



**POICOU: A Point Plasma Code Including Profile
Effects for Tokamak Power Reactors**

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I. Introduction

Although a point plasma model contains much less information than a fluid description, it is more convenient to use in carrying out preliminary fusion power reactor conceptual designs. Point computer codes require less computer time and are easier to manipulate, thus they can be used to survey over a large range of possible reactor operating parameters. The plasma density and temperature profiles assumed in getting the point kinetic equations can have an impact on the solutions obtained from solving those equations.⁽¹⁾

Physically, this is due to the fact that plasma energy balance depends on how the participating particles are distributed in configuration space as well as in velocity space. We report here a time independent point computer code for tokamak power reactors which includes effects of different particle density and temperature profiles. It was developed from a numerical model in our previous work.⁽²⁾ As an illustration, we apply it to study the gas blanket power reactor⁽⁵⁾ by assuming it has a flat density profile and a bell-shaped temperature distribution, and will see that while the average plasma density and temperature may remain the same compared with the one having both bell-shaped profiles, the thermal power output can drop by a factor of as high as 25%.

II. Numerical Model

The plasma density and temperature are assumed to vary radially according to

$$\begin{aligned} n(r) &= n_0 \left(1 - \frac{r^2}{a^2}\right)^x \\ T(r) &= T_0 \left(1 - \frac{r^2}{a^2}\right)^y \end{aligned} \quad (1)$$

the average plasma density and temperature are defined as

$$\begin{aligned} \bar{n} &= \frac{2}{a^2} \int_0^a n(r) r \, dr \\ \bar{T} &= \frac{\int_0^a n(r) T(r) r \, dr}{\int_0^a n(r) r \, dr} \end{aligned}$$

The point energy balance equations are then obtained from plasma energy transport equations as follows:

(1) For the conduction and convection terms

$$\frac{1}{r} \frac{\partial}{\partial r} (r n(r) \chi \frac{\partial T(r)}{\partial r}) \rightarrow \frac{3}{2} \frac{\bar{n} \bar{T}}{\tau_c}$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r D \frac{\partial n(r)}{\partial r} \frac{3}{2} T(r)) \rightarrow \frac{3}{2} \frac{\bar{n} \bar{T}}{\tau_p}$$

where $\tau_c = \text{energy conduction time} = \frac{L_c^2}{4\chi(\bar{n}, \bar{T})}$ (2)

$$\tau_p = \text{particle confinement time} = \frac{L_p^2}{4D(\bar{n}, \bar{T})}$$

In these expressions, L_j is the characteristic length for the particular physical process j ; for parabolic radial dependence it is taken to be the plasma minor radius a , and for non-parabolic profiles, since

$$\left\langle \frac{n}{dn} \right\rangle \propto \frac{a}{x}, \quad \left\langle \frac{T}{dT} \right\rangle \propto \frac{a}{y}$$

L_p and L_c are taken to be $\frac{a}{x}$ and $\frac{a}{y}$. If the profile is flat, L is simply assumed to be some large distance. $X(\bar{n}, \bar{T})$ and $D(\bar{n}, \bar{T})$ are plasma thermal diffusivity and diffusion coefficient and are assumed to follow trapped particle scalings. (3)

(2) The equilibration, radiation and fusion terms are evaluated by integrating over the plasma volume with the assumed profiles.

(3) Alpha particles from fusion products are divided into two energy groups: fast and thermal with averaged energies given by (see ref. (4))

$$\bar{E}_{\alpha f} = \frac{\frac{3}{2} E_{\alpha} F_{\alpha e}}{\ln\left[1 + \left(\frac{E_{\alpha}}{E_{\text{crit}}}\right)^2\right]}$$

$$\bar{E}_{\alpha t} = \frac{3}{2} \bar{T}_i$$

where

E_{α} : alpha particle energy from D-T fusion = 3.52 MeV

$F_{\alpha e}$: fraction of alpha energy given to electrons

$$F_{\alpha e} = 1 - \frac{1}{3} \frac{E_{\text{crit}}}{E_{\alpha}} \left\{ \ln \left[\frac{E_{\text{crit}} - E_{\text{crit}}^{1/2} E_{\alpha}^{1/2} + E_{\alpha}}{E_{\text{crit}} + 2E_{\text{crit}}^{1/2} E_{\alpha}^{1/2} + E_{\alpha}} \right] + 2\sqrt{3} \tan^{-1} \left[\frac{2E_{\alpha}^{1/2} - E_{\text{crit}}^{1/2}}{\sqrt{3} E_{\text{crit}}^{1/2}} + \frac{\sqrt{3}}{3} \pi \right] \right\} \quad (3)$$

$$E_{\text{crit}} = 14.8 A_{\alpha} \bar{T}_e \left(\frac{1}{\bar{n}_e \ln \Lambda_e} \sum_j \frac{\bar{n}_j Z_j^2 \ln \Lambda_j}{A_j} \right)^{2/3}$$

\sum_j is over the plasma ion species, and

A_j, Z_j are atomic mass and atomic number for the j species respectively.

Alpha particle density in each group is obtained by solving the steady state coupled equations:

$$\frac{d\bar{n}_{\alpha f}}{dt} = S_{\alpha} - \frac{\bar{n}_{\alpha f}}{\tau_{\alpha SD}} - \frac{\bar{n}_{\alpha f}}{\tau_{\alpha fP}}$$

$$\frac{d\bar{n}_{\alpha T}}{dt} = \frac{\bar{n}_{\alpha f}}{\tau_{\alpha SD}} - \frac{\bar{n}_{\alpha T}}{\tau_{\alpha TP}}$$

where S_{α} : fast alpha source from fusion

$\tau_{\alpha SD}$: fast alpha particle thermalization time

$$\tau_{\alpha SD} = \frac{6.67 \times 10^{12} \bar{T}_e^{3/2}}{\bar{n}_e \ln \Lambda_e} \ln \left(1 + \left(\frac{E_{\alpha}}{E_{\text{crit}}} \right)^{3/2} \right) \quad (\text{sec})$$

\bar{n}_e is in cm^{-3} , \bar{T}_e is in KeV.

$\tau_{\alpha fP}, \tau_{\alpha TP}$: fast and thermal alpha particle confinement time; for simplicity they are taken to be equal to each other and equal to

$$\tau_{\alpha fP} = \tau_{\alpha TP} = C \tau_{pi}$$

τ_{pi} : plasma ion particle confinement time (see equation (2))

C : correction factor due to ion density and temperature profile effects

$$C = \begin{cases} 1 & \text{if } x \geq y \\ \frac{x^2}{y^2} & \text{if } x < y \end{cases} \quad (\text{see equation (1)})$$

(4) If the reactor is operated in the driven mode, the injected beam particles are treated similarly as alphas by dividing them into fast and thermal groups.

We then obtain the ion and electron energy balance equations as follows:

$$\begin{aligned} \frac{d}{dt} \left(\frac{3}{2} (\bar{n}_i + \bar{n}_{\alpha t}) \bar{T}_i \right) &= \frac{P_{inj} F_b F_{bi}}{V} + \frac{P_{inj} F_{tct}}{E_b V} \chi_{\alpha} E_{\alpha} F_{\alpha i} \\ &+ F_{tp} (1 - F_{tp}) \overline{n_i^2 <6V> \frac{d}{dt} \chi_{\alpha} E_{\alpha} F_{\alpha i}} - \frac{3}{2} \bar{n}_i \bar{T}_i \left(\frac{1}{\tau_{pi}} + \frac{1}{\tau_{ci}} \right) \\ &- \frac{3}{2} \bar{n}_{\alpha t} \bar{T}_i \left(\frac{1}{\tau_{P\alpha}} + \frac{1}{\tau_{C\alpha}} \right) - 4.38 \times 10^{-28} C_1 \frac{(\bar{T}_i - \bar{T}_e) \bar{n}_i \bar{n}_e}{A_i \bar{T}_e^{3/2}} \\ &+ \frac{3}{2} \frac{\bar{n}_{\alpha f} \bar{T}_i}{\tau_{\alpha SD}} \quad (\text{watt/cm}^3) \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{d}{dt} \left[\frac{3}{2} \bar{n}_e \bar{T}_e \right] &= \frac{P_{inj} F_b F_{be}}{V} + \frac{P_{inj} F_{tct}}{E_b V} \chi_{\alpha} E_{\alpha} F_{\alpha e} \\ &+ F_{tp} (1 - F_{tp}) \overline{n_i^2 <6V> \frac{d}{dt} \chi_{\alpha} E_{\alpha} F_{\alpha e}} - \frac{3}{2} \bar{n}_e \bar{T}_e \left(\frac{1}{\tau_{pe}} + \frac{1}{\tau_{ce}} \right) \\ &+ 4.38 \times 10^{-28} C_1 \frac{(\bar{T}_i - \bar{T}_e) \bar{n}_i \bar{n}_e}{A_i \bar{T}_e^{3/2}} - 4.8 \times 10^{-31} C_2 \bar{n}_e^2 \bar{T}_e^{1/2} Z_{eff} \\ &- 3.20 \times 10^{-13} C_3 (1 - .0305 A)^3 \left(\frac{\bar{n}_e}{a} \right)^{.5} B_t^{2.5} \bar{T}_e^{2.1} \quad (5) \\ &- P_{zr} \quad (\text{watt/cm}^3) \end{aligned}$$

where

P_{inj} : injected beam power in watts

E_b : beam energy in KeV

$$F_b = 1 - \frac{T_i}{E_b}$$

$$E_\alpha = 3.52 \times 10^3 \text{ KeV}$$

F_{bi}, F_{be} : fraction of beam energy given to ions and electrons while slowing down (see equation (3))

V : plasma volume in cm^3

F_{tct} : fraction of beam particles undergoing fusion while slowing down

χ_α : fraction of born fast alphas which are retained in the plasma

F_{tp} : tritium fraction in plasma

$F_{\alpha i}, F_{\alpha e}$: fraction of alpha energy given to ions and electrons (see equation (3))

$\overline{n_i^2}_{<6V>dt}$: profile average of n_i^2 and $<6V>dt$

For 7. KeV $< T < 30$. KeV

$$<6V>dt = (-2.06 + .314T) \times 10^{-16} \frac{\text{cm}^3}{\text{sec}}$$

$$\overline{n_i^2}_{<6V>dt} = (-2.06 U_1 + .314 U_2 \bar{T}_i) \bar{n}_i^2 \times 10^{-16}$$

U_1, U_2 : correction factors due to profile effects

A_i : atomic mass of ions

A : aspect ratio

B_t : on axis toroidal magnetic field in Tesla

a : plasma minor radius in meter

C_1, C_2, C_3 : correction due to profile effects

\bar{n} , \bar{T} : average plasma density and temperature in cm^{-3} and KeV respectively

τ_p , τ_c : particle confinement time and energy conduction time in sec (see equation (2))

$$Z_{\text{eff}} = \frac{\bar{n}_i}{\bar{n}_e} + \frac{\bar{n}_z}{\bar{n}_e} Z^2$$

\bar{n}_z , Z : impurity density and atomic number

P_{zr} : impurity line and recombination radiation loss term

$$P_{zr} = 1.82 \times 10^{-32} C_4 Z^4 \bar{n}_z \bar{n}_e \bar{T}_e^{-1/2} + 4.13 \times 10^{-34} C_5 Z^6 \bar{n}_z \bar{n}_e \bar{T}_e^{-3/2}$$

At low temperature, it is replaced by Hinnov's formula

$$P_{zr} = 1.2 \times 10^{-26} C_6 \bar{n}_z \bar{n}_e$$

C_4 , C_5 , C_6 : profile correction factors

Ion and electron density are then obtained by fixing poloidal β_p and assuming plasma is quasi-neutral.

$$\beta_p = 1.602 \times 10^{-10} \frac{(\frac{2}{3} \bar{n}_{bf} \bar{E}_{bf} + \frac{2}{3} \bar{n}_{\alpha f} \bar{E}_{\alpha f} + \bar{n}_{\alpha t} \bar{T}_i + \bar{n}_i \bar{T}_i + \bar{n}_e \bar{T}_e)}{\frac{B_p^2}{2\mu_0}} \quad (6)$$

where \bar{n}_{bf} : average beam particle density in fast group

\bar{E}_{bf} : average beam particle energy in fast group

B_p : poloidal magnetic field in Tesla

$$\bar{n}_e = \bar{n}_i + 2\bar{n}_\alpha + Z\bar{n}_z \quad (7)$$

Our steady state point kinetic model is based on these four coupled equations (4-7). They are used to solve for \bar{T}_i , \bar{T}_e , \bar{n}_i , \bar{n}_e for a given input set of parameters using implicit looping newton's method.

III. An Application

In this section we use the previously developed point model to study the gas blanket reactor. In Table 1 we compare the solution obtained from assuming a uniform density distribution with one assuming a parabolic density distribution, both are having same parabolic temperature profiles and identical other input parameters. Since the solution for flat density distribution is not very sensitive to the particle confinement time, τ_p , for simplicity it is taken to be ten times that of the parabolic density distribution case. First, we note that although the machine size is relatively small ($R = 5$ m, $A = 4$) comparing with UWMAK I, II and III,

Table 1: Comparison Between Solutions Obtained for Flat Density Profile and Parabolic Density Profile

	Flat Density Profile	Parabolic Case
P_T	1819 MW	2596 MW
$\bar{n}_e, \frac{\bar{n}_{D+T}}{\bar{n}_e}$	$2.16 \times 10^{14} \text{ cm}^{-3}, .97$	$2.20 \times 10^{14}, .95$
\bar{T}_i, \bar{T}_e	14.2 KeV, 15.6 KeV	13.1 KeV, 15.2 KeV
β_t, β_p	7.7%, 4.0	7.7%, 4.0
$I_p, q(a)$	6.94 MA, 2.40	6.94 MA, 2.40
BT	6.0 T	6.0 T
R, a, S	5 m, 1.25 m, 1.33	5 m, 1.25 m, 1.33

it can be ignited without much difficulty even with trapped-particle transport scaling laws using a moderate on axis toroidal magnetic field of 6 Tesla. Secondly, although the temperature and density in the two cases are roughly equal, the output thermal power for flat density case is dropped by a factor of almost 30%. Comparing with the parabolic case, this is due mostly to the profile effect because $\int_{V01} ni^2 \langle 6v \rangle d\bar{r} \propto c_1 \int_{V01} ni^2 d\bar{r} + c_2 \int_{V01} ni^2 \bar{T}_i d\bar{r}$, $c_1 = -2.06$, $c_2 = .314$, for flat density profile $\langle ni^2 \rangle = \bar{n}i^2$, $\langle ni^2 \bar{T}_i \rangle = \bar{n}i^2 \bar{T}_i$ while for the parabolic density distribution case $\langle ni^2 \rangle = \frac{4}{3} \bar{n}i^2$, $\langle ni^2 \bar{T}_i \rangle = \frac{3}{2} \bar{n}i^2 \bar{T}_i$.

In Figure 1 we plot $\beta_p = \frac{\sum_j \bar{n}_j \bar{T}_j}{\frac{B_p^2}{2\mu_0}}$ as a function of electron density

for equilibrium ion and electron temperature $\bar{T}_i = 14.2$ KeV, $\bar{T}_e = 15.6$ KeV. The lower curve gives the contribution of alpha particles to plasma pressure which is roughly 10% of the total plasma pressure. In Figure 2 we show the ion energy balance varying as function of ion temperature for a fixed ion to electron temperature ratio $\frac{T_i}{T_e} = .91$ corresponding to that at equilibrium. The intersection between the ion energy gain and ion energy loss curves gives the solution; as can be seen from the graph, the equilibrium is thermally stable. In Figure 3 we show electron energy balance varying as function of electron temperature, also for the same equilibrium ion temperature to electron temperature ratio $\frac{T_i}{T_e} = .91$.

There are two intersecting points but only the upper one at $T_e = 15.6$ KeV satisfying both ion and electron energy balance equations corresponds to the real solution, as is shown; it is also thermally stable. In both Figures 2 and 3, the average electron density is $\bar{n}_e = 2.16 \times 10^{14} \text{ cm}^{-3}$. In all three figures, the plasma density profile and temperature profile are flat and parabolic respectively; other parameters are as given in Table 1. Finally, we note that this solution lies closely to the ion temperature range approximately 15 KeV which has optimum power density for a βt limited D-T reactor.

Acknowledgement

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References

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2. J. Kesner, "Space and Time Code," (unpublished); J. Kesner, R. W. Conn, Nuclear Fusion 15 (1975), 775.
3. S. O. Dean, et al., "Status and Objective of Tokamak System for Fusion Research."
4. W. Houlberg, UWFDM-103.
5. F. Engelmann, et al., Comments on Plasma Physics, 3 (77), 63.

Appendix

Structure, Input and Output Format of POICOU

1. Structure

There are eight major routines in POICOU and these are discussed briefly below:

PMAIN: inputing and outputing, solve electron energy balance equation using Newton's method.

PBETAP, PTIEQT, TEQTN: evaluate terms in poloidal beta equation, ion energy equation and electron energy equation.

TIEQTN, BETAEQ: solve ion energy equation and poloidal beta equation by Newton's method.

SCALE: calculate particle confinement time, electron and ion energy conduction time using trapped-particles scalings.

TCT: calculate fraction of injected beam undergoing fusion while slowing down.

2. Input and Output

Inputs to POICOU are free format of the following form:

DBM TAN ITRANP EDGET EDGED FATDTP

KSET MTCT BCONV TCONV

RMAJ RMIN XHIWID BETAP BETAT

BT FTB FTP Z XZ AZ

XALPH EFUS WB PINJ

DENGES TGES TSTEP NSTEP

These terms are explained below:

$$\text{DBM: } n_j = n_{j0} \left(1 - \frac{r^2}{a^2}\right)^{\text{DBM}}$$

$$\text{TAN: } T_j = T_{j0} \left(1 - \frac{r^2}{a^2}\right)^{\text{TAN}}$$

ITRANP: 0 or 1 for using average or edge temperature and density when evaluating transport coefficients.

EDGET, EDGED: no effect if ITRANP = 0, if ITRANP = 1 then evaluate transport coefficients using edge temperature = average temperature x EDGET, edge density = average density x EDGED.

FATDTP: if DBM = 0, $\tau_p = \text{FATDTP} \times \tau_p$ of parabolic.

KSET: number of retrials for driven reactor, for each new trial $P_{inj} = 1.1 \times P_{inj}$ old.

MTCT: 1 or 0, corresponding to whether or not tritium ion fraction is to be readjusted due to deuterium beam injected on cold tritium target. This is for driven reactors.

BCONV, TCONV: poloidal beta and particle energy equations converging criteria.

RMAJ, RMIN: plasma major and minor radius in m.

XHIWID: plasma height to width ratio (1 corresponding to circular plasma).

BETAP, BETAT: poloidal and toroidal beta.

BT: on axis toroidal magnetic field in Tesla.

FTB, FTP: fraction of tritium in beam and plasma.

Z, XZ, AZ: impurity atomic number, fraction and atomic weight.

XALPH: fraction of born alphas contained within the plasma chamber.

EFUS: energy released per fusion event in MEV.

WB, PINJ: injected beam particle energy in KEV and beam power in watts, respectively.

DENGES, TGES: initial particle density and temperature guess in cm^{-3} and KEV, respectively.

TSTEP, NSTEP: if the initial guess is bad, the code will retry NSTEP times with each new initial temperature guess = old temperature guess + TSTEP (KEV).

A typical input and output stream of POICOU is shown at the end. POICOU first summarized the input data, then gives various parameters corresponding to the found solution. Some of the major terms are explained below:

TAUSDA, TAUPA: alpha slowing down time and confinement time.

TAVENG: plasma energy confinement time = $\frac{\sum_j \bar{n}_j \bar{T}_j}{P_{inj} + P_\alpha}$.

TAUP: particle confinement time.

TAUXE, TAUXI: electron and ion energy conduction time.

MODE: 1 pseudo-classical scaling.

2,3 high and low collisionality trapped-electrons scaling.

4 trapped-ions scaling.

FRTBRN: fractional burn-up of plasma.

PALPMW: alpha particles power.

```

0. 1. 0. 1. 1. 1. 1. 1. 0.
0. 0. 1.E-3 1.E-3
5. 1.25 1.6 4. .077
6. .5 .5 26. 0. 55.
1. 20. 0. 0.
1.E14 30. 2. 1

```

```

DENSITY=DENMAX*(1.-(R/RMIN)**2)**DBM, DBM= .0000
TJ =TJMAX *(1.-(R/RMIN)**2)**XTAN, TAN= 1.0000
NTCT = 0
EDGED = 1.0000
RMIN(M) = 1.2500
S = 1.3242
A = 4.0000
BETAT = .0770
CUR(MA) = 6.9416
PINJ(W) = .0000
Z = 26.0000

```

***** THIS IS AN IGNITION MACHINE *****

FUSION ENERGY

```

KEV
17600.000
20000.000
YI(KEV) = 14.1762
BETAP = 4.0007
TAUSUA(S) = .2632
TAUP(S) = 13.1018
NI/NE = .9683
FTF = .5000
PALFMW(MW) = 363.8558
TERMS OF ION ENERGY EQUATION IN WATT/CM3:
BEAM INJ = .0000
THERMAL = -.6064
THERMA BIM = .0000
TERMS OF ELECTRON ENERGY EQUATION IN WATT/CM3:
BIM INJ = .0000
THERMAL = -.6783
SYNC RAD = -.1105
Z LINE RAD = .0000
TERMS OF BETAP EQUATION:
BEAM ALPHA = .0000
PASMA ALPHA = .3248

```

NEUTRON WALL LOADING

NW/M2

```

4.025
4.025
YE(KEV) = 15.5643
BETAT = .0770
TAUENG(S) = 1.0310
TAUXE(S) = 1.3102
XALPHA = 1.0000
FTB = .5000
TAUFA(S) = 1.3102
BIM FUSION = .0000
ELN-ION = .2529
TISUM = .0000
BIM FUSION = .0000
ION-ELN = -.2529
Z RECB = .0000
TESUM = .0009
FAST BEAM = .0000

```

THERMAL POWER

MW

```

1819.279
2067.363
NE(CM-3) = .2160+15
NTAU(S-CM-3) = .2572+15
MODE = 4
TAUXI(S) = 1.3102
NA/NE = .0159
FRTRN = .0328
XLOGLM = 21.4395
PASMA FUS = .3461
TERMA ALFA = .0074
PASMA FUS = 1.1285
BREMS = -.0859
Z RAD = .0000
ION-ELN = 3.6760

```

POWER DENSITY

MW/M3

```

7.373
8.379

```

XXXXXXXXXXXXXXXXXXXX

DONE

XXXXXXXXXXXXXXXXXXXX

BYE BYE

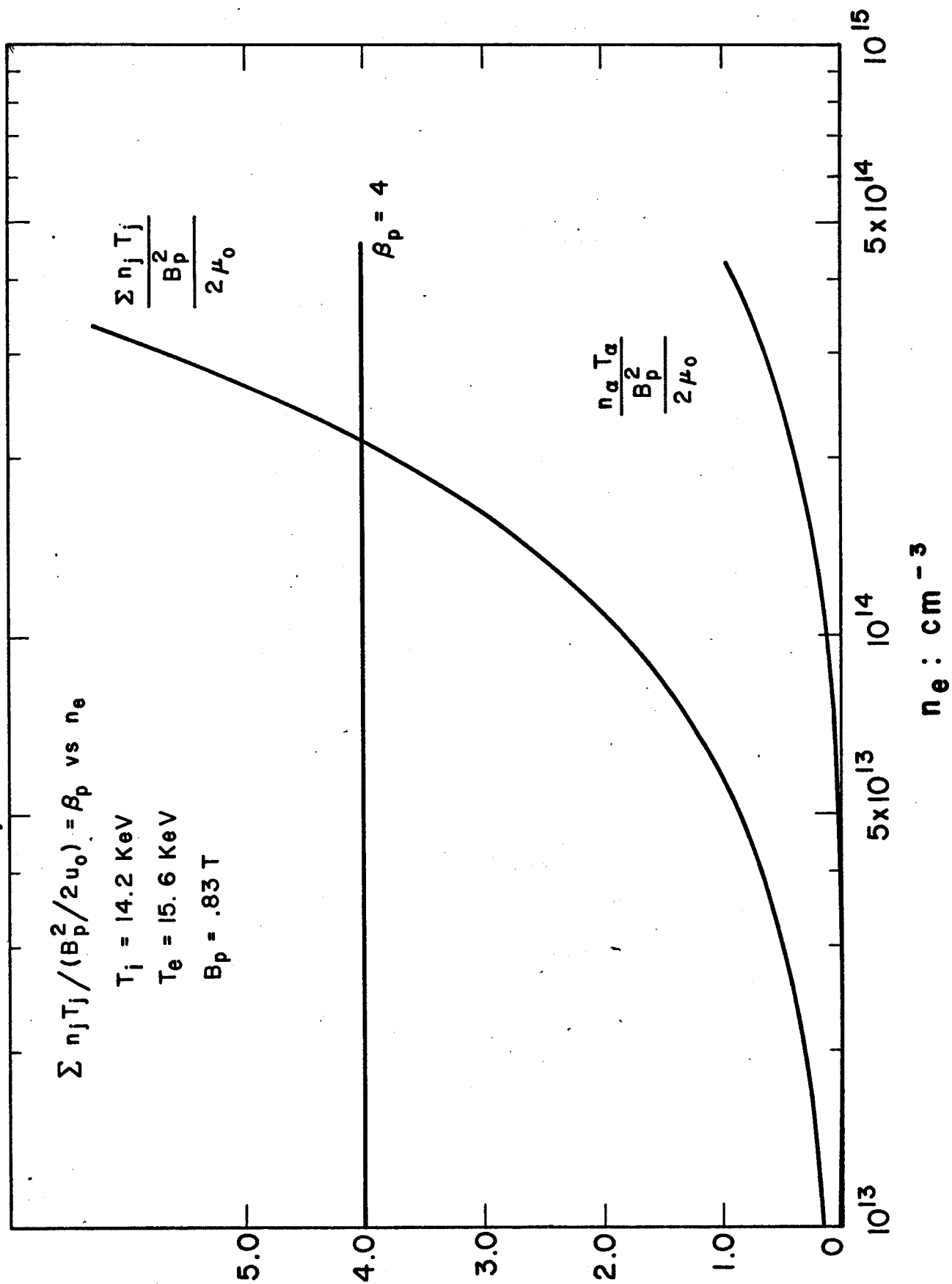


FIG. 1

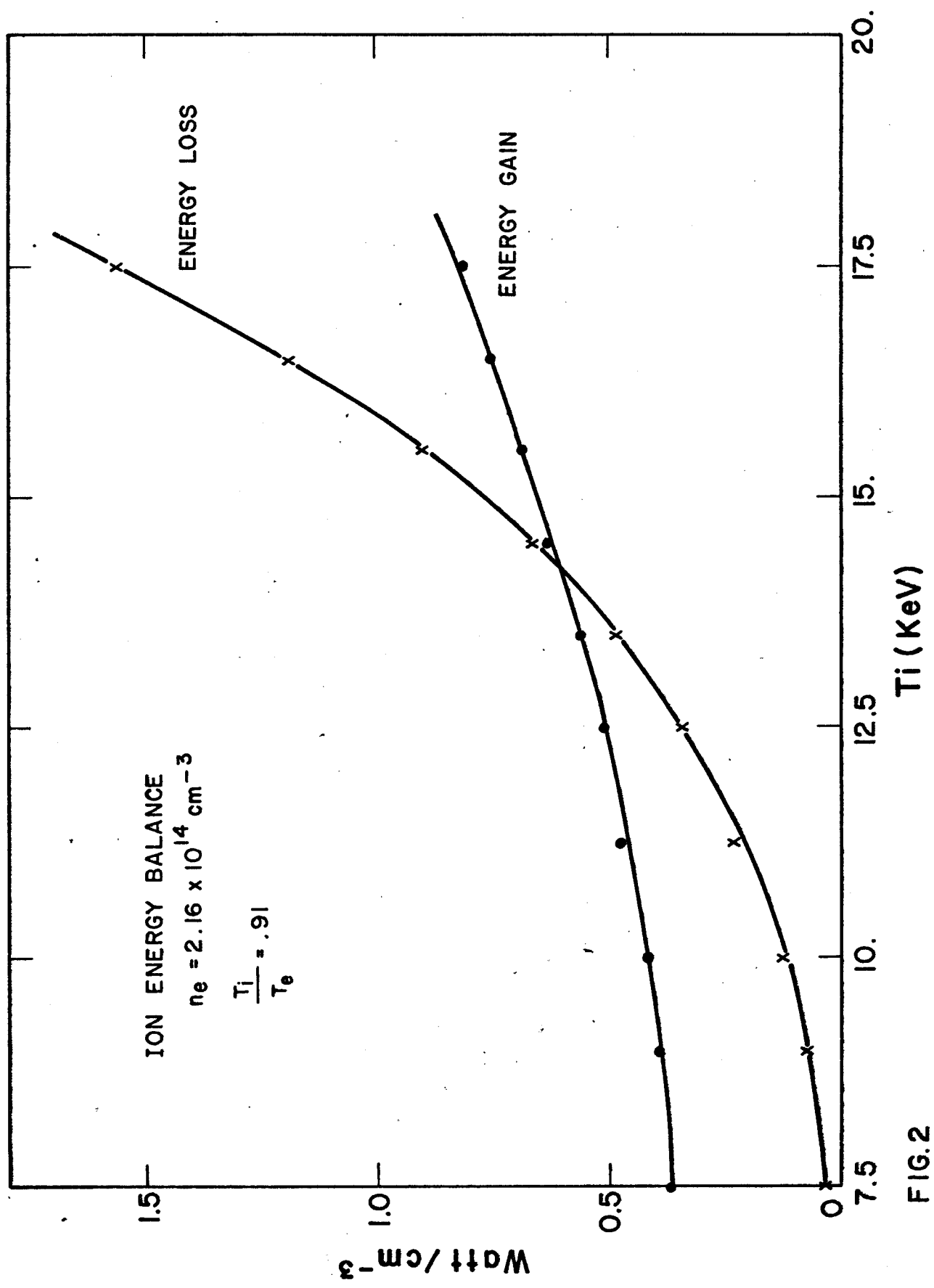


FIG. 2

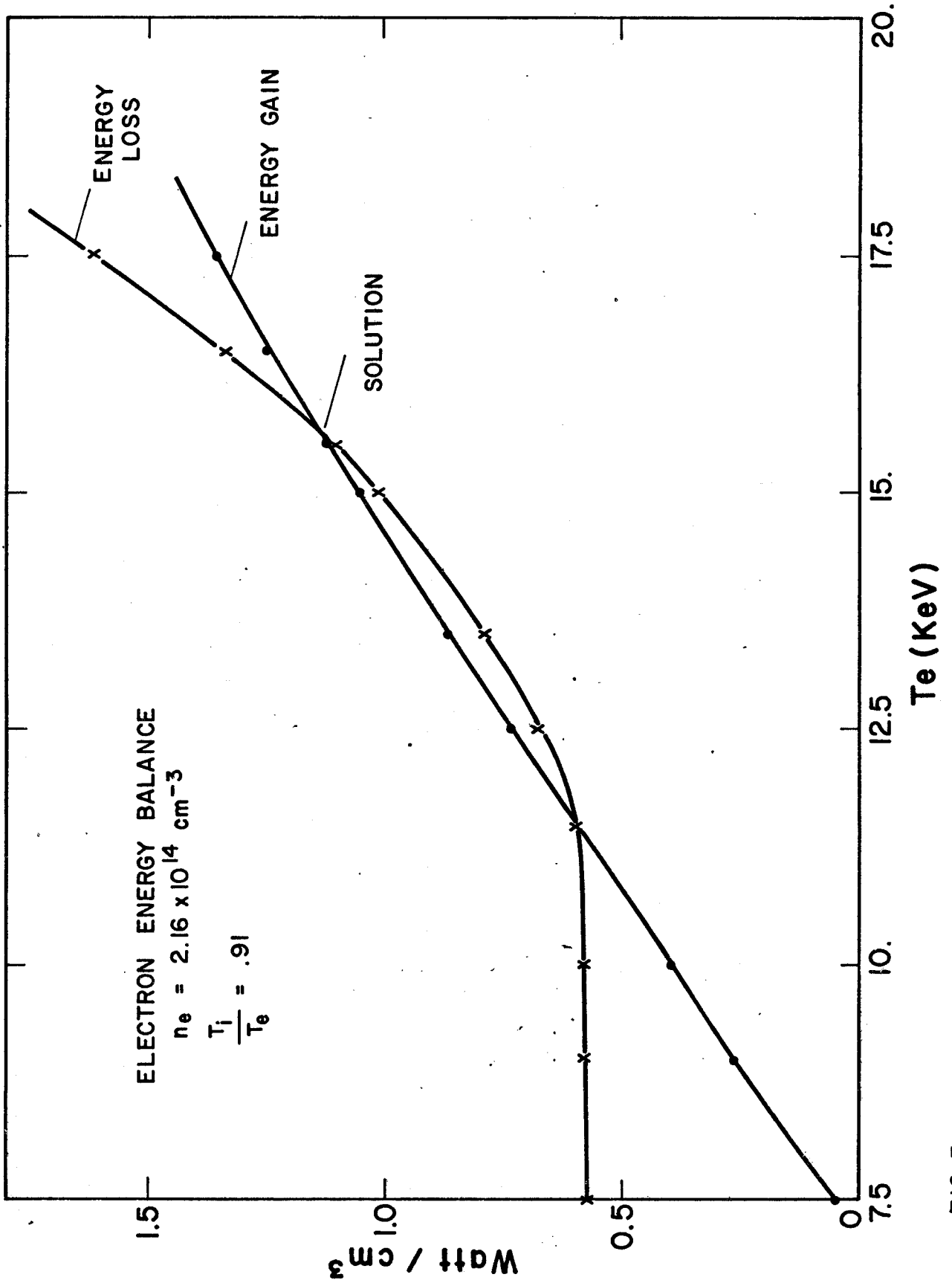


FIG. 3