

Problem 11.2

In General,

$$p_{O_2} = \text{constant (with radius)}$$

$$\begin{aligned} \overline{G}_{O_2} &= RT \ln p_{O_2} = \overline{H}_{O_2} - T \overline{S}_{O_2} \\ &= -65 + 120 RTx \quad \text{kcal/mole} \\ &\quad \text{(T in units of 1000 } ^\circ\text{K)} \end{aligned}$$

$$\text{or, } x = \frac{1}{120} \ln p_{O_2} + \frac{65}{RT} \quad 1.)$$

and

$$\frac{T - T_s}{T_o - T_s} = 1 - \frac{r^2}{R^2}$$

a.)

$$\begin{aligned} \overline{x} &= \frac{1}{R^2} \int_0^R 2r \cdot x(r) \cdot dr \\ &= \frac{r^2}{R^2}, d = \frac{2r}{R^2} dr \end{aligned}$$

$$\text{Then, } T = T_s - (T_o - T_s) \left(1 - \frac{r^2}{R^2}\right)$$

$$\bar{x} = \int_0^1 x(r) dr$$

$$\bar{x} = \frac{1}{120} \ln p_{O_2} + \frac{65}{RT_s \left(1 + \frac{T_0 - T_s}{T_s}\right)} d$$

Solving;

$$\ln p_{O_2} = 120\bar{x} - \frac{65}{R(T_0 - T_s)} \ln \frac{T_0}{T_s}$$

or since $\bar{x} = 0.05$

$$\begin{aligned} \ln p_{O_2} &= 120 \cdot 0.05 - \frac{65}{1.98(1.5)} \ln(2.5) \\ &= -14.1 \end{aligned}$$

$$p_{O_2} = 7.5 \times 10^{-7} \text{ atm } (6 \times 10^{-4} \text{ torr})$$

b.) to plot $x(r)$, use eq. 1

$$\begin{aligned} x &= \frac{1}{120} \ln p_{O_2} + \frac{65 \cdot 1}{RT} \\ &= -\frac{14.1}{120} + \frac{65}{1.98 \cdot 120 \cdot T} \\ &= -0.1175 + 0.274 \frac{1}{T} \end{aligned}$$

Use x from above with parabolic temperature Profile to get $x(R)$

c.) @ $r=R$

$$\begin{aligned} \overline{G}_{O_2} &= RT_s \ln p_{O_2} \\ &= 1.98 \cdot 1.00 \cdot (-14.1) \\ &= -28 \text{ kcal} \end{aligned}$$

in chapter 12 we will see that



1000°K -220 kcal/mole

$\overline{G}_{O_2}^\circ > \overline{G}_{Zr}^\circ$ cladding oxidized. We shall see later that oxide layer prevents oxidation from occurring too fast.