

- **Relative frequency at which molecules of different species pass through a small hole is proportional to the speed of the molecule.**

Hence the ratio of $\frac{^{35}\text{U}}{^{38}\text{U}}$ on the low pressure side is greater than the $\frac{^{35}\text{U}}{^{38}\text{U}}$ ratio on the high pressure side.

For $^{235}\text{UF}_6$ and $^{238}\text{UF}_6$, maximum α is;

$$\alpha = \sqrt{\frac{(238 + 6 \sum 19)}{235 + 6 \sum 19}}$$

$$\alpha = 1.004289$$

(more realistic value is 1.003 due to down stream back pressure and leaks)

- **Low value of α requires a very large number of steps**

Figures (2)

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- **What is the Barrier? (mostly classified)**
 - **Very thin and delicate**
 - **100's of millions of holes/cm³**
 - **≈ 20 Å diameter hole**
 - **Must exclude organic materials and air to avoid plugging**
 - **Materials reported to be sintered Ni and anodized Al**

More complete analysis of U enrichment, see;

Benedict, M, T.H. Pigford, and H.M. Levi, "Nuclear Chemical Engineering", 2nd ed., McGraw Hill Book Company, NY, 1981

Villani, S, ed., "Uranium Enrichment", Springer Verlag, NY, (1979)

Cochran, R. C. and N. Tsoulfanidis, "The Nuclear Fuel Cycle: Analysis and Management", American Nuclear Society, La Grange Park, IL, (1990)

Important Variables and Equations

**kg U Feed (F) = kg Enriched U Product (P)
+ kg U Waste (W)**

$$x_f^F F = x_p^P P + x_w^W W$$

where x_f = wt. fraction of ^{35}U in feed

x_p = wt. fraction of ^{35}U in product

x_w = wt. fraction of ^{35}U in waste

(Note: F, P, & W could be in kg or kg/unit time)

- 2 eqs. and 6 variables, F, P, W, x_f , x_p , x_w
- Trick is to solve for 2 in terms of the other 4!

1.) $x_f = 0.711\%$ now (1996)

2.) x_p = as requested by the customer

Table 1.1 and 2 figures

3.) x_w = could be between 0.2 and 0.3 %, currently in the U.S. is 0.3%

4.) P = mass of desired product

One can solve the equations above;

$$F = P \frac{(x_p - x_w)}{(x_f - x_w)}$$

$$W = P \frac{(x_p - x_f)}{(x_f - x_w)}$$

Feed factor is defined as;

$$\frac{F}{P} = \frac{(x_p - x_w)}{(x_f - x_w)}$$

Waste factor ;

$$\frac{W}{P} = \frac{F}{P} - 1$$

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How Much Energy is Required to Reach a Given Enrichment?

Define Separative Work Unit (SWU) as;

"resource required to perform the enrichment to the desired level of x_p given x_f and x_w . For gaseous diffusion this is equivalent to electrical energy"

of SWU's produced by an enrichment plant

during a time period t ,

$$SWU = \left[P \sum V(x_p) + W \sum V(x_w) - F \sum V(x_f) \right] t$$

The quantity $V(x_i)$ is called the separation potential and is given by;

$$V(x_i) = (2x_i - 1) \ln \left[\frac{x_i}{1 - x_i} \right]$$

where $i = f, p, w$

We normally quote SWU's per unit of product ($P \cdot t$) where P is feed rate.

$$S = \frac{SWU}{P \sum t} = V(x_p) + \left(\frac{W}{P} \right) \sum V(x_w) - \left(\frac{F}{P} \right) \sum V(x_f)$$

$$S = \text{"SWU" factor, } \frac{SWU}{kg}$$

Figure 3.6 plus Schematic

Problem -1

a.) What is the number of kgs of natural U that has to be provided as feed in an enrichment plant if one requests 30,000 kg of U enriched to 3% in ^{35}U ? Assume the tails assay is 0.2%.

b.) What is the number of SWU's needed for separation?

$$\frac{F}{P} = \frac{(3 - 0.2)}{(0.711 - 0.2)} = 5.479 \quad \frac{\text{kg feed}}{\text{kg product}}$$

Total feed is then;

$$F = 30,000 \cdot 5.479 = 164,370 \text{ kg U feed}$$

$$\begin{aligned} b.) \quad V(x_f) &= (2 \sum 0.00711 - 1) \ln \left[\frac{0.00711}{1 - 0.00711} \right] \\ &= 4.869 \end{aligned}$$

$$V(x_w) = (2 \sum 0.002 - 1) \ln \left[\frac{0.002}{1 - 0.002} \right]$$

$$= 6.188$$

$$\begin{aligned} V(x_p) &= (2 \sum 0.03 - 1) \ln \left[\frac{0.03}{1 - 0.03} \right] \\ &= 3.268 \end{aligned}$$

$$S = 3.268 + (5.479 - 1)(6.188) - (5.479)(4.869) = 4.307$$

Hence the total number of SWU's is then;

$$30,000 \text{ kg} \cdot 4.307 \text{ SWU/kg} = 129,210 \text{ SWUs}$$

Table 1.1
Summary of Fuel Characteristics in Fission Power Plants
(After Benedict-1981)

	BWR	PWR	HTGR	CANDU	LMFBR
MW(e)	1100	1100	330	508	1200
Thermal Eff.- %	33	33	39	30	40
Assembly Geometry	8x8 9x9	17x17	Hexagonal	Cylindrical	Hexogona l
Assembly Length-m	3.8	3.7	0.78	0.5	1
# of Assemblies	590	180	1482 -6/column	4680 12/channel	360
Core Ht-m	3.8	3.7	4.75	5.95	1
kg Fuel /assembly	270	600	22	37	80
Tot.tonne fuel in core	138	90-100	0.77- ²³⁵ U 16- ²³² Th ^a	105	29
BU-MWd per MTU	30,000	30,000	100,000	8,000	100,000
≈% Fuel Replaced/y	25	33	18	continuous	Varied
Enrichment- %	1.8	2.8	93	0.711	15-20 ²³⁹ Pu
Power Density- (kW/liter)	54	100	8	12	280
Linear Ht Rate-kW/m)	19	17	8	26	29
a-Initial Loading					

Problems Due Wed., Sept. 27, 1997

1.) An enrichment plant has a throughput of 32,000 kgU/day and produces 26,000 kgU as tails. What is the enrichment of the product if the feed is natural U and the tails are 0.25%?

2.) A gaseous diffusion method has been proposed to produce BF_3 enriched to 90% in B^{10} . How many kgs of BF_3 feed (natural B) are needed to produce 1 kg of B^{10} with 8% tails?

3.) Calculate the natural U feed and SWU factors 1 billion years into the future. Assume tails of 0.15% and 3% enriched product;

$$t_{1/2} (^{35}\text{U}) = 7.1 \times 10^8 \text{ y,}$$

$$t_{1/2} (^{38}\text{U}) = 4.51 \times 10^9 \text{ y.}$$

4.) Assuming that the price per SWU is \$80 and the cost of conversion is \$4/kgU, what is the price of the U_3O_8 (\$/lb U_3O_8) beyond which it will cost less to enrich the already mined, purified, and converted (to UF_6) tails that contain 0.2% ^{35}U rather than mine new U?

[Assume the product will be 3% enriched in either case and the new tails will be 0.1% (when the old tails are enriched). Tails stored as UF_6 cost nothing.]