Heat Load on Surfaces Due to Constant Rate of Deposition of Plasma Energy

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(SEE FDM 13)

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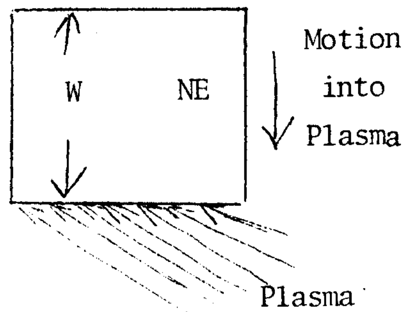
If the niobium limiter is moved gradually into the plasma a heat deposition rate Q watts per square centimeter occurs on one side. There is no heat deposition anywhere else and suppose there is no loss out of sides or back of square cross section, W by A. The thermal conductivity is 0.3 watts/(cm^{o}K). In the heat conduction equation



$$\frac{dT}{dt} = a^2 \frac{d^2T}{dx^2} , \qquad 1)$$

a^2 is in cm^2/sec, so $a^2 = \frac{K}{\rho c}$ where K is in joules/(cm DEG sec) = 0.3, ρ is GM/cm^3, and c is joules/(GM DEG) for specific heat = .27. Thus

$$a^2 = 0.3/(8.5 \times 0.27) = .132 \text{ cm}^2/\text{sec}. \qquad 2)$$

If Q is instantaneously dumped on the surface in joules/cm^2

$$T_2 - T_1 = \frac{Q}{\rho c \; a\sqrt{\pi t}} \; e^{-\frac{x^2}{4a^2t}} \qquad 3)$$

for conductor in half space to infinity. (See Sir William Thompson 1879, Math and Phys Papers Vol. II, and Beyerley 1893, "Fourier Series, etc."). With a skin depth

$$\delta = 2a\sqrt{t} = 2\sqrt{\frac{tK}{\rho c}} = .72\sqrt{t} \text{ cm} \qquad 4)$$

and if $\dot{Q}$ is a permanent source in joules/(cm^{2}sec) = watts/cm^2,

$$T_2 - T_1 = \frac{\dot{Q}}{a\rho c\sqrt{\pi}} \int_0^t \frac{e^{-\frac{x^2}{4a^2(t-\tau)}}}{\sqrt{t-\tau}} d\tau \quad . \qquad 5)$$

The surface rise at x = 0 and for times which give $\delta \hat{<}$ 1 cm (t < 2 seconds)

$$T_2 - T_1 = \frac{\dot{Q}}{a\rho c\sqrt{\pi}} \int_0^t \frac{d\tau}{\sqrt{t-\tau}} = \frac{2\dot{Q}}{a\sqrt{\pi}} \frac{\sqrt{t}}{\rho c} = 2\dot{Q}\sqrt{\frac{t}{\pi K\rho c}} \qquad 6)$$

With our numbers of 144 mega joules striking a band around the purging limiter 10^4 cm long and 2 cm wide ≡ Acm (FDM 13)

$$T_2 - T_1 = \frac{2\ (144)\ \times\ 10^6}{\Delta t\ 2\text{cm}\ 10^4\text{cm}} \sqrt{\frac{t}{\pi\ (0.3)(8.5)(.27)}}$$

or

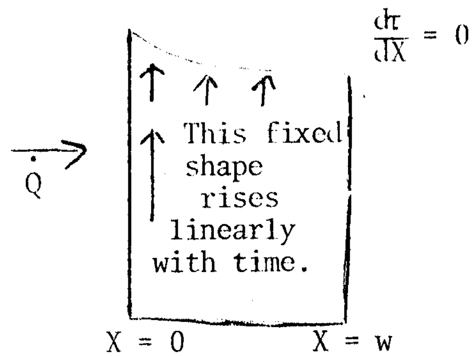
$$T_2 - T_1 = 9{,}700\ \sqrt{t}\ /\ \Delta t_p \qquad 7)$$

where Δt_p is the purge time and t < 2 sec for estimating the first surface rise. Thus

$$T_2 - T_1 \sim \frac{13{,}400}{\Delta t_p}$$

should be less than the melting point (2468° C) or $\Delta t_p > \frac{13{,}400}{2000°} \cong 7$ second absolute purging minimum to avoid surface rise melting by two seconds. (Note if this heat load would be dumped uniformly on the niobium wall, which has an area 10^4 times larger, the dump time could be .0007 seconds without melting).

For long times when the whole purger is hot but still is receiving heat from plasma the temperature distribution looks thus:



This is much more easily solved without the "AGE" equation by reformulating the problem thus: $\frac{\dot{Q}}{W}$ joules go into each cubic centimeter. The flow past a point x is $\frac{\dot{Q}}{W}$ (W-x). This must equal $-K\frac{dT(x)}{dx}$. Thus

$\dot{Q} = -\frac{KW}{W-x}\frac{dT}{dx}$ is a constant. Solving:

$$\frac{dT}{dx} = -\frac{\dot{Q}}{KW}(W-x) \qquad \text{note this is properly zero at } W = x.$$

$$T_{(x)} - T_{(x=0)} = -\frac{\dot{Q}}{KW}\left(xW - \frac{x^2}{2}\right). \qquad 8)$$

This gives a temperature drop across the bar of thickness W to be

$$T_W - T_0 = -\frac{\dot{Q}}{K}\frac{W}{2} \qquad 9)$$

If we choose four times the minimum purge time for surface melting i.e., 4 x 7 sec = 28 seconds for purge, we get

$$T_W - T_0 = \frac{144 \times 10^6 \text{ joules} \qquad 2 \text{ cm}}{2 \times .3 \text{ watts/(cmDEG)} \; 2\text{cm} \times 10^4\text{cm} \; 28 \text{ seconds}} = 840^{\circ} \text{ drop.}$$

across purger.

But with this 2 cm x 2 cm bar 10^4 cm long the average temperature rise is

$$\frac{144 \times 10^6 \text{ joules}}{4 \text{ cm}^2 \times 10^4\text{cm} \times 8.5 \text{ am/cm}^3 \; .27 \text{ joules/am DEG}} = 1{,}560^{\circ}\text{C}$$

so the peak temperature would be close to 2,000° C or to the melting point.

Thus to avoid melting the hot edge of the bar it would appear that we need something like 20 to 30 seconds purge, but assembling the equations: the temperature spread in the thickness of the purging bar is (by 9)

$$\Delta T_x = \frac{qW}{2At_p LK}$$

$$\Delta T = \frac{q}{AWL\rho c} \quad \text{is the average rise after all heat is collected,}$$

and by 6) for short time surface temperature rise: 9)

$$\Delta T_{t_s} = \frac{2q}{ALt_p}\frac{\sqrt{t_s}}{\sqrt{\pi K\rho c}} \quad \text{with} \quad \sqrt{t_s} = \frac{W}{2}\sqrt{\frac{\rho c}{K}}$$

$$\Delta T_{t_s} = \frac{qW}{ALt_p K\sqrt{\pi}} \qquad (\text{like } \Delta T_x)$$

where q is the total heat deposited, t_p is the purging time, L is the bar length, A is the bar width collecting q, W is the bar thickness or depth, K conductivity, ρ density, and c specific heat.

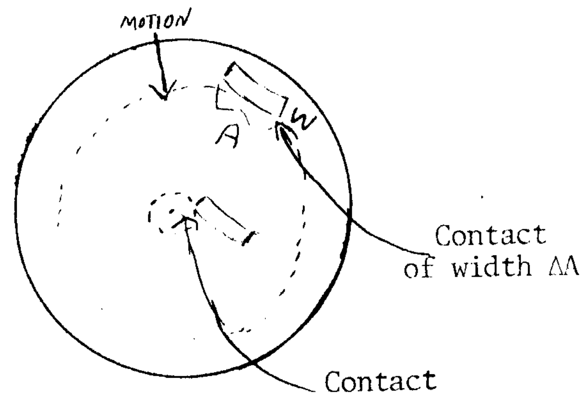
$$\left(\Delta T + \frac{\Delta_x T}{2}\right) < T_{melting}$$

or

$$\frac{q}{L}\left(\frac{1}{AW\rho c} + \frac{W}{4t_p KA}\right) < T_{melt.} \qquad 10)$$

The first term is minimized by choosing a big enough bar (LAW cm^3). The second term is minimized by a wide contact area (LA cm^2) and a minimum bar thickness, W cm, to transmit the heat through.

The second term can be kept down by shaping the purger thus: as it moves down contact is made at one edge and the contact moves across the limiter surface. If we move the limiter rapidly so the last term of 10) alone applies



$\Delta T_x < T_{melt}$ or $\dfrac{qW}{2L A t_p K} < T_{melt}$. If out of Λ only a width $\Delta\Lambda$ is struck at one time, Δq deposited is then $q\dfrac{\Delta\Lambda}{\Lambda}$ in time $\Delta t_p = t_p\dfrac{\Delta\Lambda}{\Lambda}$ and 10) becomes

$$\frac{\Delta q W}{2L\Lambda\Delta t_p K} = \frac{q\frac{\Delta\Lambda}{\Lambda}W}{2L\ \Lambda t_p\frac{\Delta\Lambda}{\Lambda}K} = \frac{qW}{2L\Delta\Lambda t_p\ K} < T_{melt}.$$

$$\frac{144\cdot 10^6\ W}{2\text{x}10^4\Delta\Lambda t_p\ 0.3} < 2000^\circ\text{C} \quad \text{or} \quad \frac{3.7}{0.3} = 12 < \frac{\Delta\Lambda t_p}{W}. \qquad 11)$$

So if we can strike a band as wide as 1 cm, $t_p/W > 12$. If W 0.2 cm, t_p = 2.4 sec. But for $\delta \sim .2$ cm by 4), $t_{skin} = \frac{1}{12}$ sec or we have a sweep to reach "Λ" of $\frac{2.4\text{ sec}}{1/12\text{ sec}}$ = 30 times $\Delta\Lambda \Rightarrow 30$ cm = Λ total. Note that the assumption of ΔΛ = 1 cm is unsupported and vital to the operation described. One does not know how wide a band will be struck by the plasma.