Analytical Approximation to the Fusion Cross Section

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I. Introduction

In the numerical work previously performed at Wisconsin[1] on the steady state operation of a Tokamak like CTR system, ion temperature solutions, T_i, were restricted to a range of $7\ \mathrm{KeV} \leq T_i \leq 30\ \mathrm{KeV}$. This limitation resulted from the use of a linear approximation to the fusion cross section, $\langle\sigma v\rangle$, of the form,

$$\langle\sigma v\rangle = \gamma_1 + \gamma_2 T_i$$

In this report, analytical expressions are given which approximate the fusion cross section in the ion temperature range between 10 KeV and 100 KeV.

II. Numerical Approach

The deuterium-tritium fusion cross section data given by Rose[2] was used in the calculation. Using the 17 discrete values provided by Rose, the least squares program on the Datacraft 6024/3 was used to determine the best polynomial fit to the data. Polynomials of degree one through four were examined.

III. Results

Suitable accuracy was obtained by using two fourth order polynomials. The first,

$$\langle\sigma v\rangle = (\alpha_1 + \alpha_2 T_i + \alpha_3 T_i^2 + \alpha_4 T_i^3 + \alpha_5 T_i^4) \times 10^{-22}$$

where

$$\alpha_1 = 2.86718$$
$$\alpha_2 = -8.77608\times10^{-1}$$
$$\alpha_3 = 1.02399\times10^{-1}$$
$$\alpha_4 = -3.74327\times10^{-3}$$
$$\alpha_5 = 4.93374\times10^{-5}$$

T_i in KeV

$\langle\sigma v\rangle$ in $M^3 sec^{-1}$

applies in the range 10 KeV $\leq T_i \leq$ 25 KeV. The second,

$$\langle\sigma v\rangle = (\gamma_1 + \gamma_2 T_i + \gamma_3 T_i^2 + \gamma_4 T_i^3 + \gamma_5 T_i^4) \times 10^{-22}$$

where

$$\gamma_1 = -5.60777$$
$$\gamma_2 = 7.28349\times10^{-1}$$
$$\gamma_3 = -1.39106\times10^{-2}$$
$$\gamma_4 = 1.25966\times10^{-4}$$
$$\gamma_5 = -4.53356\times10^{-7}$$

T_i in KeV

$\langle\sigma v\rangle$ in M^3 sec^{-1}

applies in the range 25 KeV $\leq T_i \leq$ 100 KeV. The two polynomials give results which compare to those of Rose within 1%. For comparison, the data of Rose and the analytical approximations are shown on Figure I.

If a single polynomial fit is desired, an accuracy of 5% is attained using a sixth degree polynomial of the form,

$$\langle\sigma v\rangle = (\lambda_1 + \lambda_2 T_i + \lambda_3 T_i^2 + \lambda_4 T_i^3 + \lambda_5 T_i^4 + \lambda_6 T_i^5 + \lambda_7 T_i^6) \times 10^{-22}$$

where

$$\lambda_1 = -1.03036$$
$$\lambda_2 = 3.48484 \times 10^{-2}$$
$$\lambda_3 = 2.52089 \times 10^{-2}$$
$$\lambda_4 = -9.47712 \times 10^{-4}$$
$$\lambda_5 = 1.49257 \times 10^{-5}$$
$$\lambda_6 = -1.10368 \times 10^{-7}$$
$$\lambda_7 = 3.13088 \times 10^{-10}$$

T_i in KeV

$\langle\sigma v\rangle$ in M^3 sec^{-1}

This polynomial applies in the full range 10 KeV $\leq T_i \leq$ 100 KeV.

References

1. R. Conn, D.G. McAlees and G.A. Emmert, "Self Consistent Energy Balance Studies for CTR Tokamaks", Fusion Design Memo 19, University of Wisconsin (July 1972).

2. D.J. Rose, Nucl. Fusion 9, 183 (1969).

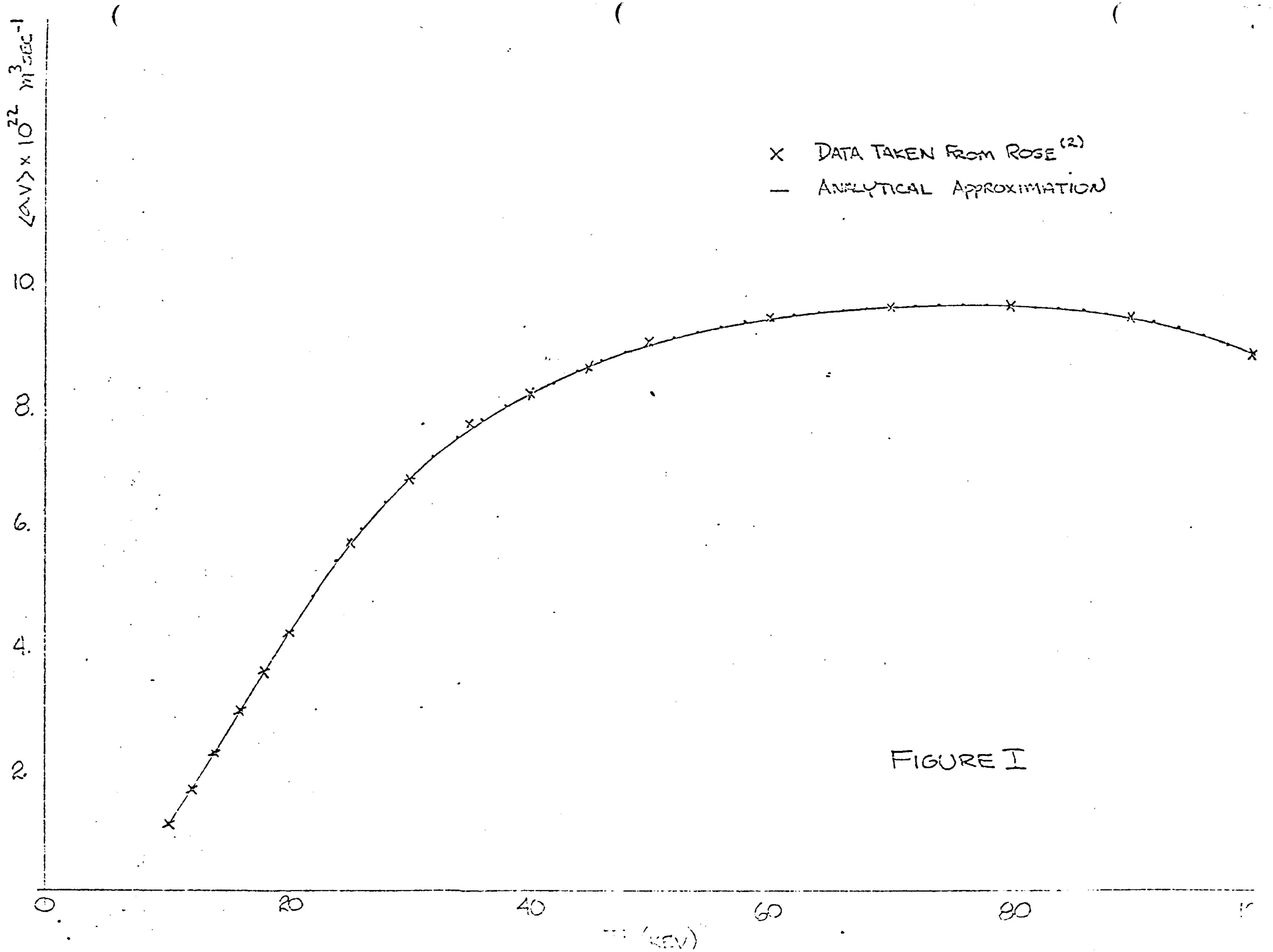
× DATA TAKEN FROM ROSE (2)
— ANALYTICAL APPROXIMATION
⟨σv⟩ × 10^22 m^3 sec^-1
10.
8.
6.
4.
2.
0
20
40
60
80
(KEV)

FIGURE I