Alpha Particle Heating in CTR Plasmas: Energetics and Time Dependence

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ALPHA PARTICLE HEATING IN CTR PLASMAS:
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Abstract

The Fokker-Planck equation has been used to examine the properties of alpha particle slowing down in D-T plasmas at CTR conditions. The alpha particle is treated as a test particle while the background plasma distribution functions are assumed to be Maxwellian. The results where comparable are in good quantitative agreement with the work of Sigmar and Joyce and qualitatively agree with the work of Rose. The main results include:

1) The alpha particle slowing down time and the fraction of the initial α-energy deposited in the electrons versus in the ions is dependent only on Te. It is insensitive to variations in Ti.

2) The fraction of the initial α-energy deposited in electrons and ions is insensitive to the absolute value of the density for values of density in the range Ni = 5×10^{13}/cc to 3×10^{14}/cc. However, the results are sensitive to the ratio Ne/Ni.

3) At low Te, the alpha particles deposit most of their energy in the electrons. The electron temperature at which equal amounts of the initial alpha energy are deposited in electrons and ions is 40 KeV when Ne = Ni. When Ne = 1.2 Ni, this 50-50 point is shifted to 46 KeV, which is a 15% increase in Te. These results agree well with recent calculations of Sigmar and Joyce.

4) The time, t_{SD}^{α}, for the alpha particle to slow down from Eo = 3500 KeV to approximate 5 Te ranges from $\sim$.45 sec at Te = 10 KeV to $\sim$2.2 sec at Te = 80 KeV. Because of approximations made, the results have not been extended to examine complete thermalization. However, this extension can readily be performed if necessary.

5) A comparison of t_{SD}^{α} with typical confinement times for CTR Tokamaks $\sim$[illegible]0 sec in some cases) indicates the α-particles will completely thermalize before escaping from the plasma. They will therefore constitute a third ion species with temperature, T_{α}, approximately equal to the D-T ion temperature.

6) The approach discussed herein can be used to analyze energy deposition questions as regards heating by neutral particle injection.

I. Introduction

This memo briefly describes a model, based on the Fokker-Planck equation, for the treatment of test particle slowing down in a CTR palsma. The theory follows, in part, the treatment of Shkarofsky, Johnston, and Bachynski[1] with some minor exceptions. Part of the motivation for this work is to provide the capability, at Wisconsin, for performing this type of calculation and will be useful input to those carrying on studies in areas such as energy balance calculations, fusion reactor control, and ignition. The next section describes the basis for the calculational model. Those interested in the results may safely pass on to section III.

II. Basic Equations

One can derive[1,2] from the Fokker-Planck equation for the test particle distribution function (a delta function in this case) that the rate of change of the energy of the test particle due to interactions with a background of field particles having a Maxwellian distribution of speeds is given by

$$\left(\frac{dE_\alpha}{dt}\right)_{\alpha j} = \frac{4\pi M_\alpha}{v_\alpha} \left(\frac{Z_\alpha Z_j e^2}{4\pi\epsilon_o M_\alpha}\right)^2 L_{\alpha j}\left(\left(J_{-1}^{o}\right)_j - \frac{M_\alpha}{M_j}\left(I_o^o\right)_j\right). \quad (1)$$

The notation is:

E'_α = energy of test particle

j = jth species of background (ions or electrons)

M_α = mass of test particles

M_j = mass of background particle

V_α = speed of test particles

Z_α, Z_j = charge on test particle and background, respectively

e = electric charge

Also, we have the functions

$$(I_o^o)_j = n_j(\phi(X_j)-S_j\phi'(X_j)) \quad (2)$$

$$(J_{-1}^o)_j = n_j(X_j\phi'(X_j)) \quad (3)$$

where n_j = density of species j,

$$X_j = \frac{V_\alpha}{\sqrt{\frac{2KT_j}{M_j}}} = \frac{v_\alpha}{\langle V_j \rangle} \quad (4)$$

and ϕ and ϕ' are the error function and its derivative, respectively. $L_{\alpha j}$ is the coulomb logarithm.

We now specialize to the case of a D-T plasma and consider alpha particles as the test particle. The alpha particle resulting from a D-T fusion reaction is very energetic, being born with initial energy of 3.52 MeV. Our interest is in how this reaction particle gives up its energy to the ions and electrons constituting the background and in the time scales involved.

For most of the slowing down range (3.5 MeV to the temperature of the background plasma) the speed of the alpha particle is much greater than the average ion speed. Thus, $X_I >> 1$ under this condition, $\phi(X_i)$ and $\phi'(X_i)$ can be taken as

$$\phi(X_i) \approx 1 \quad (5)$$

$$\phi'(X_i) \approx \frac{2}{\sqrt{\pi}} e^{-X_i^2}. \quad (6)$$

he other hand, because $M_e << M_\alpha$, $X_e << 1$ even though $E >> KT_e$. For
< 1, one can use

$$\phi(X_e) \approx \frac{2}{\sqrt{\pi}} \left(X_e - \frac{X_e^3}{3}\right) \tag{7}$$

$$\phi'(X_e) \approx \frac{2}{\sqrt{\pi}} (1-X_e^2). \tag{8}$$

se approximations have been used in the studies to be discussed in section
I. Such approximations allow one to investigate the important energy and
me ranges involved in the alpha slowing down process with minimal difficulty.
wever, one can readily study the thermalization regime ($E_\alpha \approx KT$) by simply
t approximating $\phi(X_i)$.

If now a schematic is provided to determine $L_{\alpha i}$ and $L_{\alpha e}$, one can turn to
he question of calculations. In standard notation, $L_{\alpha j}$ is

$$L_{\alpha j} = \ell n \Lambda_{\alpha j} \tag{9}$$

where

$$\Lambda_{\alpha j} = \frac{1}{(X_o)_{\alpha j}} \tag{10}$$

and $(X_o)_{\alpha j}$ is the cutoff angle for small angle scattering between species α and
j. It is given by[3]

$$X_o = \text{larger of} \quad \begin{cases} \dfrac{b_o(90°)}{\lambda d} & \text{(classical determination)} \\ \dfrac{\lambda}{\lambda d} & \text{(quantum determination)} \end{cases} \tag{11}$$

where $b_o(90°)$ = 90° impact parameter

λ = center of mass wavelength in two-body scattering

and λ_d = Debye length.

Expressions for $b_o(90°)$ and λ are

$$b_o(90°) = \frac{q_\alpha q_j}{4\pi\varepsilon_o M_r v_r^2} \tag{12}$$

$$\lambda = \frac{\hbar}{M_r v_r} \tag{13}$$

where M_r is the reduced mass and v_r is the relative speed in center-of-mass coordinates. One can readily show that under CTR conditions, the quantum determination of the minimum scattering angle is required. Note that for alpha ion scattering and $X_i >> 1$, $v_r \approx V_\alpha$ whereas for alpha electron scattering and $X_e << 1$, $v_r \approx V_e$. Under these conditions, it is found that

$$L_{\alpha i} = \frac{1}{2} \ln \left[\frac{2M_r^2 \varepsilon_o (KT_e) E_\alpha}{M n_e \hbar^2 e^2}\right] \tag{14}$$

$$L_{\alpha e} = \frac{1}{2} \ln \left[\frac{2M_e \varepsilon_o (KT_e)(KT_i)}{n_i Z_i^2 \hbar^2 e^2}\right] \tag{15}$$

with K is equal to Boltzmann's constant.

Equations (1), (5)-(8), (14) and (15) are now in a form suitable for use in studying test particle, and specifically alpha particle, slowing down in CTR plasmas.

II. Results

The equations in section I have been used to study alpha particle slowing down in D-T CTR plasmas at conditions projected for Tokamak like systems. A computer program is available to design team members who may need it.

Before proceeding to the computational results, let us examine some simple, but instructive, approximations for the alpha particle slowing down time, t_{SD}^{α}. If we neglect terms of order X_e^3 and higher in equation (7), the equation for $\frac{dE_\alpha}{dE}$ can be written schematically as

$$\frac{dE_\alpha}{dt} = - \frac{\Lambda_i}{\sqrt{E_\alpha}} - \Lambda_e E_\alpha \tag{16}$$

which can be readily solved[4] to yield

$$t = \frac{2}{3\Lambda_e} \ln \left(\frac{\Lambda_i + \Lambda_e E_o^{3/2}}{\Lambda_i + \Lambda_e E_\alpha^{3/2}}\right). \tag{17}$$

E_o is the initial alpha particle energy and E_α is the alpha energy after time t. Setting $E_\alpha = 0$ and $E_o = 3.5$ MeV then gives an estimate of the time for an alpha particle to give up all its energy. We denote this slowing down time as t_{SD}^{α}. For $n_i = n_e = 10^{14}$/cc, $T_i = T_e = 15$ KeV, $\ln \Lambda_{\alpha i} = 20$, $\ln\Lambda_{\alpha e} = 15.6$, and a D-T plasma with $M_i = 2.5$, one finds

$$t_{SD}^{\alpha} = .655 \text{ sec.}$$

To compare this with other relevant time scales, we calculate the e-folding time for rethermalization of electrons with electrons, ions with ions, and electrons with ions, using appropriate formulae from Spitzer[3]. For the conditions above, one finds

$$\tau_{ee} = .4 \text{ msec}$$

$$\tau_{ii} = 23 \text{ msec}$$

$$\tau_{ei} = 1120 \text{ msec} = 1.12 \text{ sec}$$

By comparison with t_{SD}^{α}, the following ordering is found.

$$\tau_{ee} << \tau_{ii} < t_{SD}^{\alpha} < \tau_{ei} .$$

This implies that the ions and electrons adjust adiabatically to the presence of energetic alpha particles and remain Maxwellian. However, since $t_{SD}^{\alpha} < \tau_{ei}$, alpha particle slowing down is a driving mechanism that can maintain unequal electron and ion temperatures. If electrons are heated preferentially, T_e will rise and bremsstrauhlung losses will increase. As such, electron-ion rethermalization will not be as effective in heating plasma ions.

In addition, $t_{SD}^{\alpha} \approx$ 1/2 sec is short compared with confinement times now being calculated for CTR Tokamaks[5] (which may be the order of 60 sec). If $t_{SD}^{\alpha} << \tau$(confinement), the alpha particles will completely thermalize before escaping. Since $M_{\alpha} \sim M_i$, one can show that the alpha particle temperature in a CTR system will approximately equal to the ion temperature.

Turning now to the computational results, figure 1 illustrates the alpha particle energy as a function of time for several values of T_e. All results to be presented from here on are insensitive to T_i because the ion temperature occurs only in the argument of the logarithm in equation (1). The alpha particles slow down less rapidly as T_e increases since the electrons become less effective scatterers.

Figure 2 indicates the time required for alpha particles to slow down from 3.5 MeV to $5T_e$, as a function of T_e. The results agree with estimates based on equation (17). To follow $E_{\alpha}(t)$ to complete thermalization, one must use equations (2) and (3) rather than (5) and (6). Approximations (7) and (8) would remain adequate.

Another important parameter for CTR studies is the fraction of the initial test particle energy that is deposited in the background electrons and ions as the test particle slows down. For 3.5 MeV alpha particles, the fraction of this energy going to the electrons, $U_{\alpha e}$, and the fraction going to the ions, $U_{\alpha i}$, has been calculated as a function of the electron temperature. Figure 3 illustrates typical results when $n_i = n_e$. The results are not sensitive to the absolute value of n_i and n_e over the range, 5×10^{13} part/cc to 5×10^{14}/cc, which are typical densities for Tokamak CTR's. The results agree reasonably well with the recent calculations of Sigmar and Joyce[6] over the entire T_e range. The results of Rose[4] differ quantitatively from those of Sigmar and Joyce and from the results in figure 3 but all results agree qualitatively. That is, all results indicate that alpha particles preferentially heat electrons at low T_e. The work of Sigmar and JOyce[6], incidently, is based on the use of the Balescu-Lenard kinetic equation[7].

In figure 4, the effect of a 20% increase in n_e over n_i (for example, due to the presence of the alpha particles themselves) is examined. One can see that $U_{\alpha e}$ is increased and $U_{\alpha i}$ decreased compared to the equal density case at any given T_e. This will exacerbate effects based on the preferential heating of electrons by alphas when $T_e \lesssim 45$ KeV.

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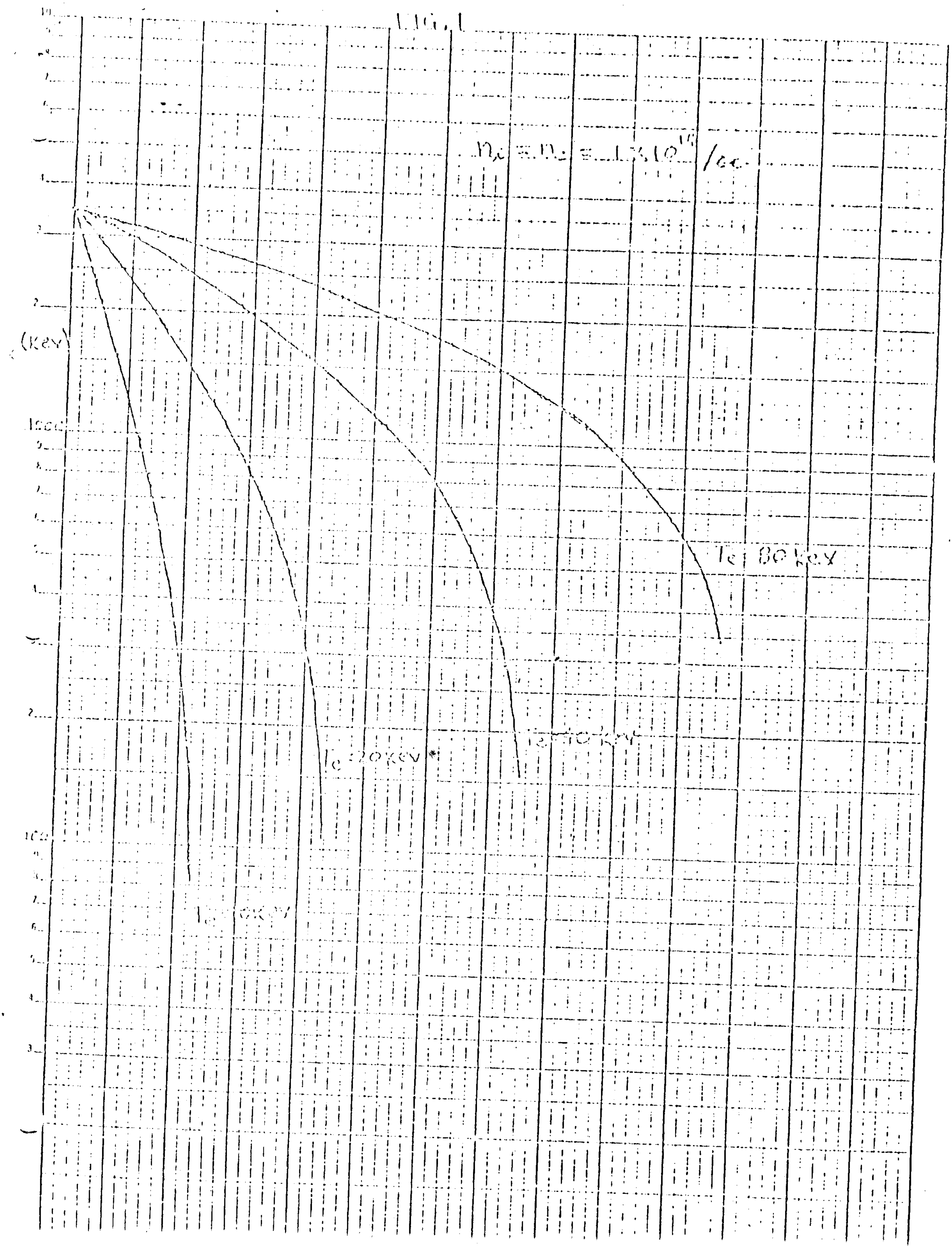
FIG. 1



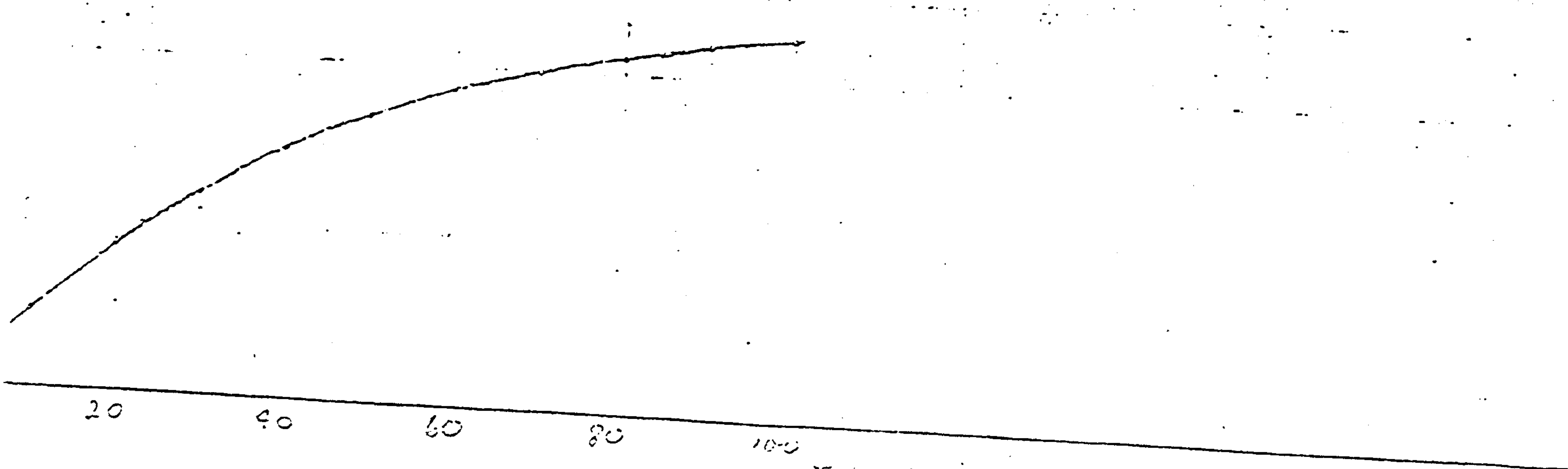
$n_i = n_e = 1 \times 10^{15}$ /cc
(keV)
1000
100
Te 80 keV
Te = 40 keV
Te = 20 keV
Te = 10 keV

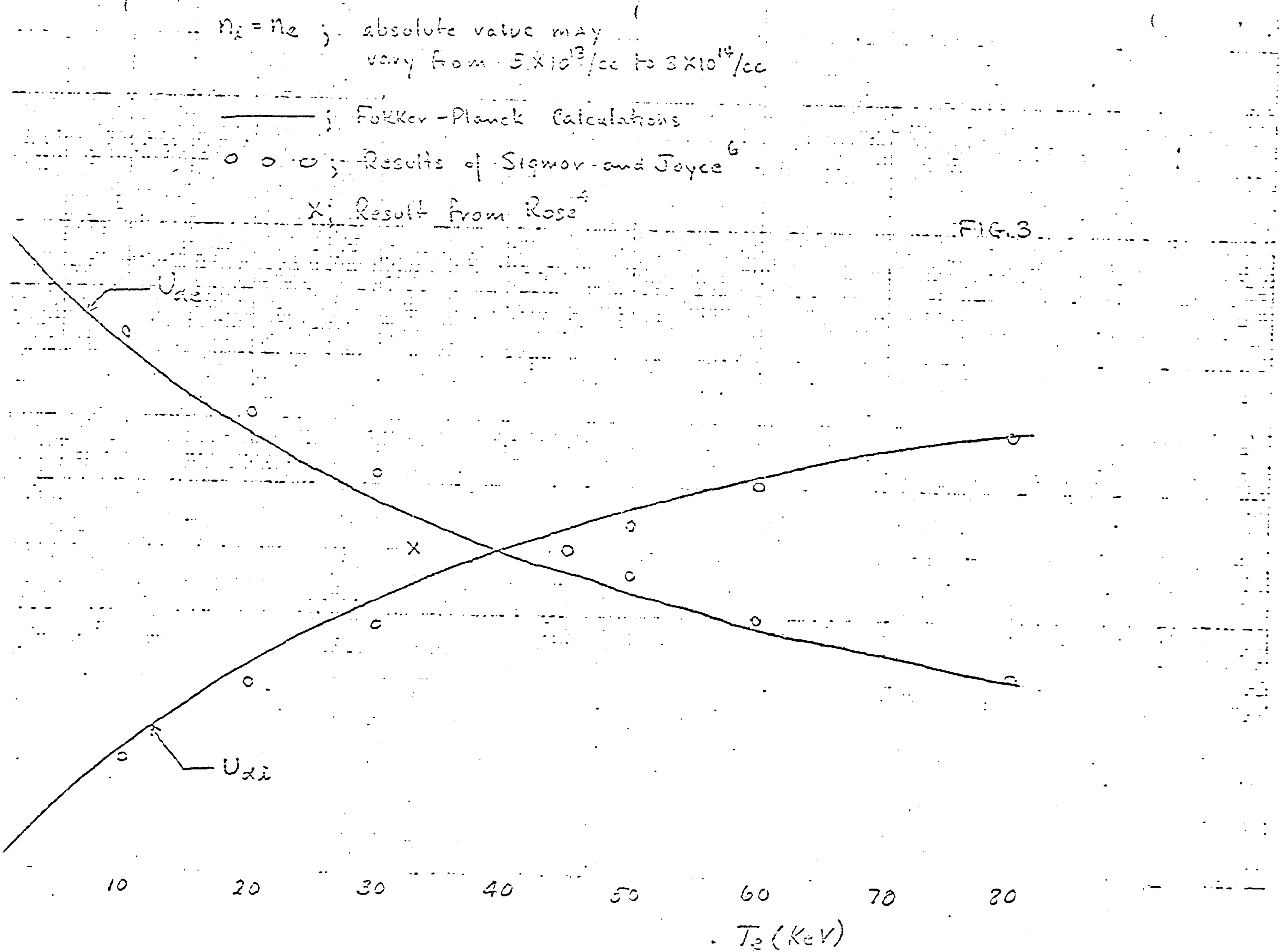
time for α-particle to slow down to

$\epsilon = 5 \times T_e$, vs. T_{e0} $\quad E_0^{\alpha} = 3.5$ MeV

FIG. 2

τ_0 (sec)





$n_i = n_e$; absolute value may vary from 5×10^{13}/cc to 3×10^{14}/cc
——— ; Fokker-Planck Calculations
o o o ; Results of Sigmov and Joyce[6]
X ; Result from Rose[4]
$U_{\alpha e}$
$U_{\alpha i}$
10
20
30
40
50
60
70
80
T_e (KeV)

FIG. 3

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— — x $n_i = 5\times10^{13}$ or $n_i = 1\times10^{14}$; $n_e = n_i$

—— $n_i = 5\times10^{13}$ or 1×10^{14} ; $n_e = 1.2\, n_i$

Implications

a) Shows results are not sensitive to absolute value of n_i in density range of interest

b) A density difference, e.g., due to α-particles, of 20% has < 20% effect on curve, but not negligible. Shifts 50-50 point from 40 keV to ~45 keV

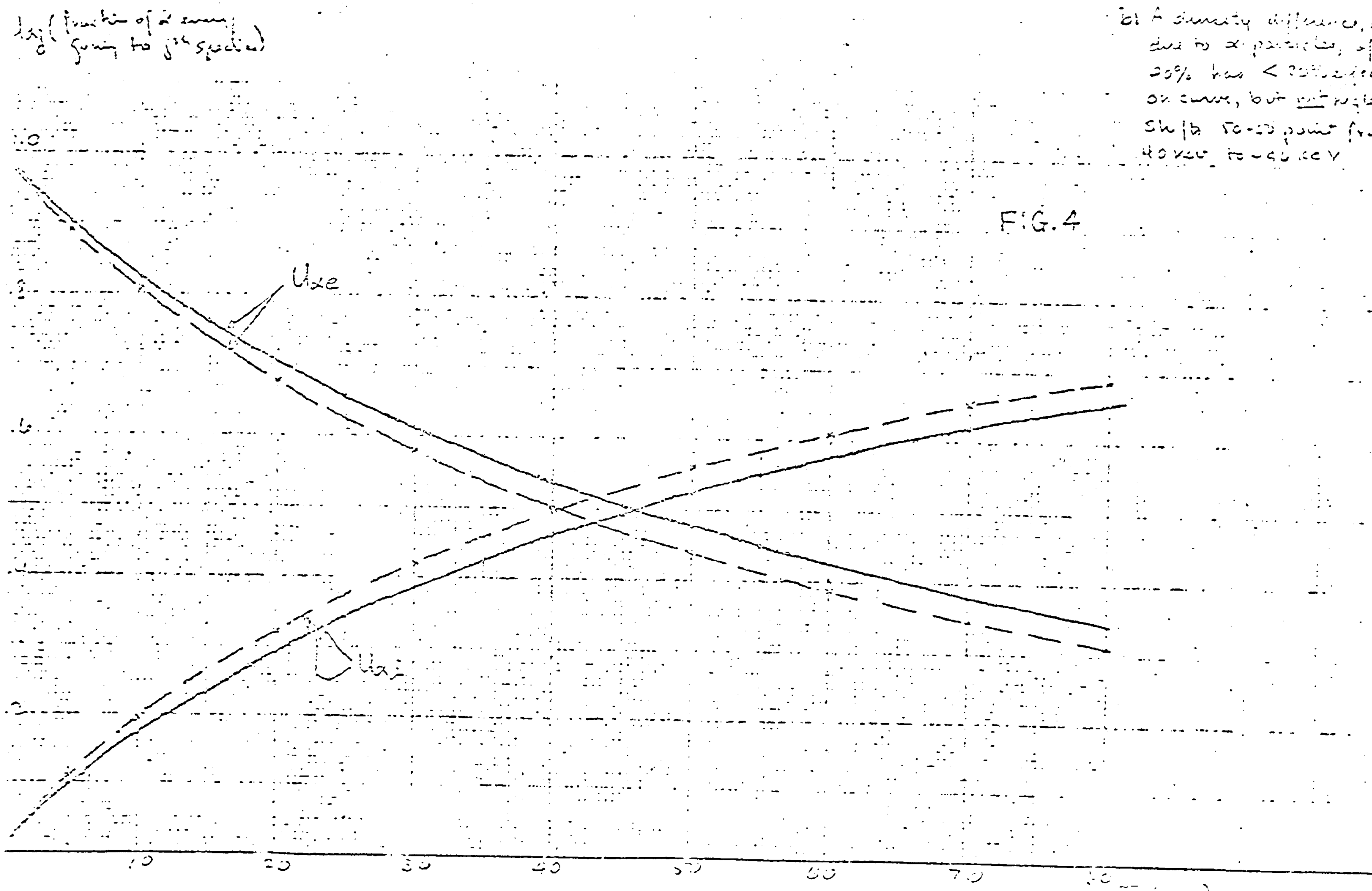

FIG. 4