Space Travel Overview

You Can Get There from Here!

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Lecture 6

Resources from Space

NEEP 533/ Geology 533 / Astronomy 533 / EMA 601

University of Wisconsin

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Outline

- Laws of motion and gravitation
- Interplanetary trajectories
- Rocket physics

Newton's Laws of Motion

- The fundamental laws of mechanical motion were first formulated by Sir Isaac Newton (1643-1727), and were published in his *Philosophia Naturalis Principia Mathematica*.

- Calculus, invented independently by Newton and Gottfried Leibniz (1646-1716), plus Newton's laws of motion are the mathematical tools needed to understand rocket motion.

Newton's Laws of Motion

- Every body continues in its state of rest or of uniform motion in a straight line except insofar as it is compelled to change that state by an external impressed force.
- The rate of change of momentum of the body is proportional to the impressed force and takes place in the direction in which the force acts.

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}$$

- To every action there is an equal and opposite reaction.

Newton's Law of Gravitation

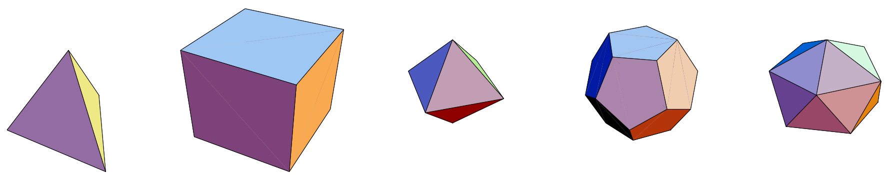
- Every particle of matter attracts every other particle of matter with a force directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

$$\mathbf{F} = \frac{-GMm}{r^2}\,\hat{\mathbf{r}}$$

$G = 6.67\times10^{-11}\ \mathrm{m^3\ s^{-2}\ kg^{-1}}$

is the *gravitational constant.*

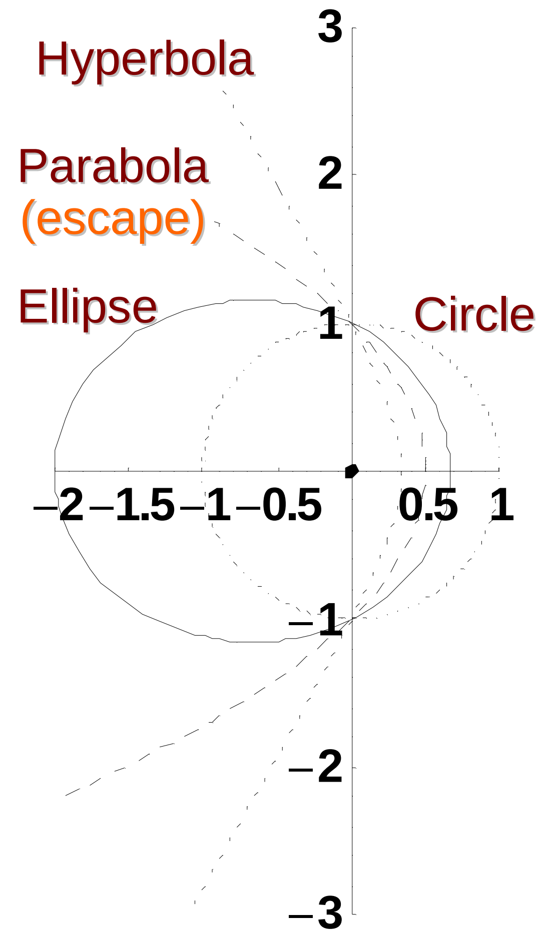
Kepler's Laws of Planetary Motion



- The planets move in *ellipses* with the sun at one focus.
- Areas swept out by the radius vector from the sun to a planet in equal times are equal.
- The square of the period of revolution is proportional to the cube of the semimajor axis.

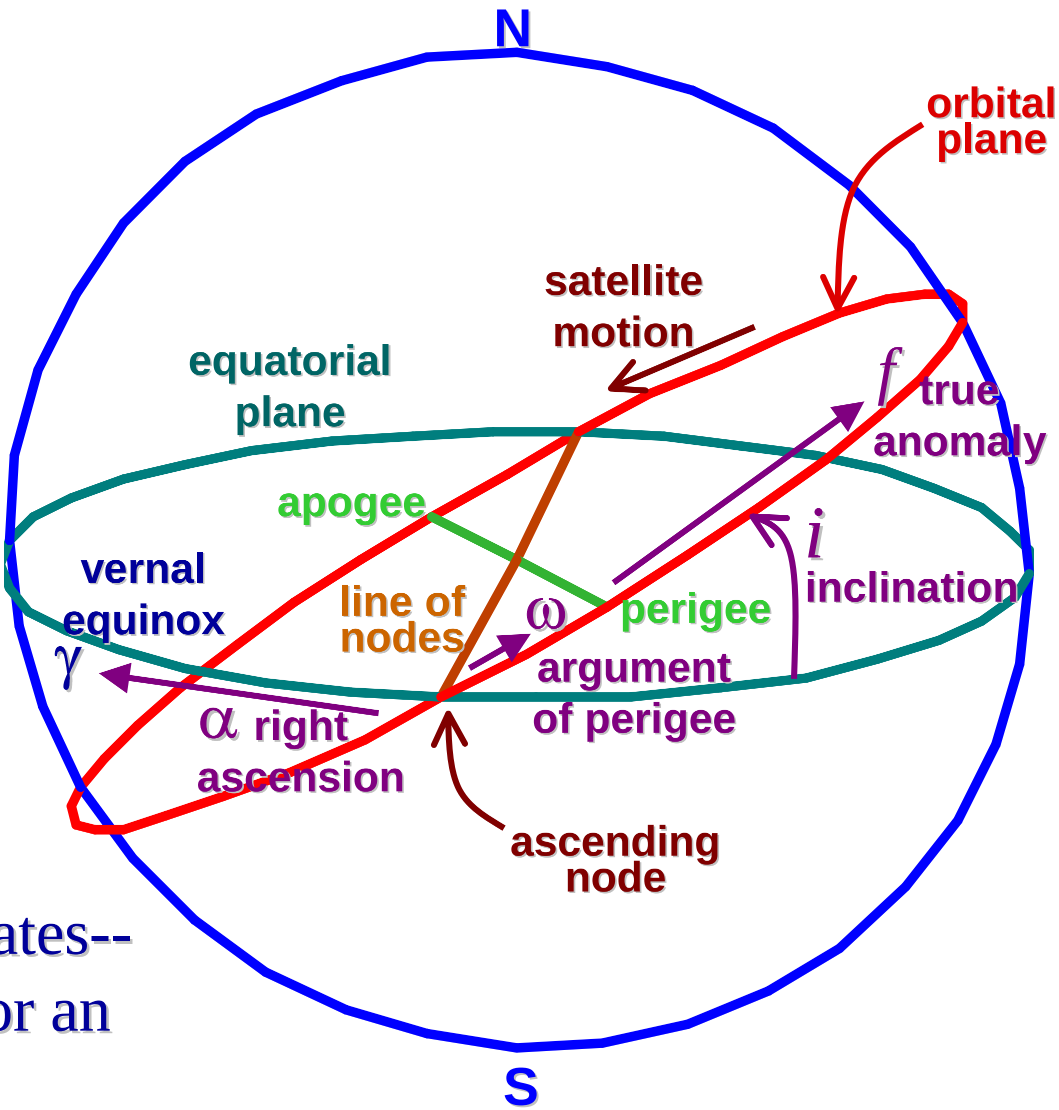
$$T^2 \propto a^3$$

Conic sections



Defining a Celestial Body's Position Requires Six Parameters

- The six parameters appear in purple.
- Two parameters are not shown:
  - *a*, semimajor axis
  - *e*, eccentricity



- Note: these are *equatorial* coordinates--most appropriate for an Earth satellite

Some Useful Orbital Dynamics Formulas

Circular velocity

$$v_{cir} = \left(\frac{GM}{r} \right)^{1/2}$$

Escape velocity

$$v_{esc} = \left(2\frac{GM}{r} \right)^{1/2}$$

Velocity of a body on an elliptical orbit

$$v_{ellipse} = \left[GM \left(\frac{2}{r} - \frac{1}{a} \right) \right]^{1/2}$$

$G = 6.67 \times 10^{-11}\ m^3\ s^{-2}\ kg^{-1}$

Escape from Earth

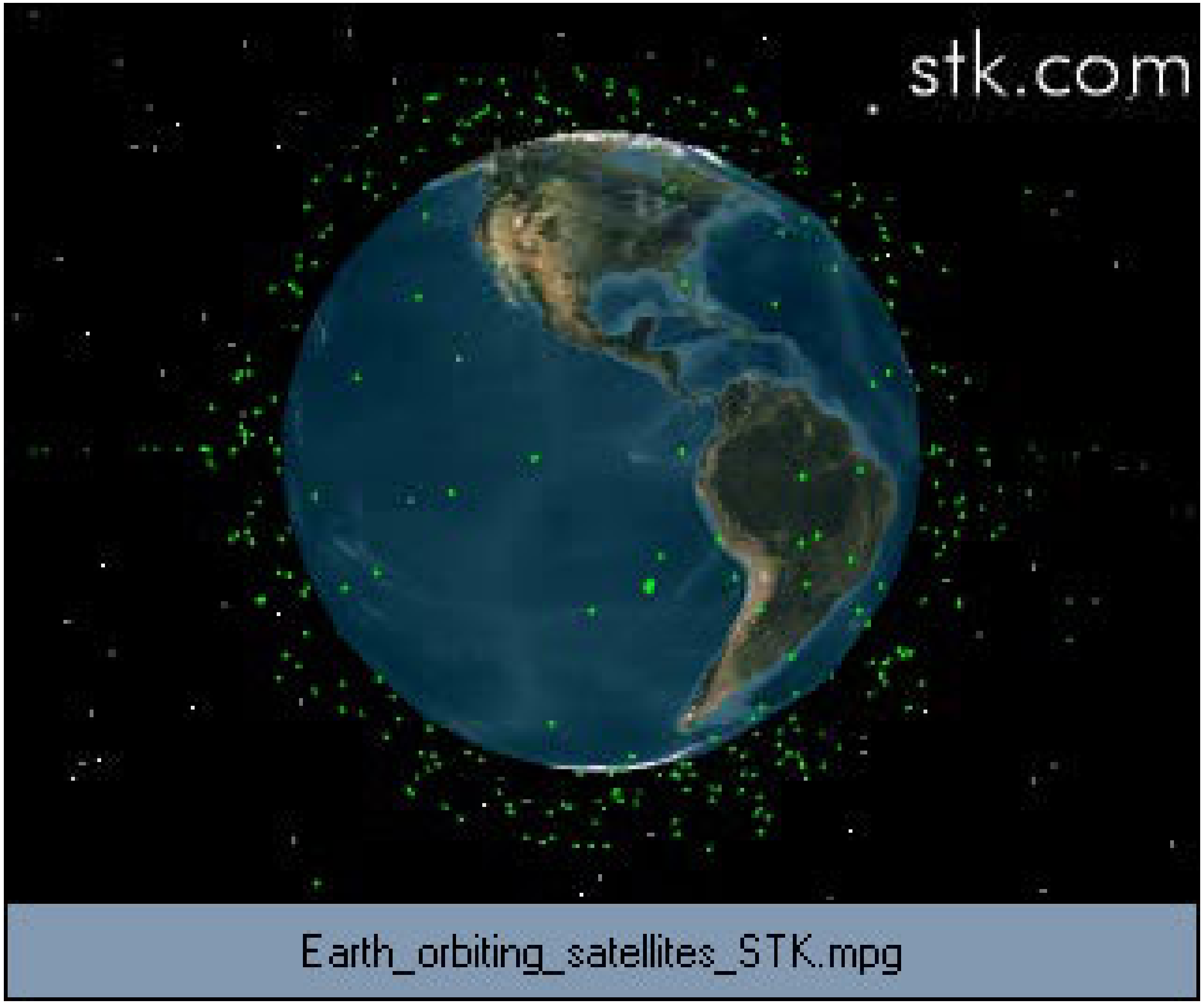
- Earth's mass = 6 x 10^{24} kg
- Earth's average radius = 6380 km = 6.38 x 10^6 m
- $G=6.67\times10^{-11}$ m^3 s^{-2} kg^{-1}

$$v_{esc} = \left(2\frac{GM}{r}\right)^{1/2} = \left(\frac{2\times6.67\times10^{-11}\times6\times10^{24}}{6.38\times10^{6}}\right)^{1/2}$$

$$= 11.2 \text{ km/s}$$

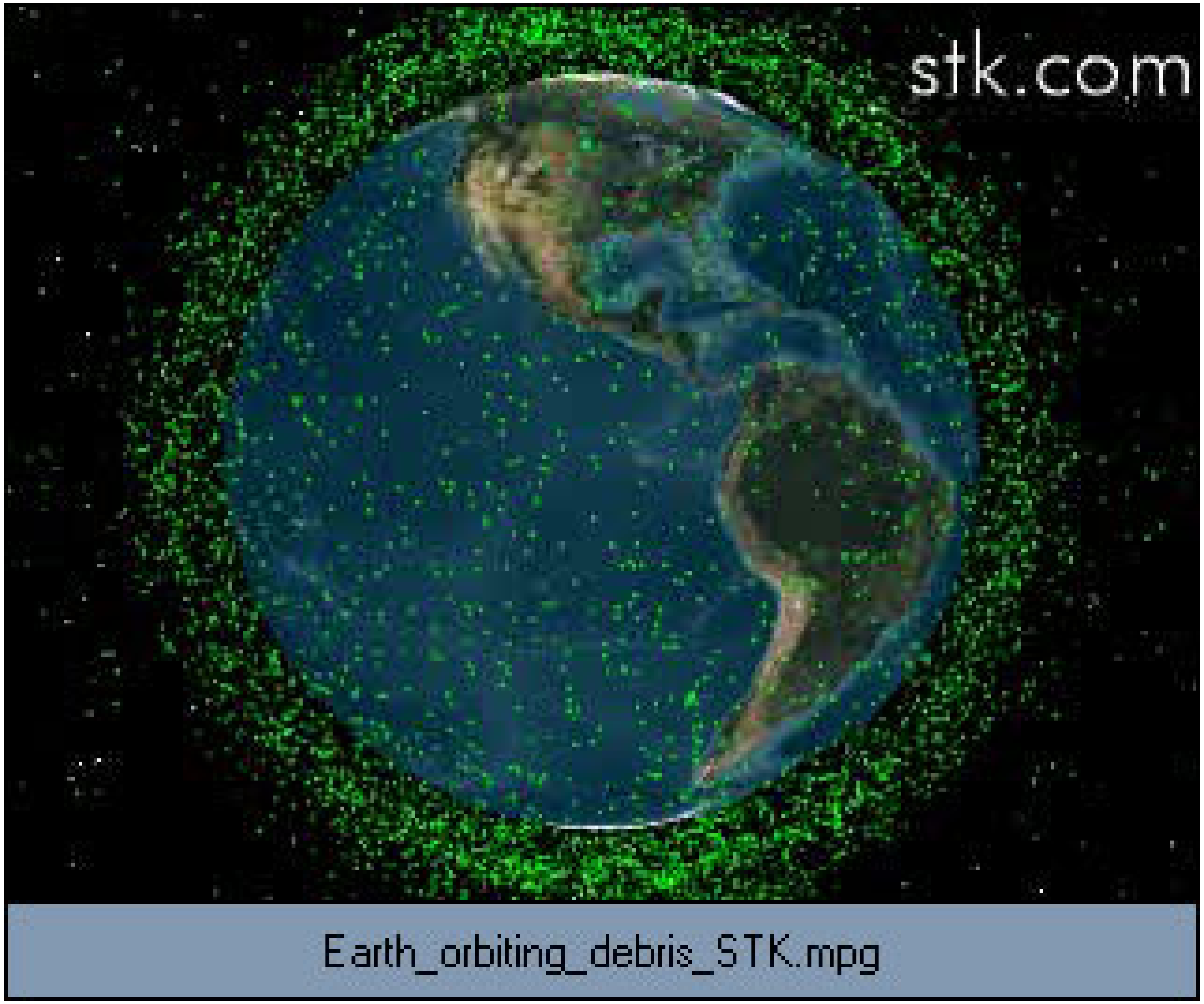
- The actual velocity increment required to escape from Earth is somewhat higher, because of the finite time of flight and atmospheric friction.
- Ground to low-Earth orbit (LEO) requires ~9.5 km/s.

Satellites Orbiting Earth



Note: Satellite Toolkit (STK) software (http://www.stk.com)

Debris in Earth Orbit



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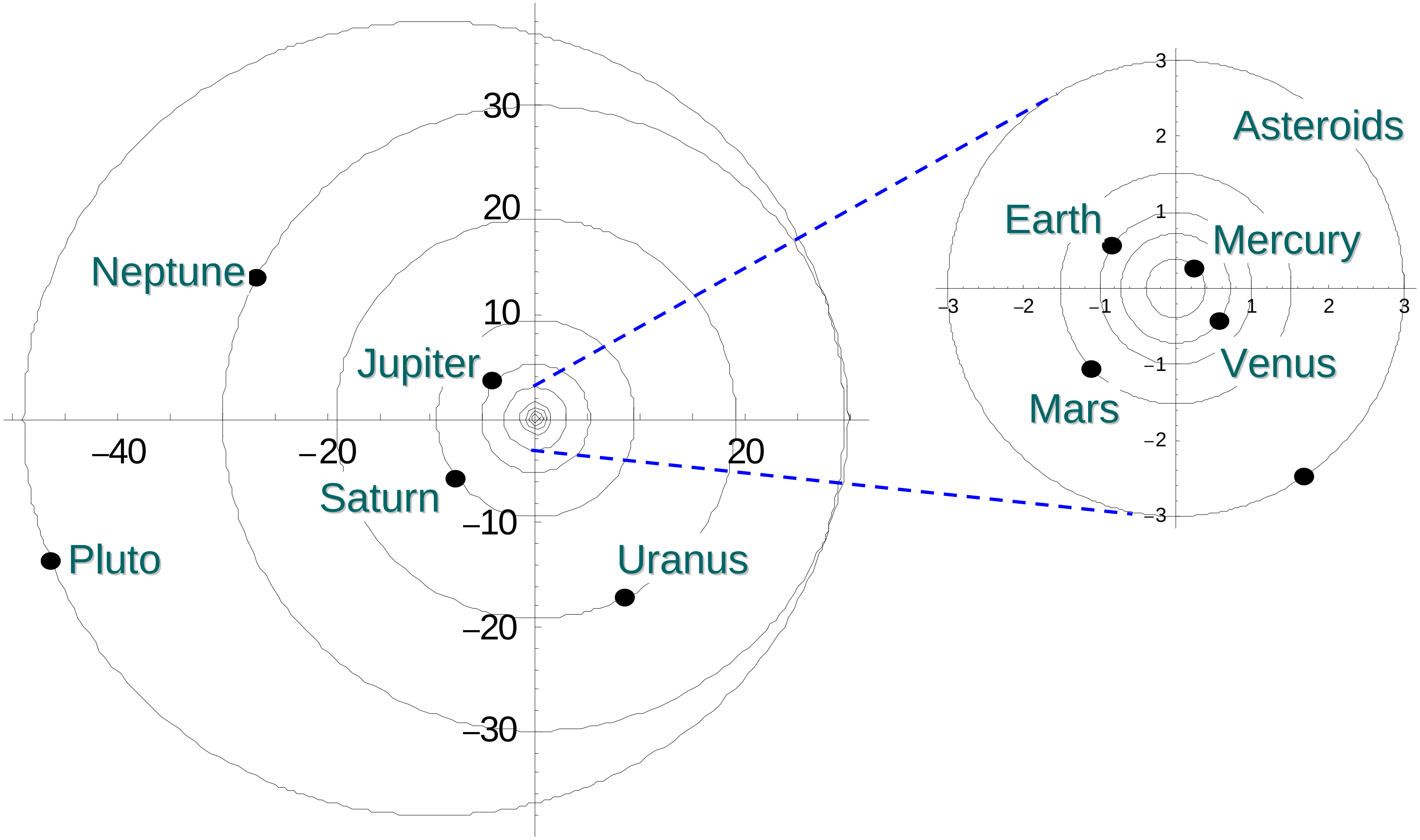
The Solar System

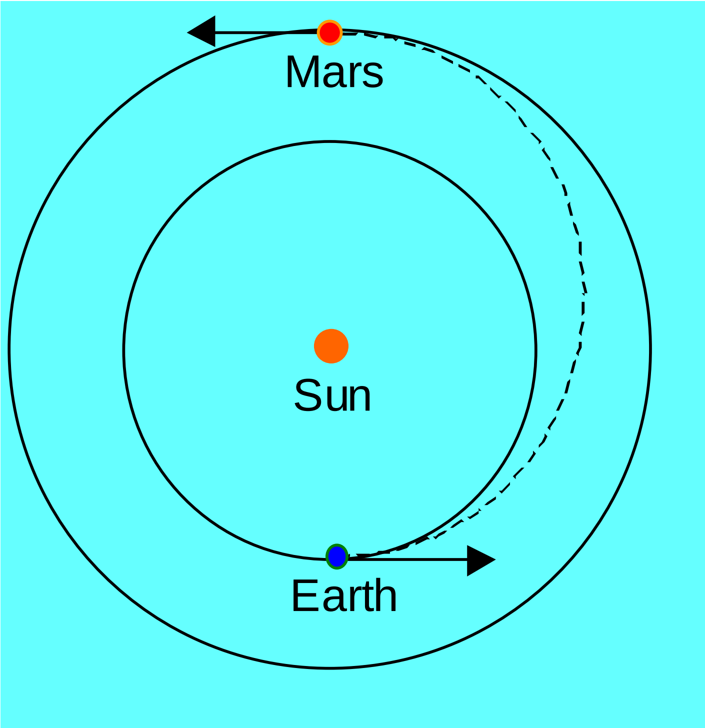


- Distances in astronomical units: 1 AU=1.5x10^8 m

Hohmann's Minimum-Energy Interplanetary Transfer

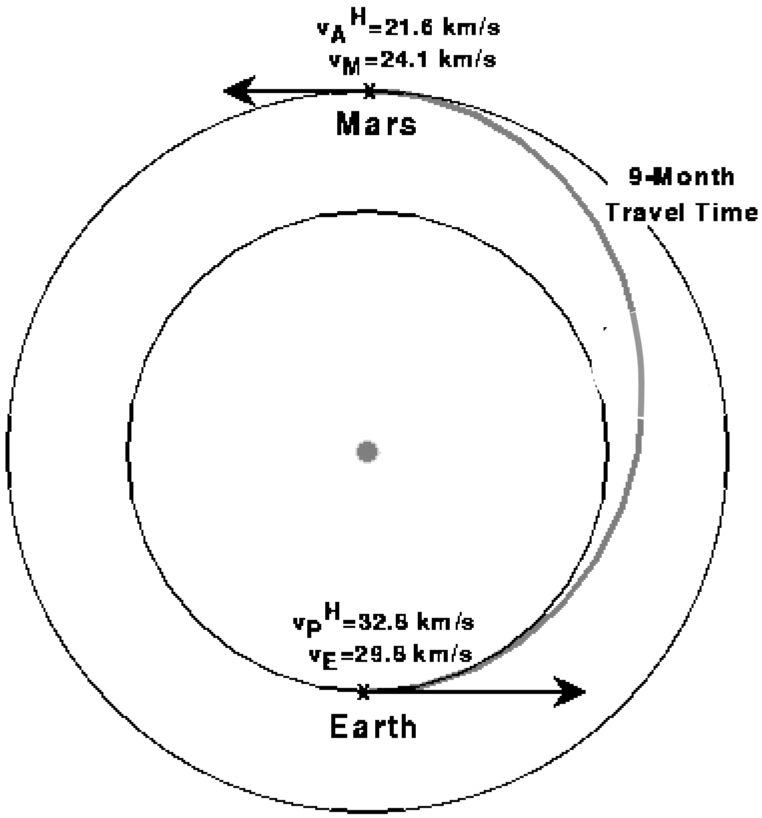
- The minimum-energy, two-impulse transfer between circular orbits is an elliptical trajectory called the Hohmann trajectory, shown at right for the Earth-Mars case.
- A Hohmann transfer consists of three phases:
  1. Large impulse to leave circular orbit
  2. Long coast phase on an elliptical orbit
  3. Large impulse to match target planet's velocity

Earth-Mars Hohmann transfer



Hohmann's Minimum-Energy Interplanetary Transfer Can Be Calculated Easily

- The Hohmann trajectory appears at right for the Earth-Mars case, where the minimum total delta-v expended is 5.6 km/s.
- The values of the energy per unit mass on the circular orbit and Hohmann trajectory are shown, along with the velocities at perihelion (closest to Sun) and aphelion (farthest from Sun) on the Hohmann trajectory and the circular velocity in Earth or Mars orbit.



- The sum of these differences between the velocities is the total delta-v value used in the rocket equation.

Hohmann Transfer Time

- Kepler's third law, $T^2 \propto a^3$, can be used to calculate the time required to traverse a Hohmann trajectory by raising to the 3/2 power the ratio of the semimajor axis of the elliptical Hohmann orbit to the circular radius of the Earth's orbit and dividing by two (for one-way travel):

$$\frac{T_1}{T_2} = \left(\frac{a_1}{a_2}\right)^{3/2}$$

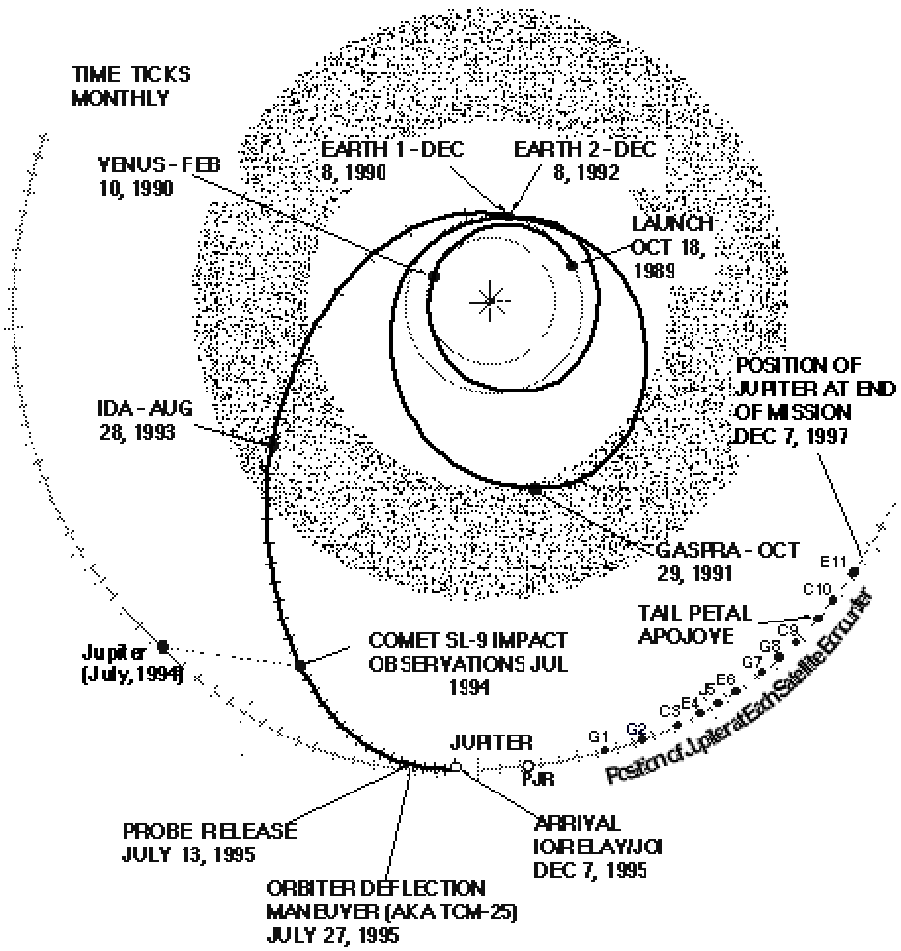
- For example, call the travel time for an Earth-Mars trip T and the semimajor axis of the Hohmann ellipse a:

a = (1 AU + 1.5 AU)/2 = 1.25 AU

T = 0.5 (a / 1 AU)$^{3/2}$ years
= ~0.7 years = ~8.4 months

Many Spacecraft Trajectories Can Be Approximated by Hohmann Orbits

Galileo Spacecraft's Trajectory

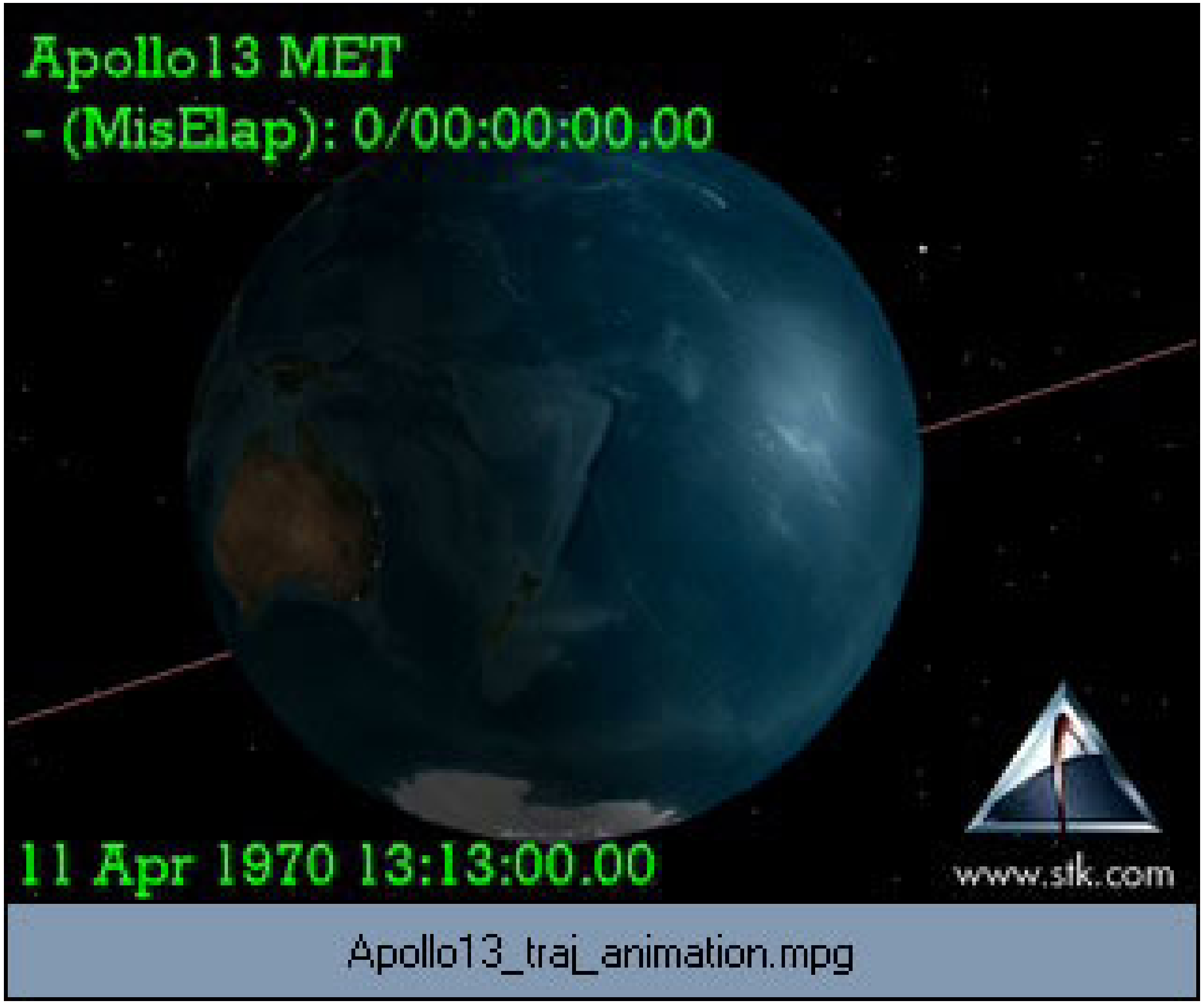


a = (1 AU + 5.2 AU)/2
= 3.1 AU

T = 0.5 (a / 1 AU)$^{3/2}$
years = ~2.7 years

The final phase (nearly elliptical) of Galileo's trip to Jupiter took ~3 years.

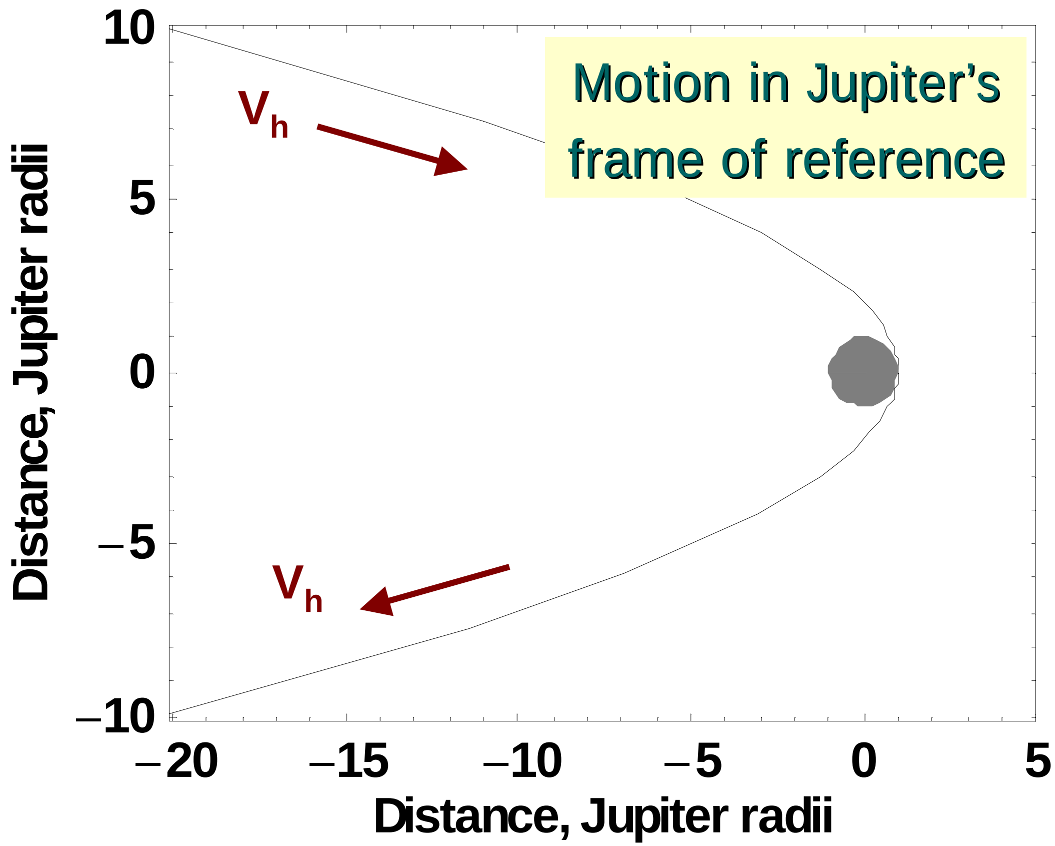
Apollo Mission Trajectories Were Figure Eights



Note: Satellite Toolkit (STK) software (http://www.stk.com)

Gravity Assists Enable or Facilitate Many Missions

- In the planet's frame of reference, a spacecraft enters and leaves the sphere of influence with a so-called hyperbolic excess velocity, v_h, equal to the vector sum of its incoming velocity and the planet's velocity.



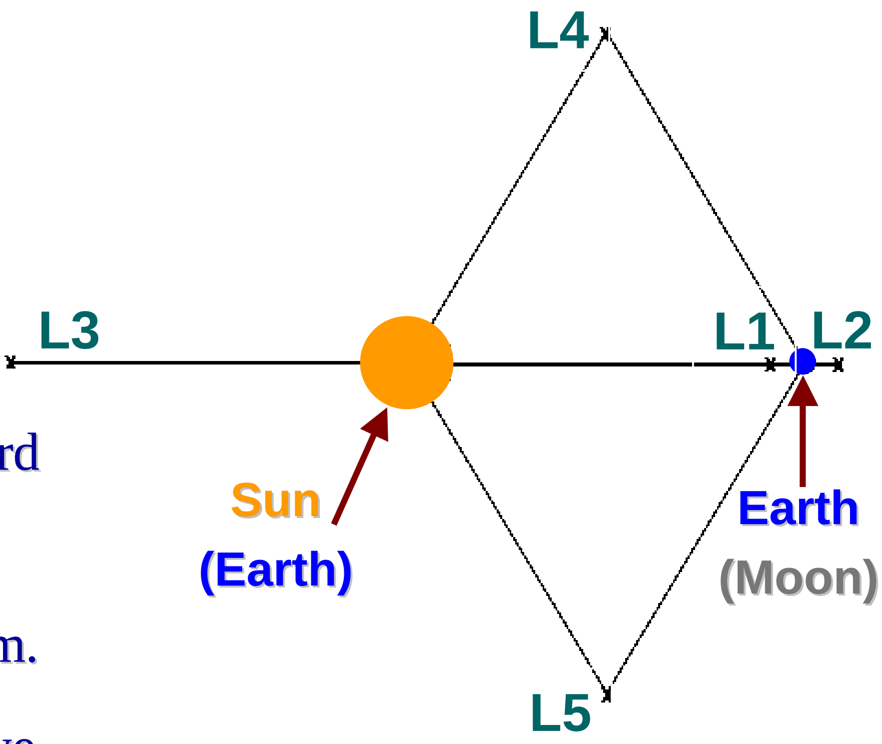
Gravity Assists
Enable or Facilitate Many Missions

- In the planet's frame of reference, the direction of the spacecraft's velocity changes, but not its magnitude. In the spacecraft's frame of reference, the net result of this trade-off of momentum is a small change in the planet's velocity and a very large delta-v for the spacecraft.

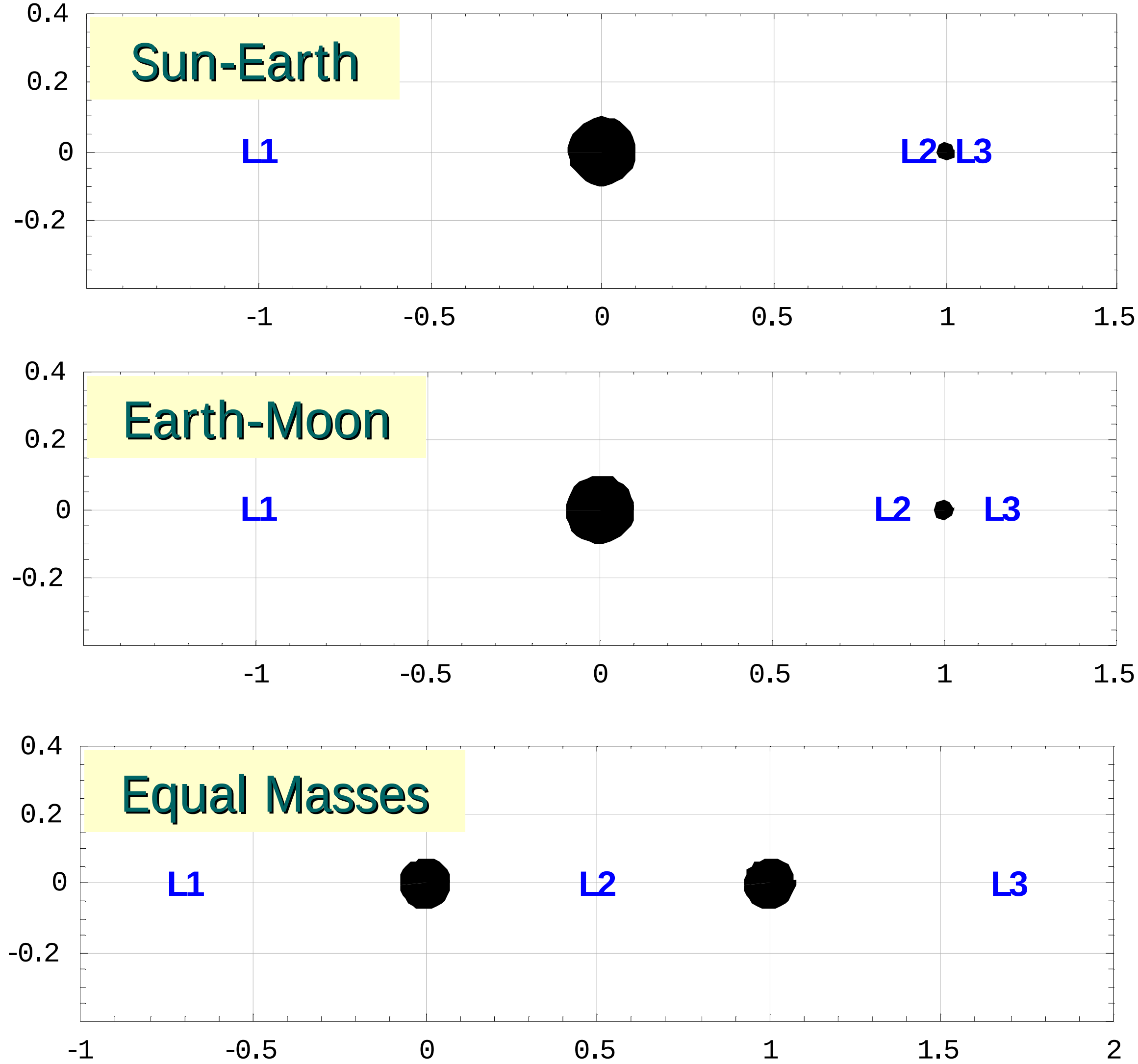
- Starting from an Earth-Jupiter Hohmann trajectory, a flyby of Jupiter at one Jovian radius has these parameters:
  - Hyperbolic excess velocity v_h~5.6 km/s
  - Angular change in direction ~160^o
  - Δv~13 km/s
  - Final velocity = 13+5.6 km/s=18.6 km/s (greater than Solar-System v_{esc})

Lagrange Points are Equilibrium Positions in a Three-Body System

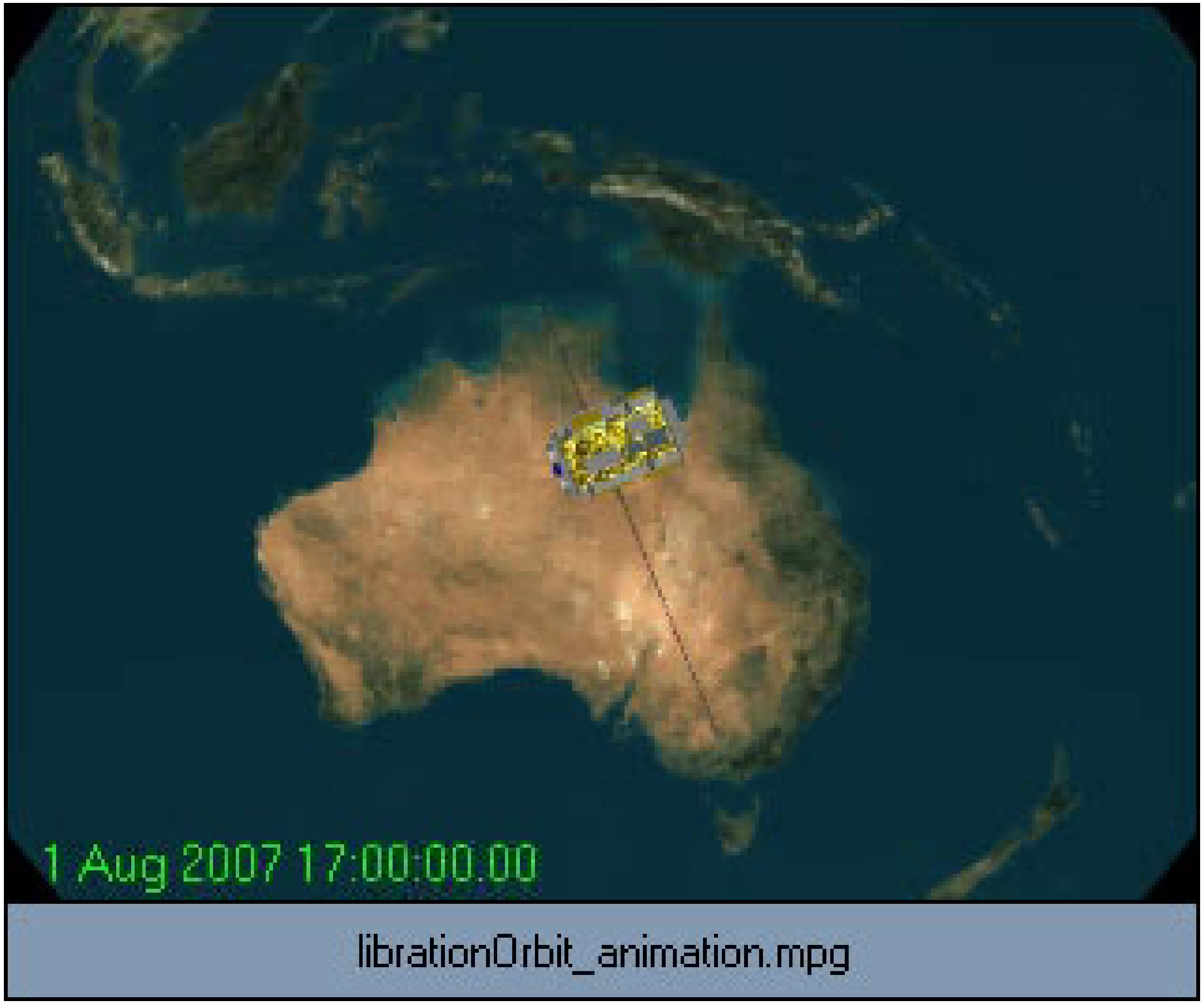
- The points L1, L2, and L3 lie on a straight line through the two main bodies and are points of unstable equilibrium. That is, a small perturbation will cause the third body to drift away.
- The L4 and L5 points are at the third vertex of an equilateral triangle formed with the other two bodies; they are points of stable equilibrium.
- In general, the three bodies can have arbitrary masses.



Lagrange Points for the Sun-Earth, Earth-Moon, and Equal-Mass Systems



Libration Orbits Are One Way to Utilize Lagrange Points



Note: Satellite Toolkit (STK) software (http://www.stk.com)

Tsiolkowsky's Rocket Equation

- Conservation of momentum leads to the so-called rocket equation, which shows the dependence of payload fraction on exhaust velocity.
- It assumes short impulses with coast phases between them, such as used for chemical and nuclear-thermal rockets.
- First derived by Konstantin Tsiolkowsky in 1895 for straight-line rocket motion with constant exhaust velocity, it also applies to elliptical trajectories with only initial and final impulses.

Konstantin Tsiolkowsky

Tsiolkowsky's Rocket Equation

- Assuming constant exhaust velocity, conservation of momentum for a rocket and its exhaust leads to

$$\frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt} = 0$$

$$\Rightarrow \frac{dm}{m} = \frac{-dv}{v_{ex}}$$

$$\Rightarrow \frac{m_f}{m_i} = \exp\left(\frac{-\Delta v}{v_{ex}}\right)$$

$m_f \equiv$ final mass

$m_i \equiv$ final mass

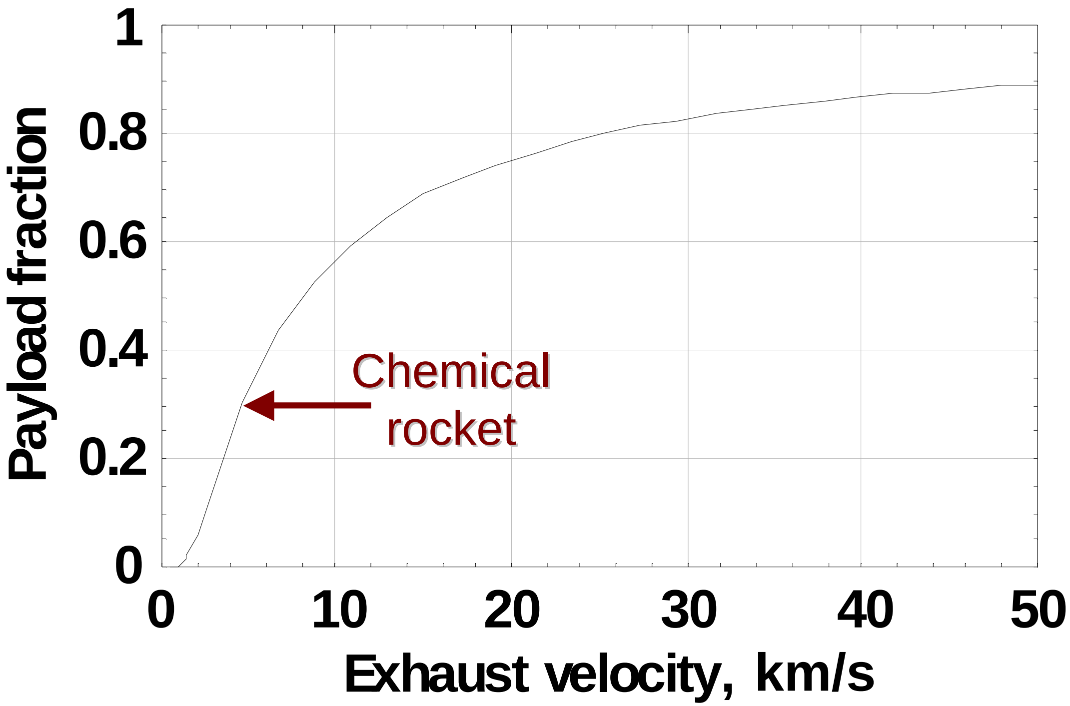
$\Delta v \equiv$ velocity increment

$v_{ex} \equiv$ exhaust velocity

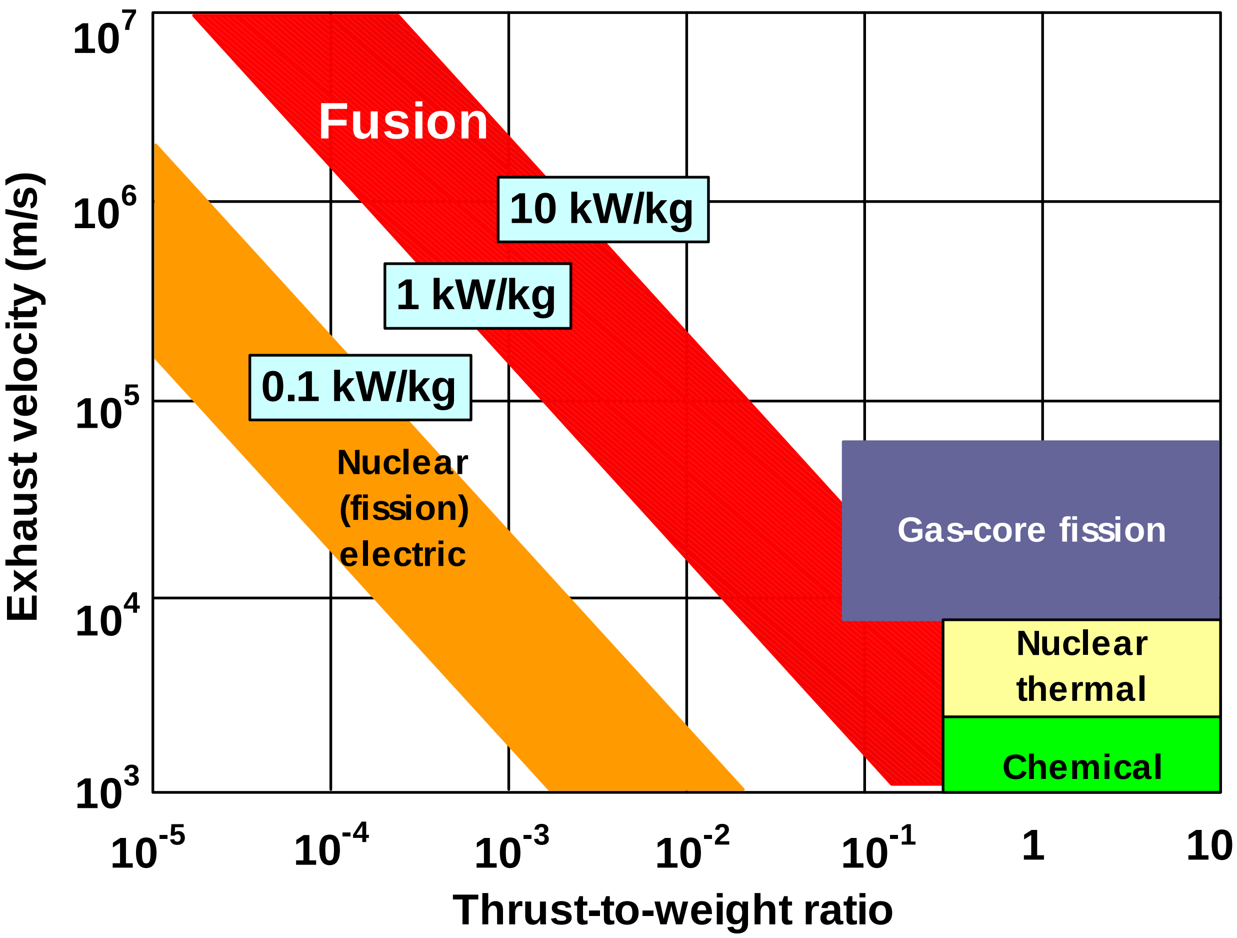
High Exhaust Velocity Gives Large Payloads

- This plot of the rocket equation shows why high exhaust velocity historically drives rocket design: payload fractions depend strongly upon the exhaust velocity.

Earth-Mars one-way trip: Δv~5.6 km/s



Comparison of Propulsion Options

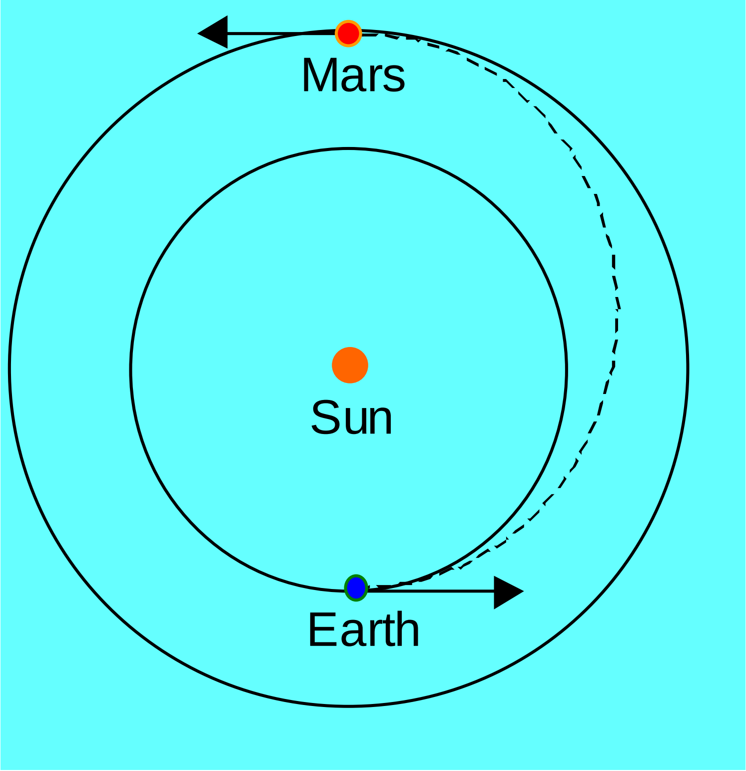


How Do Separately Powered Systems Differ from Chemical Rockets?

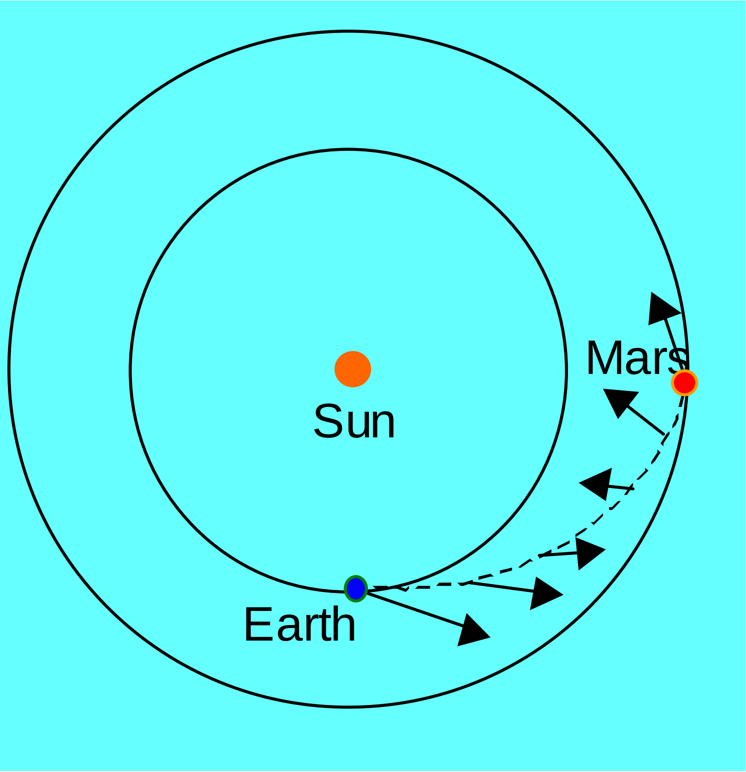
- Propellant not the power source.
- High exhaust velocity ($\geq 10^5$ m/s).
- Low thrust ($\leq 10^{-2}$ m/s} $\equiv 10^{-3}$ Earth gravity) in most cases.
- Thrusters typically operate for a large fraction of the mission duration.
- High-exhaust-velocity trajectories *differ fundamentally* from chemical-rocket trajectories.

Taking Full Advantage of High Exhaust Velocity Requires Optimizing Trajectories

Chemical rocket trajectory (minimum energy)



Fusion rocket trajectory (variable acceleration)



Note: Trajectories are schematic, not calculated.