Problem 11.2

In General,

$$\mu_{O_2} = \text{constant (with radius)}$$

$$\overline{G_{O_2}} = RT \ln \mu_{O_2} = \overline{H_{O_2}} - T\,\overline{S_{O_2}}$$

$$= \text{-65} + 120\,RTx \qquad \text{kcal/mole}$$
$$(\text{T in units of } 1000\,°K)$$

$$\text{or, } x = \frac{1}{120} \ln \mu_{O_2} + \frac{65}{RT} \qquad\qquad 1.)$$

and

$$\frac{T - T_s}{T_o - T_s} = 1 - \left(\frac{r}{R}\right)^2$$

a.)

$$\overline{x} = \frac{1}{R^2} \int_0^R 2\,r \cdot x(r) \cdot dr$$

$$= \int \left(\frac{r}{R}\right)^2, d\left(\frac{r}{R}\right)^2 = \int \frac{2r}{R^2}\,dr$$

Then, $T = T_s - (T_o - T_s)(1 -)$

$$\bar{x} = \int_0^1 x(\ )d$$

$$\bar{x} = \frac{1}{120} \int_0^1 \left[\ln \ _{O_2} + \frac{65}{RT_s \ \left[1 + \frac{T_0 - T_s}{T_s}(1 - \ )\right]} \right] d$$

Solving;

$$\ln \ _{O_2} = 120\bar{x} - \frac{65}{R(T_0 - T_s)} \ln \frac{T_0}{T_s}$$

or since $\bar{x} = 0.05$

$$\ln \ _{O_2} = 120 \cdot 0.05 - \frac{65}{1.98(1.5)} \ln(2.5)$$

$$= -14.1$$

$$_{O_2} = 7.5 \times 10^{-7} \ atm \quad (6 \times 10^{-4} \ torr)$$

b.) to plot x(r), use eq. 1

$$x = \frac{1}{120} \left[\ln \ _{O_2} + \frac{65 \cdot 1}{RT} \right]$$

$$= -\frac{14.1}{120} + \frac{65}{1.98 \cdot 120 \cdot T}$$

$$= -0.1175 + 0.274 \ \frac{1}{T}$$

Use x from above with parabolic temperature Profile to get x(R)

c.) @ r=R

$$\overline{G_{O_2}} = RT_s \ln \, {}_{O_2}$$

$$= 1.98 \cdot 1.00 \cdot (-14.1)$$

$$= -28 \text{ kcal}$$

in chapter 12 we will see that

$$Zr + O_2 \text{ (g)} \rightarrow ZrO_2$$

$$1000°K \quad -220 \text{ kcal/mole}$$

$$\overline{G_{O_2}^{\circ}} > \overline{G_{Zr}^{\circ}} \quad \text{cladding oxidized. We shall}$$

see later that oxide layer prevents oxidation from occurring too fast.