Limitation on Fractional Burnup Posed by Alpha Pressure

H.K. Forsen and T. Yang

May 1972

UWFDM-14

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Fusion Technology Institute
University of Wisconsin
1500 Engineering Drive
Madison, WI 53706

http://fti.neep.wisc.edu

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PESSIMISTIC EFFECTS OF LONG CONFINEMENT AND ALPHA PRESSURE

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Many of us have been concerned with the excessive fractional burnup that is suggested to occur in the neoclassical banana regime of low β toroidal reactors[1]. Because of this and the possible limitation on β_p in tokamaks[2], we decided to see the effect on the reaction rate parameter first suggested by Mills[3]. In this we write

$$\beta_p = \frac{nk(T_e+T_i+P_\alpha)}{B_p^2/2\mu_o} \tag{1}$$

and use

$$P_\alpha = n_\alpha \bar{U}_\alpha = f_b n \bar{U}_\alpha \tag{2}$$

where $f_b \simeq \frac{1}{2} n\tau\langle\sigma v\rangle$ and $\bar{U}_\alpha = 2U_{\alpha o}/3 = 2.35$ MeV. This does not necessarily assume the α particles thermalize but that they have a pressure (P_α) which can be represented as defined. That is, Eq. (1) refers to the energy density of the plasma and this represents a pressure in a Maxwell Boltzman sense but the α's and the electrons and ions are not at the same temperature (T).

Writing the reaction rate per unit volume in a D-T plasma where $n \neq n(r)$ and $T_i \neq T_i(r)$ we have

$$R_{DT} = \frac{n^2}{4} \langle\sigma v\rangle_{DT} \, . \tag{3}$$

Solving Eq. (1) for n, using Eq. (2) and inserting in Eq. (3) we obtain by Strelkov and Popkov[6], the confinement scaling is

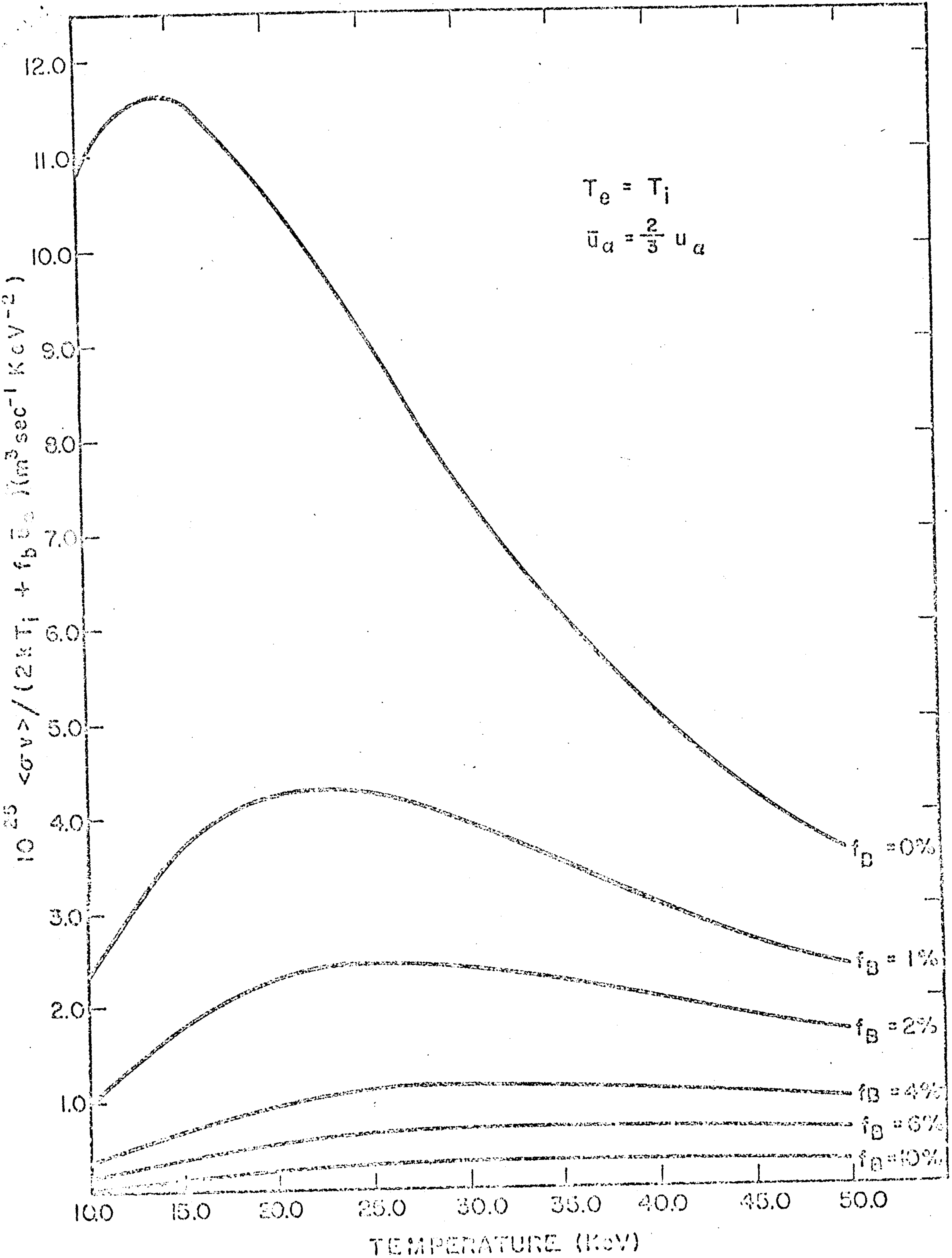
$$n\tau = 0.26\ B_T^{\ 2} a^2 T^{\frac{1}{2}} / A^{3/2} q^2 .$$

For a reactor of the size suggested by Golovin et al.[7] at 20 keV, this gives $n\tau = 2.1 \cdot 10^{16} \mathrm{cm}^{-3}\mathrm{sec}$ and a corresponding fractional burnup of over 100% which obviously cannot occur without adding fuel. There are at least two things wrong with this calculation and they are that the approximate formula for f_b is incorrect beyond about 15% burnup and that the density and temperature cannot be kept constant over the 70 second confinement period.

An approximate formula for the increase in plasma pressure is suggested by Eq. (4) and is useful for comparison. That is if $T_e \approx T_i \approx 15$ keV, then the plasma energy density scales as

$$P = Cn(1 + 78f_b) \tag{5}$$

Where $C = 4.8 \cdot 10^{-15}$ and n is in m^{-3} giving P in MKS. This shows that a small fractional burnup results in a large pressure increase under the assumption of this paper or that the α's are confined.

$T_e = T_i$
$\bar{u}_\alpha = \frac{2}{3} u_\alpha$
$10^{25} <\sigma v>/(2kT_i + f_b E_\alpha)(m^3 sec^{-1} KeV^{-2})$
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
$f_B = 0\%$
$f_B = 1\%$
$f_B = 2\%$
$f_B = 4\%$
$f_B = 6\%$
$f_B = 10\%$
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
TEMPERATURE (KeV)