Use the Semi-Major Axis of the Ellipse to Calculate Hohmann Trajectory Parameters

\[ \Delta v = 5.6 \text{ km/s} \]

Jupiter

\[ \Delta v = 8.8 \text{ km/s} \]

Earth

5.2 AU

Semi-major axis

2a

\[ \frac{T_1}{T_2} = \left( \frac{a_1}{a_2} \right)^{3/2} \]

Use Earth as reference:

\[ T_2 = 1 \text{ year; } a_2 = 1 \text{ AU} \]

Earth-Jupiter:

\[ a = \frac{(1 \text{ AU} + 5.2 \text{ AU})}{2} = 3.1 \text{ AU} \]

\[ T = 0.5 \left( \frac{a}{1 \text{ AU}} \right)^{3/2} \text{ years} = \sim 2.7 \text{ years} \]
## Characteristics of Hohmann Transfers from Earth into the Solar System

<table>
<thead>
<tr>
<th>Planet</th>
<th>Total $\Delta v$ (km/s)</th>
<th>Travel time (year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>-17.2</td>
<td>0.29</td>
</tr>
<tr>
<td>Venus</td>
<td>-5.27</td>
<td>0.40</td>
</tr>
<tr>
<td>Mars</td>
<td>5.56</td>
<td>0.71</td>
</tr>
<tr>
<td>Asteroid belt</td>
<td>11.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Jupiter</td>
<td>14.4</td>
<td>2.7</td>
</tr>
<tr>
<td>Saturn</td>
<td>15.7</td>
<td>6.0</td>
</tr>
<tr>
<td>Uranus</td>
<td>15.9</td>
<td>16</td>
</tr>
<tr>
<td>Neptune</td>
<td>15.7</td>
<td>30</td>
</tr>
<tr>
<td>Pluto</td>
<td>15.5</td>
<td>45</td>
</tr>
</tbody>
</table>
# Hohmann Transfer $\Delta v$ Values

Drop Slightly for Outermost Planets

<table>
<thead>
<tr>
<th>Planet</th>
<th>First $\Delta v$ (km/s)</th>
<th>Second $\Delta v$ (km/s)</th>
<th>Total $\Delta v$ (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturn</td>
<td>10.30</td>
<td>5.44</td>
<td>15.7</td>
</tr>
<tr>
<td>Uranus</td>
<td>11.30</td>
<td>4.66</td>
<td>15.9</td>
</tr>
<tr>
<td>Neptune</td>
<td>11.60</td>
<td>4.05</td>
<td>15.7</td>
</tr>
<tr>
<td>Pluto</td>
<td>11.80</td>
<td>3.69</td>
<td>15.5</td>
</tr>
</tbody>
</table>
Nuclear Thermal Rocket
Trajectories and
Payload Fractions
Rocket Payload Fraction Increases Exponentially with the Inverse of the Exhaust Velocity

Rocket equation

\[
\frac{m_f}{m_i} = \exp\left(\frac{-\Delta v}{v_{ex}}\right)
\]

- **Chemical rocket**
  - \(v_{ex} \approx 4.5 \text{ km/s (I}_{sp}=450 \text{ s).}\)
- **Nuclear thermal rocket**
  - \(v_{ex} \approx 9.0 \text{ km/s (I}_{sp}=900 \text{ s).}\)
- **Nuclear electric rocket**
  - \(v_{ex} \approx 100 \text{ km/s (I}_{sp}=10,000 \text{ s).}\)

\(m_f \equiv \text{final mass}\)
\(m_i \equiv \text{initial mass}\)
\(\Delta v \equiv \text{velocity increment}\)
\(v_{ex} \equiv \text{exhaust velocity}\)
Nuclear Thermal Rockets Enable Larger Payloads than Chemical Rockets Can Provide

Earth-Mars one-way trip: $\Delta v \approx 5.6 \text{ km/s (Hohmann)}$
Considering a Round Trip Improves the Advantage of Nuclear Thermal Rockets Over Chemical Rockets

Earth-Asteroids round trip: $\Delta v \approx 11.7$ km/s (Hohmann, each way)

Outbound: Earth to Asteroids

$$\frac{m_l + m_{p2}}{m_l + m_{p1} + m_{p2}} = \exp\left(\frac{-\Delta v_1}{v_{ex}}\right)$$

Inbound: Asteroids to Earth

$$\frac{m_l}{m_l + m_{p2}} = \exp\left(\frac{-\Delta v_2}{v_{ex}}\right)$$

- Solve these simultaneously to find payload ratio:
  - Chemical rocket: 0.0055
  - Nuclear thermal rocket: 0.074

\[ \frac{m_l}{m_l + m_{p1} + m_{p2}} \]
Nuclear Thermal Rockets Follow Partial Conic-Section Trajectories

- Potential NTR trajectory
  - $a=1.27$, $e=0.231$
  - Total $\Delta v = 8.3$ km/s
  - Total time $\sim 6$ months
- For this trajectory, the rocket travels only part of an ellipse.
- This trajectory is faster than a Hohmann trajectory, but it requires more energy ($\Delta v$).
Accessing the Moons of the Solar System
Launching from Moons Is Relatively Easy, but Planetary Gravitational Fields Can Be Large

<table>
<thead>
<tr>
<th>Moon</th>
<th>Surface escape $\Delta v$ (km/s)</th>
<th>Planet escape $\Delta v$ (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moon (Earth)</td>
<td>2.37</td>
<td>1.44</td>
</tr>
<tr>
<td>Io (Jupiter)</td>
<td>2.56</td>
<td>24.5</td>
</tr>
<tr>
<td>Europa (Jupiter)</td>
<td>2.02</td>
<td>19.4</td>
</tr>
<tr>
<td>Ganymede (Jupiter)</td>
<td>2.74</td>
<td>15.4</td>
</tr>
<tr>
<td>Callisto (Jupiter)</td>
<td>2.44</td>
<td>11.6</td>
</tr>
<tr>
<td>Titan (Saturn)</td>
<td>2.54</td>
<td>7.88</td>
</tr>
</tbody>
</table>
Escape from Planetary “Surfaces” Can Be Difficult

<table>
<thead>
<tr>
<th>Planet</th>
<th>Surface escape $\Delta v$ (km/s)</th>
<th>Sun escape $\Delta v$ (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>4.25</td>
<td>67.7</td>
</tr>
<tr>
<td>Venus</td>
<td>10.4</td>
<td>49.6</td>
</tr>
<tr>
<td>Earth</td>
<td>11.2</td>
<td>42.1</td>
</tr>
<tr>
<td>Mars</td>
<td>5.03</td>
<td>34.1</td>
</tr>
<tr>
<td>Jupiter</td>
<td>59.6</td>
<td>18.5</td>
</tr>
<tr>
<td>Saturn</td>
<td>35.5</td>
<td>13.7</td>
</tr>
<tr>
<td>Uranus</td>
<td>21.4</td>
<td>9.62</td>
</tr>
<tr>
<td>Neptune</td>
<td>23.8</td>
<td>7.69</td>
</tr>
<tr>
<td>Pluto</td>
<td>0.97</td>
<td>6.71</td>
</tr>
</tbody>
</table>
Jupiter’s Satellites Exist in an Extreme Radiation Environment

- 1 Mrad environment at Europa
Travel to the Asteroids

Painting by Denise Watt
How Can Main-Belt Asteroids Best Be Moved to Earth?
Aim at Jupiter and Get a Gravity Assist!

- Jovian fly-by’s can give a very large $\Delta v$ (~10 km/s) to the spacecraft.
- Earth-main asteroid belt
  - $\Delta v \sim 11.7$ km/s
  - Many asteroids require much higher $\Delta v$, because of inclination or eccentricity.
- Asteroid-Jupiter
  - $\Delta v \sim 4.0$ km/s
  - Hohmann travel time = 4.1 years
Many Asteroids Exist
Outside of the Main Belt

- Selected asteroid trajectories

![Diagram showing asteroid trajectories and inclinations](#)

---

\[ \begin{align*}
\text{Toutatis} & : (a \rightarrow 2.51, e \rightarrow 0.634, i \rightarrow 0.47) \quad (* \text{Toutatis} *) \\
\text{1998 FW4} & : (a \rightarrow 2.5, e \rightarrow 0.73, i \rightarrow 3.6) \quad (* \text{1998 FW4} *) \\
\text{Earth} & : (a \rightarrow 1, e \rightarrow 0.017, i \rightarrow 0) \quad (* \text{Earth} *) \\
\text{Ida} & : (a \rightarrow 2.86, e \rightarrow 0.045, i \rightarrow 1.14) \quad (* \text{Ida} *) \\
\text{Icarus} & : (a \rightarrow 1.08, e \rightarrow 0.83, i \rightarrow 22.9) \quad (* \text{Icarus} *) \\
\text{Eros} & : (a \rightarrow 2.71, e \rightarrow 0.22, i \rightarrow 10.8) \quad (* \text{Eros} *) \\
\text{Ceres} & : (a \rightarrow 2.77, e \rightarrow 0.79, i \rightarrow 10.6) \quad (* \text{Ceres} *) \\
\text{Pallas} & : (a \rightarrow 2.77, e \rightarrow 0.23, i \rightarrow 34.8) \quad (* \text{Pallas} *) 
\end{align*} \]
Asteroid 2004FH Recently Approached Very Near the Earth

• Point of closest approach was 49,000 km

Orbital period:
\[ a \approx 0.82 \text{ AU} \]
\[ T = \left(\frac{a}{1 \text{ AU}}\right)^{3/2} \text{ years} \]
\[ = \sim 0.74 \text{ years} \]
\[ = \sim 8.8 \text{ months} \]

Perihelion:
- \[ r = a(1-e) = 0.58 \text{ AU} \]
- \[ v = 44.3 \text{ km/s} \]

Aphelion
- \[ r = a(1+e) = 1.05 \text{ AU} \]
- \[ v = 24.5 \text{ km/s} \]
An Impact of Asteroid 2004FH with Earth Would Not Have Caused Significant Damage

- Point of closest approach was 49,000 km
  - Velocity of 2004 FH at aphelion was 24.5 km/s
  - Earth’s orbital velocity is 30 km/s
  - Velocity difference is $v_{\text{impact}} \approx 5.5$ km/s
- Energy of 2004 FH impact would have been modest
  - $M \approx 3000 \frac{\text{kg}}{\text{m}^3} \times \frac{4}{3} \pi (15 \text{ m})^3 \approx 4 \times 10^7 \text{ kg}$
  - $E = \frac{M}{2} v^2 \approx 2 \times 10^{14} \text{ J}$
  - This energy equals $\approx 0.05$ megatons of high explosive.
Large Asteroids Create Significant Damage

• Asteroids and comets ~ 10 m diameter:
  ➢ Hit Earth approximately once a year.
  ➢ Break up in the atmosphere and do little damage.

• Objects of ~100 m diameter:
  ➢ Hit Earth at ~300 year intervals.
  ➢ Do significant, localized damage.
    □ The 1908 Tunguska object was of this class.

• Objects of ~1 km diameter can produce catastrophic global effects.
Deflecting Asteroids Poses Difficulties

- A $\Delta v$ perpendicular to the asteroid’s orbital plane cause it to oscillate about it original orbit.
  - Why? The asteroid’s energy increases, but farther orbits have slower orbital speeds, so the equilibrium orbit becomes more elliptical.

- The most effective deflection technique is to give the asteroid a parallel $\Delta v$.
  - Adds or subtracts velocity magnitudes, as opposed to vector addition in perpendicular case.
  - Changes semi-major axis and period of orbit.
  - 1 Earth-radius deflection requires $\Delta v \sim 0.1 \text{ m s}^{-1} / \tau \text{ years}$ for an initially circular orbit.
Large Eccentricities Lead to Large $\Delta v$’s for Single-Impulse Deflection of Asteroid Orbits

- Toutatis: $a=2.51$ AU, $e=0.63$, $i=0.5^\circ$

- Speed on an elliptic orbit is

\[
v = \left[ GM \left( \frac{2}{r} - \frac{1}{a} \right) \right]^{1/2}
\]

- Speed of Toutatis as it crosses Earth’s orbit is $v_1 = 37.7$ km/s.
- Earth’s velocity is $v_2 = 29.4$ km/s ($e=0.017$ included).
- Angle between velocity vectors is $38^\circ$.
- Adding vectors gives $\Delta v = 10.8$ km/s.
Large Inclinations Substantially Increase $\Delta v$’s for Single-Impulse Deflection of Asteroid Orbits

- Icarus: $a=1.08$ AU, $e=0.83$, $i=22.9^\circ$

- Velocity of Icarus as it crosses Earth’s orbit is $v_1 = 30.5$ km/s.
- Earth’s velocity is $v_2=29.4$ km/s ($e=0.017$ included).
- Angle between velocity vectors is $60^\circ$.
- Adding vectors in plane gives $\Delta v=27.9$ km/s.
- Inclination of orbits adds another $23.2$ km/s
- Total $\Delta v=51$ km/s.
Hohmann Trajectories Can Give Low $\Delta v$’s for Some Asteroids

• For asteroids with nearly circular orbits of low eccentricity and inclination, $\Delta v$’s can be < 2 km/s.
  - For example, 1999 FA (~330 m diameter) has
    $a=1.078$, $e=0.133$, $i=12$.
      - The corresponding Hohmann trajectory to Earth has $\Delta v=1.8$ km/s.
  - Some asteroid resource literature quotes $\Delta v$’s of 100’s of m/s, but I have not personally verified these values.

• Note: the Moon can gravity assist with ~1 km/s.
Taking Full Advantage of High Exhaust Velocity Requires Optimizing Trajectories

- Power limitations typically mean that high exhaust velocity leads to low thrust.
- Low-thrust trajectories have the rockets powered on for much of the trip.
  - Such operation leads to high efficiency and relatively easy plane changes.

Note: Trajectory is schematic, not calculated.