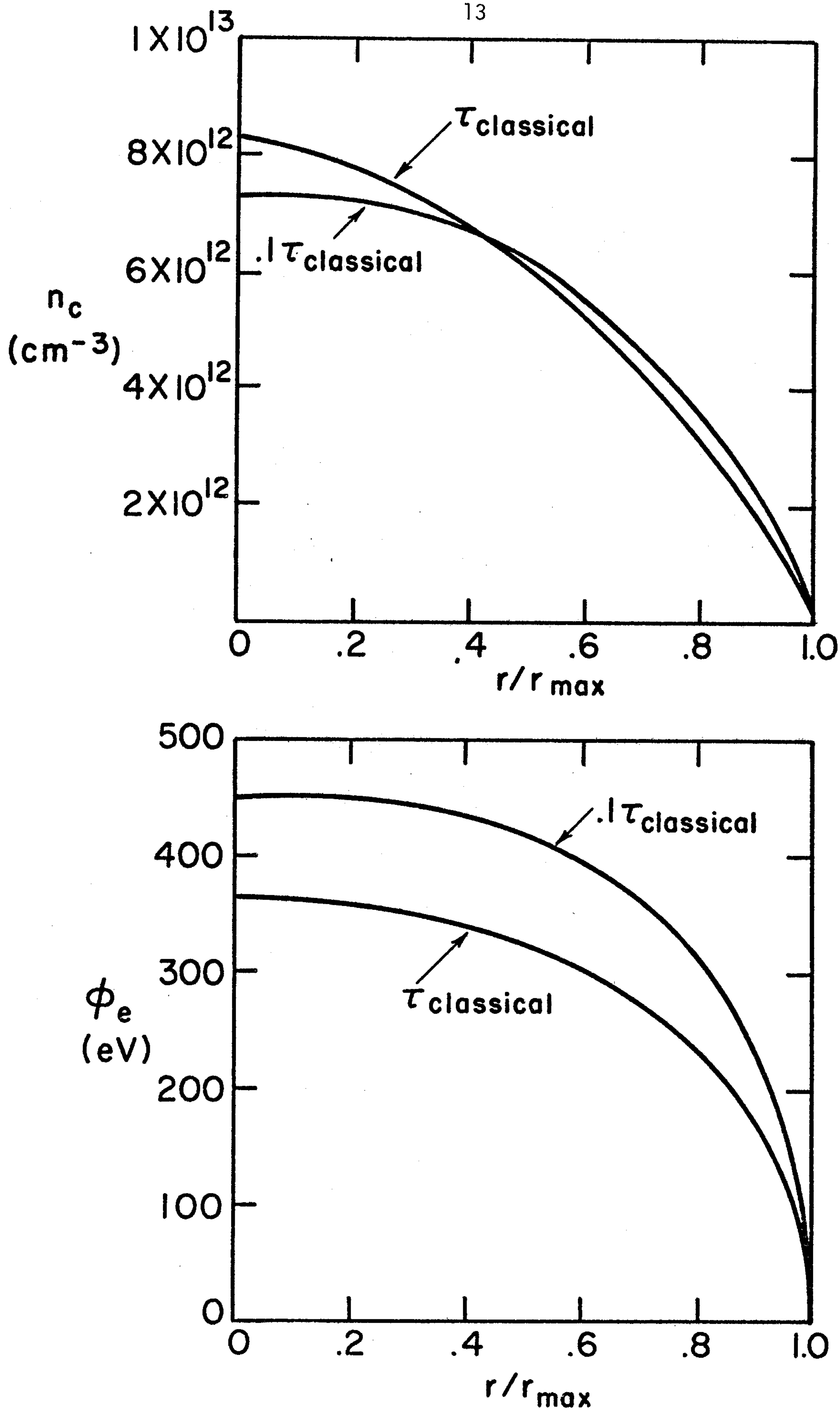


Fig.2b Equilibrium profiles for PHAEDRUS, α=1., β= 0.1, fixed boundary conditions

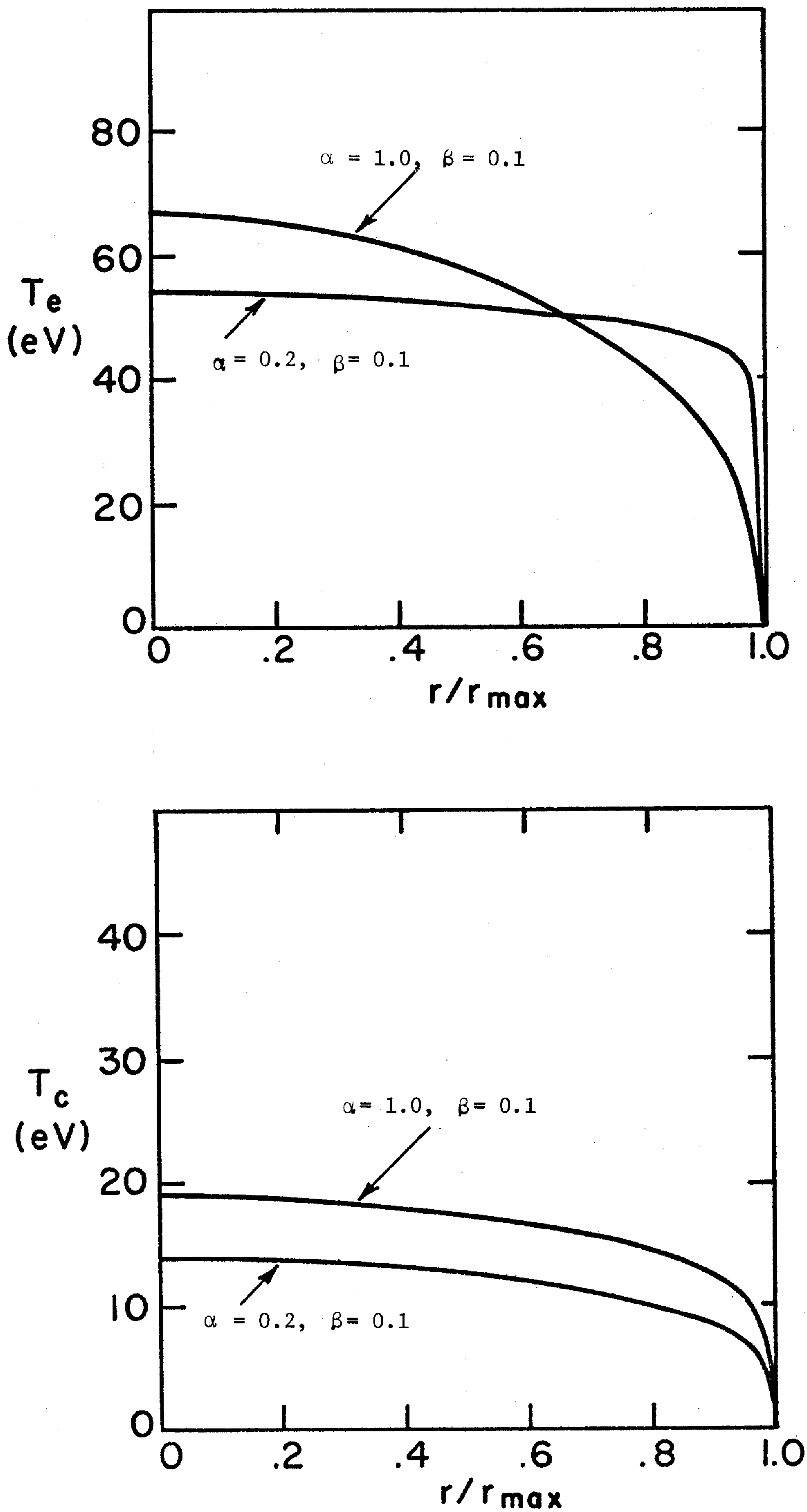
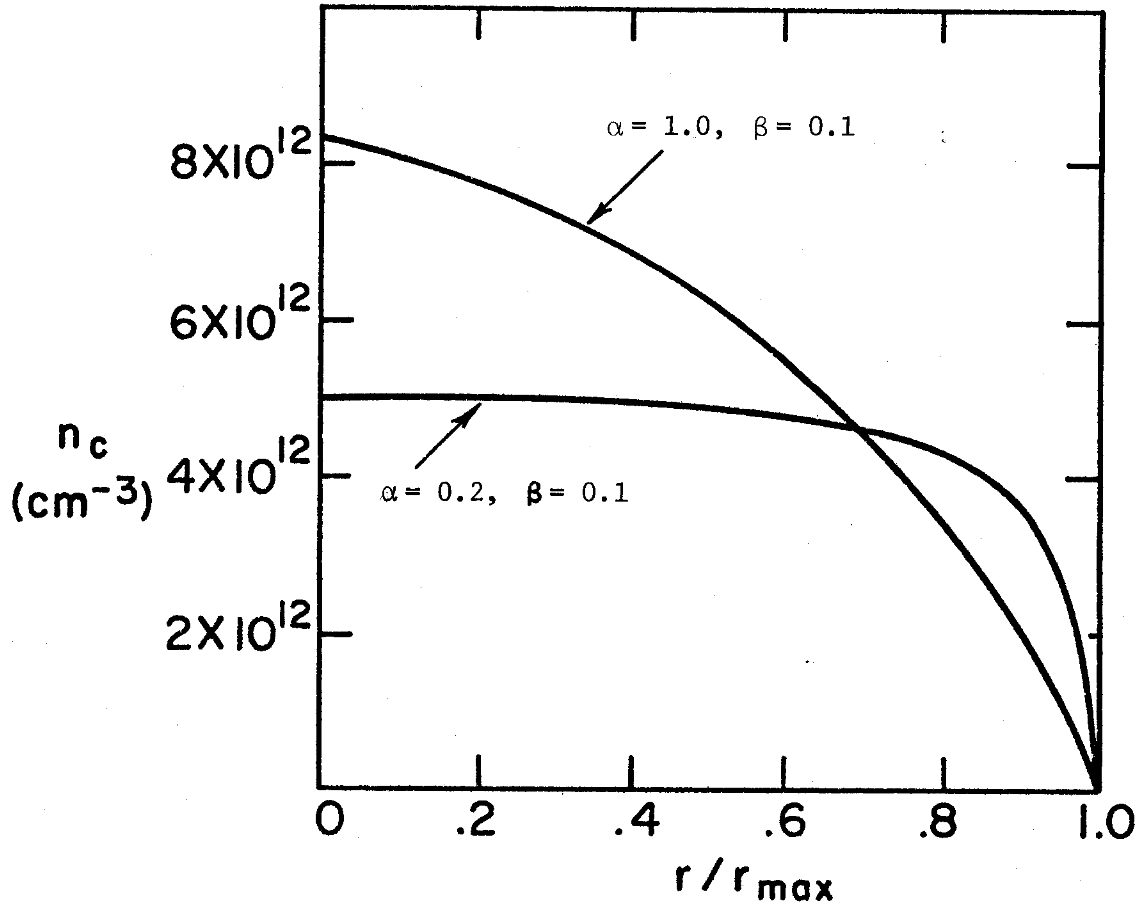


Fig. 3a Equilibrium profiles for PHAEDRUS, classical transport, fixed boundary conditions.



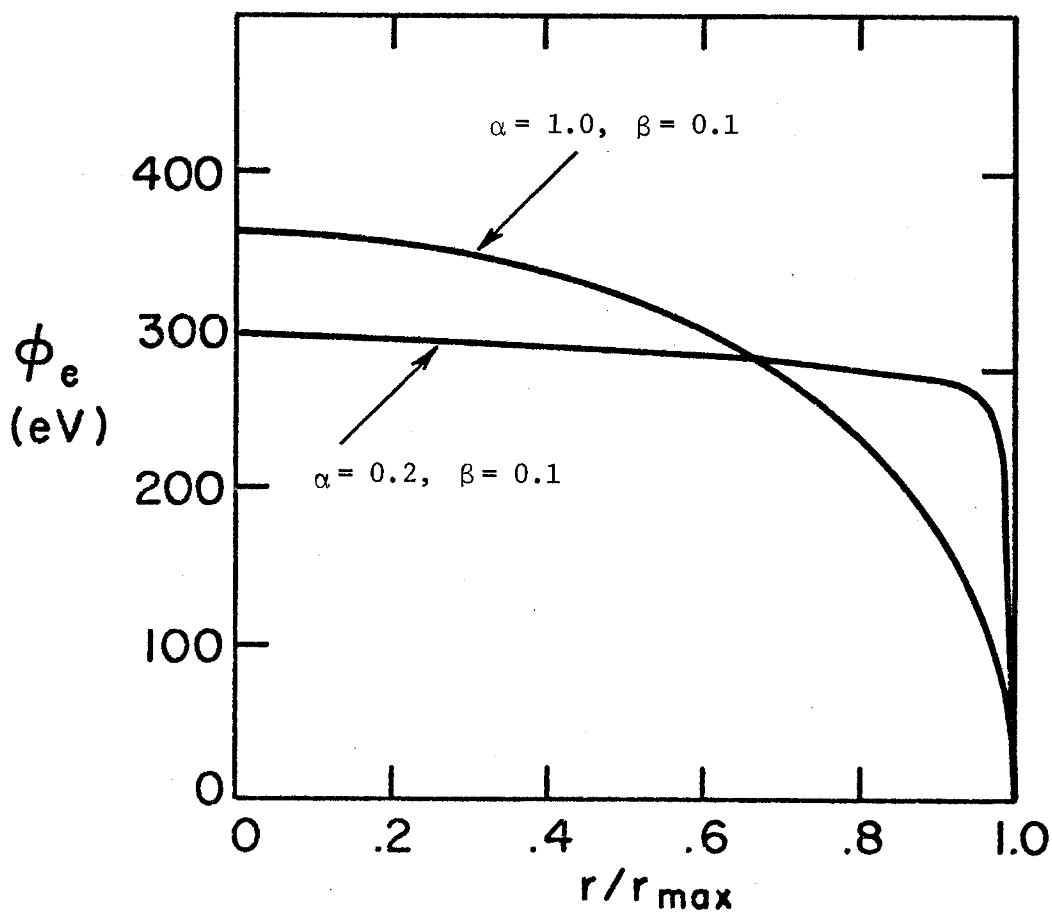


Fig. 3b. Equilibrium profiles for PHAEDRUS, classical transport, fixed boundary conditions.