Limit on Poloidal Beta in a Tokamak

G.A. Emmert

April 1972

UWFDM-12

FUSION TECHNOLOGY INSTITUTE

UNIVERSITY OF WISCONSIN

MADISON WISCONSIN

Limit on Poloidal Beta in a Tokamak

G.A. Emmert

Fusion Technology Institute
University of Wisconsin
1500 Engineering Drive
Madison, WI 53706

http://fti.neep.wisc.edu

April 1972

UWFDM-12

LIMIT ON POLOIDAL BETA IN A TOKAMAK

by

G. A. Emmert

April 1972

FDM 12

University of Wisconsin

These FDM's are preliminary and informal and as such may contain errors not yet eliminated. They are for private circulation only and are not to be further transmitted without consent of the authors and major professor.

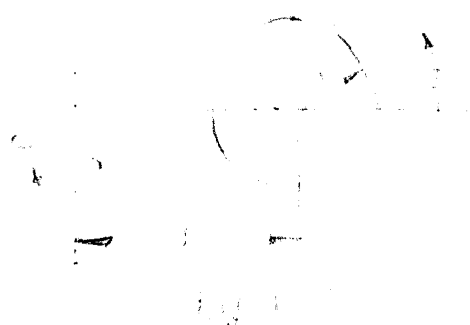
An important concern regarding the suitability of the tokamak for a fusion reactor is the limit on beta (either poloidal or toroidal beta). Since the power density is proportional to β^2, economic considerations suggest that β should be as large as possible. Since the poloidal beta, β_p, is proportional to the toroidal beta, for a given stability factor and aspect ratio, it is sufficient to consider the limit on β_p.

Shafranov,[1] Strauss,[2] and Callen and Dory[3] have argued that, for a tokamak plasma contained in a conducting shell, MHD equilibrium does not impose a limit on β_p. A conducting shell, however, is not suitable for a long time equilibrium since the surface currents in the shell decay in time. An externally imposed uniform vertical magnetic field can provide long-term equilibrium but, according to Shafranov,[1] this imposes the limit $\beta_p \lesssim A$, where A is the aspect ratio. What happens is that the separatrix between the vertical field and the poloidal field of the plasma shrinks to the plasma surface as $\beta_p \to A$. One cannot go higher in β_p without having field lines in the plasma connect to infinity; equilibrium is lost. This argument is based on the assumption that the vertical field is uniform over the cross-section of the plasma; one might suspect that a suitably designed nonuniform vertical field (which is what a conducting shell produces) will allow higher β_p.

Galeev and Sagdeev[4] have argued for a different limit on β_p in a steady-state tokamak because of a non-MHD effect--the "bootstrap current". To obtain their result, it is convenient to start with a relation between the inductive electric field E_θ, the toroidal current density J_θ, and the poloidal magnetic field B_ϕ obtained by Rosenbluth et al;[5]

$$J_\theta = \sigma_s\left(1-1.95\sqrt{\frac{r}{R}}\right)E_\theta - \frac{4.88T}{B_\phi}\sqrt{\frac{r}{R}}\frac{dn}{dr} \qquad (1)$$

where σ_s is the Spitzer conductivity, n is the density and T is the temperature ($T_i = T_e$, for simplicity). The coordinate system is shown in Fig. 1. The second term in (1) is the "bootstrap current"; it arises because of the tensor nature of diffusion in a magnetic field. Since the poloidal field B_ϕ is created by currents in the plasma



$$\vec{\nabla}\times\vec{B} = \mu_o\vec{J}$$

or, for large aspect ratio,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rB_\phi\right) = \mu_o J_\theta \qquad (2)$$

Inserting (1), we get

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rB_\phi\right) = \mu_o\sigma_s\left(1-1.95\sqrt{\frac{r}{12}}\right)E_\theta - \frac{4.88\mu_o T}{B_\phi}\sqrt{\frac{r}{R}}\frac{dn}{dr} \qquad (3)$$

Let us put $E_\theta = 0$ (stationary tokamak) and assume $n(r) = n_o(1 - r^2/a^2)$. The solution to (3) is

$$B_\phi^2(r) = \frac{4.34\mu_o T n_o}{a^2\sqrt{R}} \; r^{5/2}$$

We define β_p by

$$\beta_p = \frac{2n_o T}{\dfrac{B_\phi^2(a)}{2\mu_o}}$$

Thus

$$\beta_p = .92\sqrt{\frac{R}{a}} = .92\sqrt{A}$$

Hence, if one can get a tokamak to $\beta_p = .92\sqrt{A}$, then the required electric field to maintain the current is zero. The "bootstrap current" is sufficient to maintain the poloidal magnetic field. In order to get to higher β_p, one needs to reduce the current density J_θ near the edge ($r \approx a$); this can be done by having E_θ negative at the edge and zero at the center. Since this electric field profile is not curl-free it can be maintained only for times shorter than that for diffusion of the poloidal magnetic field. A uniform E_θ opposed to J_θ is not possible since from (3),

$$B_\phi(-E_\theta) = -B_\phi(E_\theta);$$

the entire current profile and poloidal field reverses in direction so that J_θ is always parallel to E_θ.

The physical origin of the bootstrap current is rather interesting. For large aspect ratio, we can approximate the toroid by a cylinder as shown in Fig. 2.

The radial diffusion flux is $\Gamma_r = - D \frac{dn}{dr}$. The corresponding diffusion velocity is $v_r = \Gamma_r/n$. Let us put this velocity into a Langevin equation for the z-motion of the electrons

$$m_e \frac{dv_t}{dt} = -ev_r B_\phi - m_e \nu_{ei} v_z$$

where ν_{ei} is the electron-ion collision frequency. For steady-state, d/dt = 0. Thus

$$v_z = - \frac{eB_\phi}{m_e \nu_{ei}} v_r = \frac{eB_\phi}{m_e \nu_{ei}} \frac{D}{n} \frac{dn}{dr}$$

The radial diffusion coefficient D in the banana regime is[5]

$$D = 2.24\, \nu_{ei}\, \rho_e^{\,2} \left(\frac{B_z}{B}\right)^2 \sqrt{\frac{r}{R}}\ ,$$

where $\rho_e^{\,2} = \dfrac{2m_e T}{e^2 B_z^{\,2}}$.

Thus

$$v_z = \frac{4.48}{eB_\phi} \frac{T}{n} \frac{dn}{dr} \sqrt{\frac{r}{R}}$$

The current density J_z is

$$J_z = -env_z = -\frac{4.48T}{B_\phi} \frac{dn}{dr} \sqrt{\frac{r}{R}}\ ,$$

which, except for a small difference in the numerical factor, is identical to the "bootstrap current" in (1).

Conclusion

Since there seems to be no way of getting around the "bootstrap" current limitation on β_p in a steady-state tokamak reactor, it is best to design for $\beta_p \leq \sqrt{A}$. A short pulse reactor may operate at higher β_p because of a reversed skin effect. This is an "iffy" business for design since neoclassical theory predicts a strong but experimentally unobserved skin effect in present experiments.

References

1. V.S. Mukhovatov, V.D. Shafranov, Nuclear Fusion 11, 605 (1971).
2. H.R. Strauss, Phys. Rev. Lett. 26, 616 (1971).
3. J.D. Callen and R.A. Dory, ORNL-TM-3430 (1971).
4. A.A. Galeev, R.Z. Sagdeev, JETP Lett. 13, 113 (1971).
5. M.N. Rosenbluth, R.D. Hazeltine, F.L. Hinton, Phys. Fluids 15, 116 (1972).