Synchrotron Radiation Loss in Tokamak

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This report seeks to study and calculate the synchrotron radiation loss from electrons in a tokamak with the following parameters,

R_o (reactor major radius) = 18 m

a (plasma minor radius) = 2 m

N (plasma density) = 10^{20} particles/m^3

B (toroidal magnetic field) = 50 kgauss

T_e (electron temperature) = 10 to 50 keV.

The synchrotron radiation in a tokamak for the non-relativistic limit, i.e. for v << c, has been formulated by Rosenbluth[1] taking into account the ∇B broadening. His result is given as

$$L = \sum_{i=1,2} \sum_{n=1}^{\infty} \frac{T}{N} \frac{(n\Omega)^3}{(2\pi)^3} \frac{8}{R_o} \frac{\pi}{4} \frac{Z_n^i \Lambda_n^i}{1 + Z_n^i \Lambda_n^i} \quad , \tag{1}$$

or

$$L = \frac{2.5\times10^{-16}\ NT^3}{\beta_e(\beta_e R_o B)} \sum_{i=1,2} \sum_{n=1}^{\infty} n^3 \frac{Z_n \Lambda_n^i}{1 + Z_n \Lambda_n^i} \quad , \tag{2}$$

where

$$Z_n \Lambda_n^i = 1.25\times10^{-3} \ (\beta_e R_o B) \ (10^{-3}T)^{n-2} \ f_n^i \quad , \tag{3}$$

with

$$f_n^1 = (ne)^{n-2} \quad , \quad f_n^2 = f_n^1/(2n+1) \quad , \tag{4}$$

and

$$\beta_e = 1.6\times10^{-9} \ \frac{8\pi NT}{B^2} \quad . \tag{5}$$

Here B is in gauss, N in particles per cm^3, R_o in cm and T in keV. n is the harmonic number ω/Ω and Ω is electron cyclotron frequency. The $Z_n\Lambda_n{}^i$ are strongly dependent functions of n and decreasing steeply as n increases. Rosenbluth plotted the quantity $10^{17}(\beta_e R_o B)^{\frac{1}{2}}\ \beta_e L/NT^3$ vs. T for constant $\beta_e BR_o$. Some of his curves were reproduced and shown in Fig. 1.

Since Eq. (1) was derived for a low temperature plasma, in order to study its validity at relatively higher temperature the rapidly decreasing functions $Z_n\Lambda_n{}^i$ must be studied. The ratio of the loss by Black-body radiation at each harmonic will also be calculated to determine the cut-off frequency $\omega^{*1/2}$ or the number of harmonics that contribute to the total loss.

By integrating the Black-body intensity over all $\vec{k}$ values and exit angles using the same technique and coordinates given by Rosenbluth, the Black-body radiation is given as

$$L_B = \frac{1}{\pi r N}\int_o^{2\pi} d\theta_o \int_o^{\infty} \frac{k^2 dk}{(2\pi)^3}\ 8 \int_o^{\pi/2} d\alpha \int_o^{\pi/2} d\phi\ I_B$$

$$= \sum_{i=1,2}\ \sum_{n=1}^{\infty} \frac{T}{N}\ \frac{(n\Omega)^3}{(2\pi)^3}\ \frac{8}{R_o}\ , \qquad (6)$$

where $I_B = T$. The ratio of the synchrotron radiation to Black-body radiation at each harmonic is simply

$$R(n) = \frac{L(n)}{L_B(n)} = \frac{\pi}{4}\ \frac{Z\Lambda_n{}^i}{1 + Z\Lambda_n{}^i}\ . \qquad (7)$$

The functions $Z_n\Lambda_n^{\ i}$ and the ratio R(n) for values of temperature between T = 1 and T = 15 keV have been computed. Those for T =5, 10 and 15 keV are listed in Tables I and II as typical examples.

From Table I one can see that the $Z_n\Lambda_n^{\ i}$ are steeply decreasing functions of n for temperatures below 10 keV. They decrease slowly as functions of n above 10 keV and even begin to rise again at 15 keV after a minimum at n = 11. This indicates that the equation (1) is not valid above 10 keV.

Table II shows that the plasma radiates as Black-body at the first few harmonics. However, the ratio of synchrotron and Black-body radiations at the fundamental frequency and first few harmonics are only 78%, which indicates that Eq. (1) may underestimate the loss at low frequencies.

Synchrotron radiation losses for T from 1 to 9 keV were computed by summing over n from 1 to the value where $Z_n\Lambda_n^{\ i}$ no longer decrease steeply. For temperatures from 10 to 15 and 20 keV the radiation from first 5 harmonics were computed and the total loss for each temperature was doubled which will be explained later. The spectra for temperatures 5, 7, 9 and 10 keV and that of the Black-body radiation were plotted and shown in Fig. 2, where the radiation loss of each harmonic is normalized to $\Omega^3T/4\pi^3NR_o$. One can see from Fig. 2 that n = 5 is about the peak position of the spectrum at 9 keV, and the peak of the spectrum is shifting toward larger n and increasing in magnitude as the temperature increases. The slow decreasing tail of the spectrum for 10 keV again shows that $Z_n \Lambda_n^{\,i}$ are not valid functions of n for temperatures higher than 9 keV, therefore the contributions from harmonics above n = 5 should not be included. The radiation losses calculated by taking the double of the sum of contributions of first 5 harmonics for temperatures above 9 keV are considered as lower limits only.

The synchrotron radiation loss and the normal bremsstrahlung[4] radiation for temperatures 1 to 20 keV were plotted and shown in Fig. 3, and the ratio of the two is shown in Fig. 4. The dashed portion of the curves (from 9 keV to 20 keV) is uncertain due to the reasons discussed above. One can see from Fig. 3 that the total synchrotron radiation loss rises exponentially up to 9 keV. If we try to extrapolate the values above 9 keV, the energy radiated could be extremely high. However, even with the lower limit estimation as shown by the dashed portion or the curve, the loss is still greater than that due to bremsstrahlung at temperatures above 14 keV and are about the same order between 10 and 14 keV.

We can conclude that the synchrotron radiation loss of relatively warm plasma (above 10 keV) may be significant. In order to obtain more reliable results for mildly warm plasmas, relativistic effects must be considered. The work of calculating relativistic synchrotron radiation, taking into account VB broadening and possible collision and/or Doppler broadening, is underway.

TABLE I

	T = 5 keV		10 keV		15 keV	
n	$z\Lambda_n^1$	$z\Lambda_n^2$	$z\Lambda_n^1$	$z\Lambda_n^2$	$z\Lambda_n^1$	$z\Lambda_n^2$
1	6.7 E+4	2.2 E+4	6.7 E+4	2.2 E+4	6.7 E+4	2.2 E+4
2	9.1 E+2	1.8 E+3	1.8 E+3	3.6 E+2	2.7 E+3	5.4 E+2
3	3.7 E+2	5.3 E+1	1.5 E+2	2.1 E+1	3.3 E+2	4.7 E+1
4	2.7 E+0	3.0 E-1	2.1 E+1	2.4 E+0	7.2 E+1	8.1 E+0
5	2.8 E-1	2.6 E-2	4.5 E+0	4.1 E-1	2.3 E+1	2.1 E+0
6	4.0 E-2	3.1 E-3	1.3 E+0	9.8 E-2	9.7 E+1	7.5 E-1
7	7.1 E-3	4.7 E-4	4.5 E-1	3.0 E-2	5.1 E+0	3.4 E-1
8	1.5 E-3	8.8 E-5	1.9 E-2	1.1 E-2	3.3 E+0	1.9 E-1
9	3.7 E-4	2.0 E-5	9.5 E-3	5.0 E-3	2.4 E+0	1.3 E-1
10	1.1 E-4	5.0 E-6	5.4 E-3	2.6 E-3	2.1 E-0	9.9 E-2
11	3.4 E-5	1.5 E-6	3.5 E-3	1.5 E-3	2.0 E+0	8.7 E-2
12	1.2 E-5	4.8 E-7	2.5 E-3	9.9 E-4	2.1 E+0	8.5 E-2
13	4.7 E-6	1.8 E-7	1.9 E-3	8.2 E-4	2.5 E+0	9.3 E-2
14	2.0 E-6	7.0 E-8	1.7 E-3	5.8 E-4	3.3 E+0	1.2 E-1
15	9.5 E-7	3.1 E-8	1.6 E-3	5.0 E-4	4.6 E+0	1.5 E-1

TABLE II

n	5 keV R(n)	10 keV R(n)	15 keV R(n)
1	0.780	0.780	0.78
2	0.780	0.780	0.78
3	0.710	0.720	0.78
4	0.380	0.650	0.74
5	0.100	0.440	0.64
6	0.017	0.260	0.55
7	0.003	0.160	0.43

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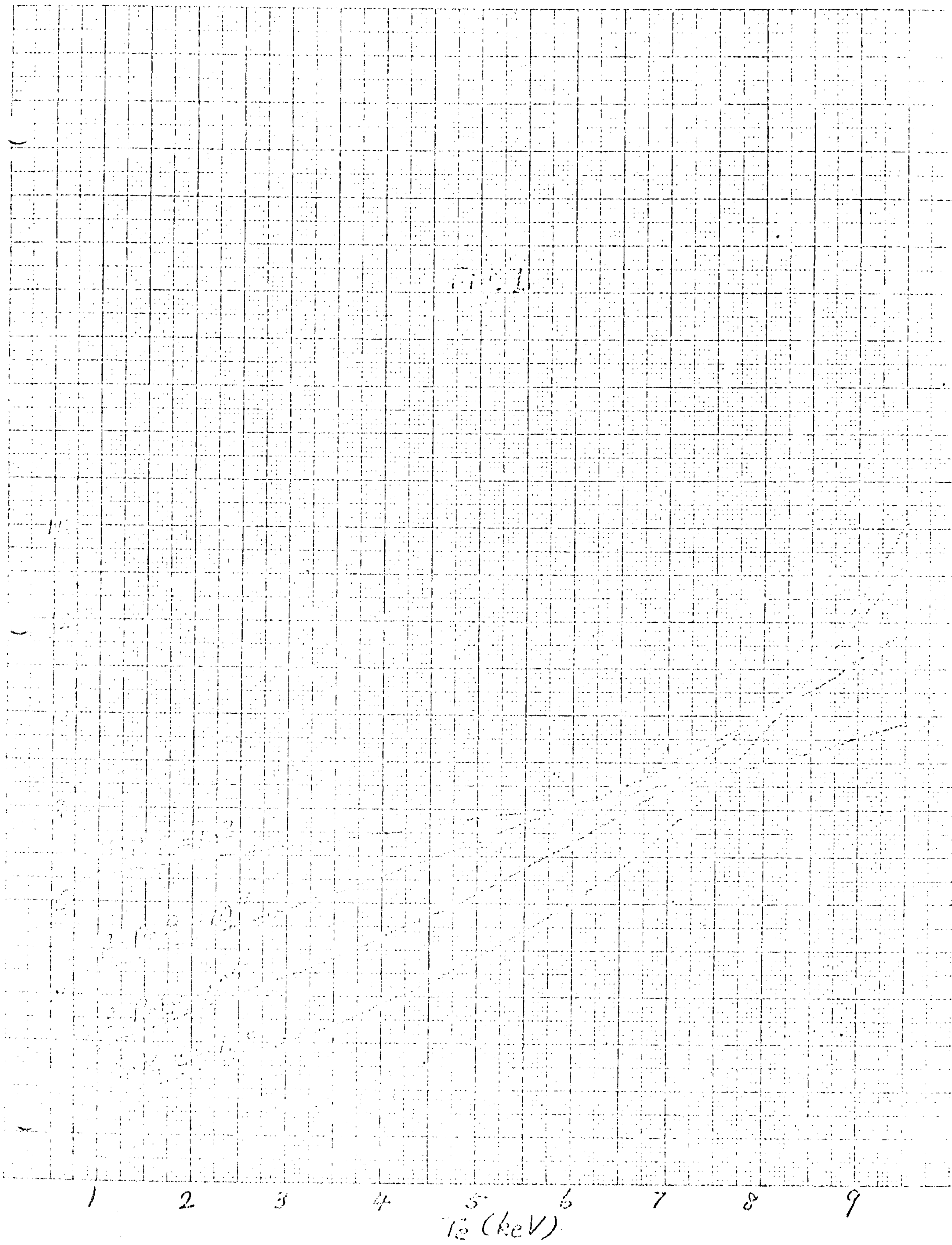
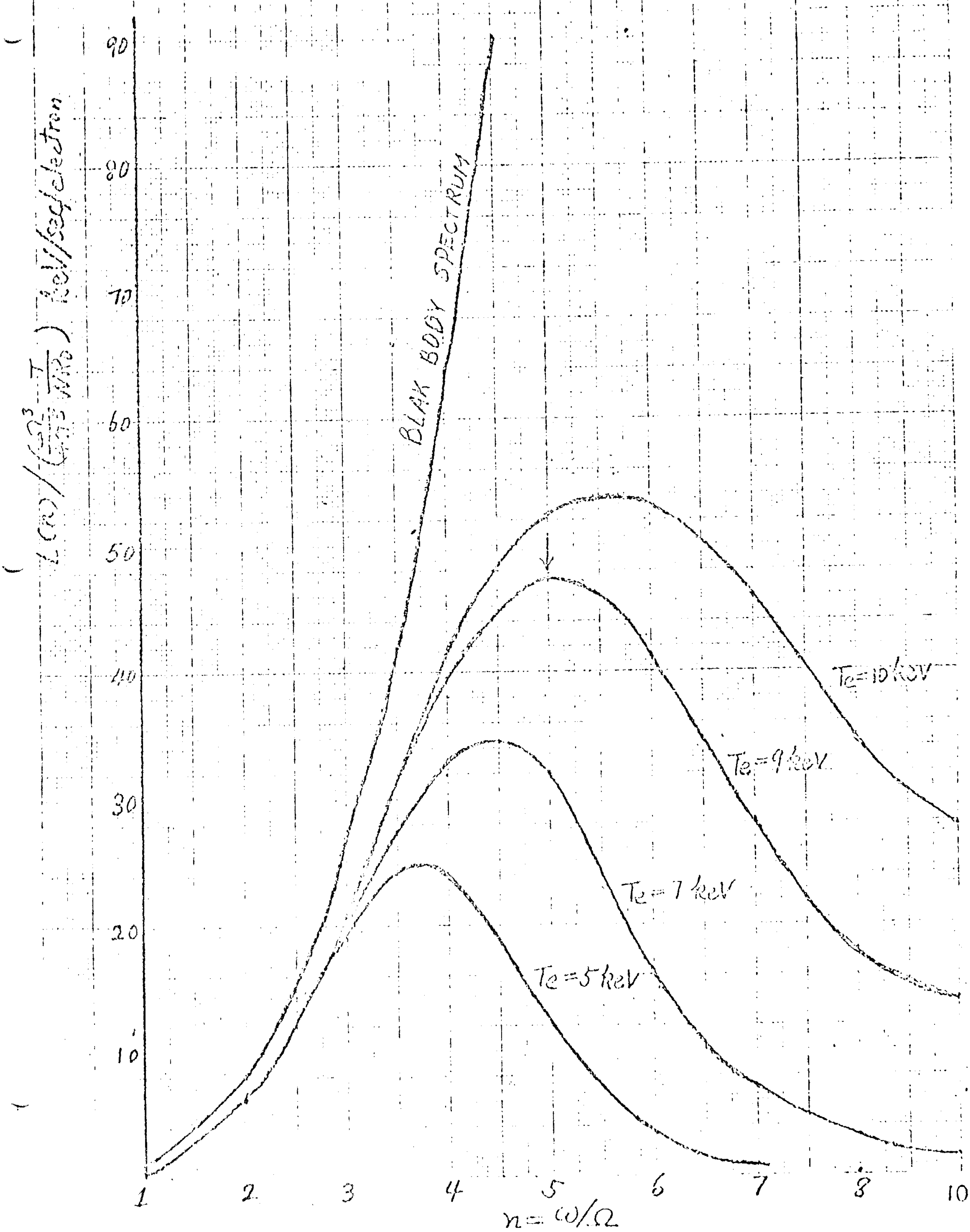

1
2
3
4
5
6
7
8
9
T_e (keV)

Fig 2



BLAK BODY SPECTRUM
Te=10 keV
Te=9 keV
Te=7 keV
Te=5 keV
keV/sec/electron
90
80
70
60
50
40
30
20
10
1
2
3
4
5
6
7
8
10
n = ω/Ω

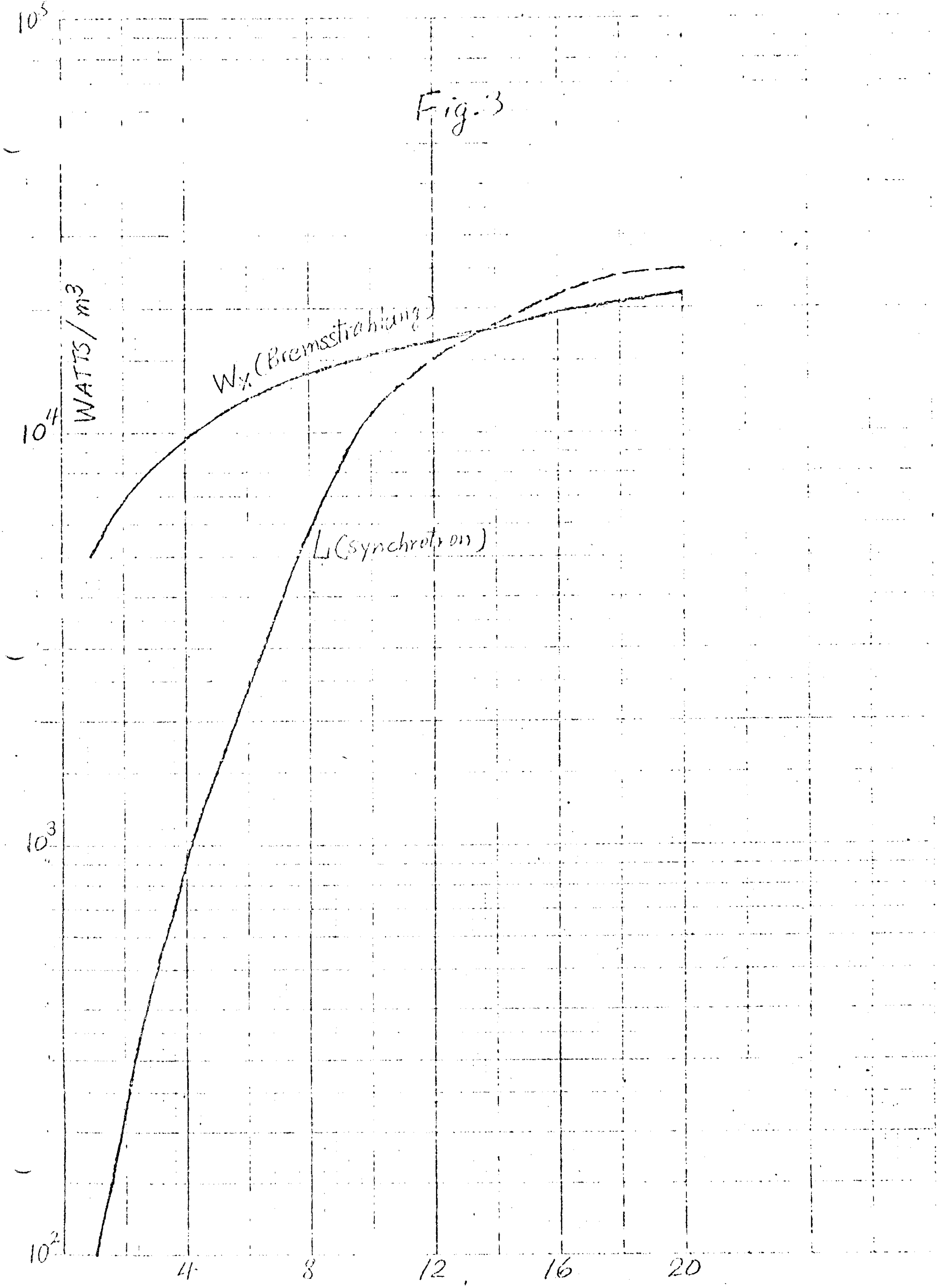
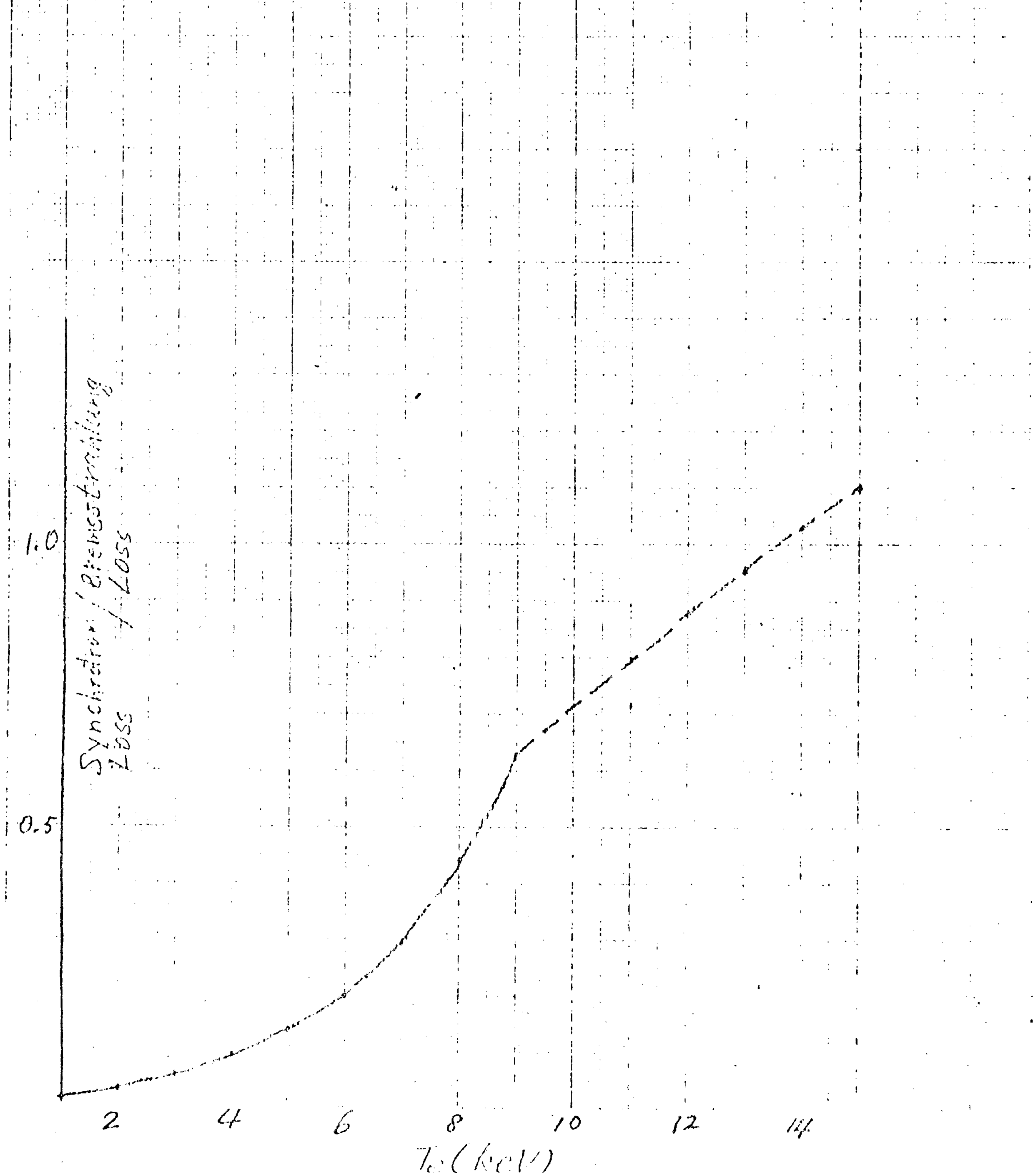


Fig. 3
10^5
10^4
10^3
10^2
WATTS/m^3
W_X (Bremsstrahlung)
L (synchrotron)
4
8
12
16
20

Fig. 4


Synchrotron / Bremsstrahlung
Loss / Loss
1.0
0.5
2
4
6
8
10
12
14
Te (keV)