<u>Problem</u> <u>10.2</u>

Assume: • ***Fast reactor fuel pin characteristics as in Table 10.2***
- ***Gap closure***
- ***Only He fill gas***
- ***linear power, R , is symmetric***
- $\int kdT$ ***from eq. 10.20***
- $h_{coolant} = \dfrac{12 Watts}{cm^2 \circ C}$
- $k_c = 0.22 \dfrac{Watts}{cm \circ C}$

Wanted: T_o ***and*** T_s ***@ midplane (before core restructuring)***

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Schematic

(Diagram)

See Hard Copy of Class Notes

$$T_{Na}\ (\text{midplane}) = 470 + \frac{650 - 470}{2} \approx 560°C$$

Na Film Temperature Drop
(at midplane)

q_{Na} = heat flux from clad surface to coolant

$$q_{Na} = \frac{P}{2\pi R_c} = h_{Na}(T_{co} - T_{Na})$$

or

$$T_{co} - T_{Na} = \frac{550\,\frac{Watts}{cm}}{2\pi \bullet (0.315\,cm) \bullet 12\,\frac{Watts}{cm^2\,{}^\circ C}}$$

or,

$$T_{co} = 560 + 23 = 583°C$$

Temperature Drop Through Clad

q_c = heat flux through cladding (ave)

$$q_c = \frac{P}{2\pi\left[R_c - \left(\frac{t_c}{2}\right)\right]} = \left(\frac{k_c}{t_c}\right) \bullet (T_{ci} - T_{co})$$

for stainless steel, $k_c = 0.22\,\frac{Watts}{cm\,{}^\circ C}$

$$\frac{k_c}{t_c} = \frac{0.22}{0.04} = 5.5\,\frac{Watts}{cm^2\,{}^\circ C}$$

$$\text{So,}\quad T_{ci} - T_{co} = \frac{550}{2\pi(0.315 - 0.020)\bullet 5.5} = 54^\circ C$$

$$\text{or,}\quad T_{ci} = 583 + 54 = 637\ ^\circ C$$

Temperature Across the Gap

q_F = Surface heat flux from the fuel

$$= \frac{P}{2\pi R} = \left(\frac{k_g}{t_g}\right)\bullet\left(T_s - T_{ci}\right)$$

from (10.63) $\displaystyle\int_{T_s}^{T_o} k\,dT = \frac{P}{4\pi}$

$$\text{or,}\quad \int_0^{T_o} k\,dT - \int_0^{T_s} k\,dT = \frac{P}{4\pi} = \frac{550}{4\pi} = 43.8\,\frac{W}{cm} \qquad 1)$$

For (He) gas conductivity use Eq. 10.102

$$k_g \approx 15.8 \times 10^{-6}\,(920)^{0.79} = 3.5 \times 10^{-3}\ W/cm^\circ C$$

Approximate gas T, °K

now,

$$T_s - T_{ci} = t_g \bullet \frac{P}{k_g 2\pi R}$$

$$= \frac{(0.007\,cm)}{(0.0035 \frac{W}{cm\,°C})} \bullet \frac{87.6 \frac{W}{cm}}{0.268\,cm}$$

Ts = 637 +654 = 1291 °C

w/o gap closure

Gap Closure

Fuel Thermal Expansion = $\Delta R/R = \alpha_F(\overline{T_f} - T_a)$

T_a = Fabrication T ≈ 25 °C

Clad Thermal Expansion = $\Delta R_c / R_c = \alpha_c (\overline{T_c} - T_a)$

Hot fuel Radius = $R_f^{hot} = R_f^{cold}[1+\alpha_f(\overline{T_f} - T_a)]$

Hot inner clad radius = $R_c^{hot} - t_c^{hot}$

gap width $t_g^{hot} = R_{ci}^{hot} - R_f^{hot}$

$t_g^{cold} = R_{ci}^{cold} - R_f^{cold}$

so,

$$t_g^{hot} - t_g^{cold} = R_{ci}^{cold}\alpha_c\left(\overline{T_c} - T_a\right) - R_f^{cold}\alpha_f\left(\overline{T_f} - T_a\right)$$

Inserting numbers

$$\overline{T_c} = \frac{(637 + 583)}{2} = 610°C$$

$\alpha_f \approx 1.2 \times 10^{-5}$ °C^{-1} (fig. 10.8) ≈1600°C (Pu-15%)

$\alpha_c \approx 1.8 \times 10^{-5}$ °C^{-1} (fig 10.9)

R_f^{cold}= 0.315-0.04-0.007 = 0.268 cm

R_{ci}^{cold} = 0.315 - 0.04 = 0.275 cm

$$t_g^{hot} - t_g^{cold} = 0.275 \bullet 1.8 \times 10^{-5}\,(610-25) - 0.268 \times 1.2 \times 10^{-5}\,(\overline{T_f} - 25)$$

$$t_g^{hot} - t_g^{cold} = 2.9 \times 10^{-3} - 3.22 \times 10^{-6}\,(\overline{T_f} - 25) \qquad 2)$$

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Solve for $\overline{T_f}$

- Use an iterative procedure, start with T_s= 1291°C w/o gap closure

From figure 10.20 $\int_0^{1291} k\,dT = 51$ W/cm

From Equation 1

$$\int_0^{T_o} k\,dT = 51 + 43.8 = 94.8\ \text{W/cm}$$

from figure 10.20

$$T_o \approx 2700\ °C$$

Since fuel has not restructured assume parabolic profile

$$\frac{(T - T_s)}{(T_0 - T_s)} = 1 - \left(\frac{r}{R}\right)^2$$

$$T = T_s + (T_0 - T_s)\left[1 - \left(\frac{r}{R}\right)^2\right] = T_0 - [T_0 - T_s]\left(\frac{r}{R}\right)^2$$

Then $$\overline{T_f} = \left(\frac{1}{\pi R^2}\right)\int_0^R \left\{T_0 - [T_0 - T_s]\left(\frac{r}{R}\right)^2\right\} 2\pi r\,dr$$

$$= T_0 - \frac{T_0 - T_s}{2} = \frac{T_0 + T_s}{2}$$

or,

$$\overline{T_f} = \frac{1291 + 2700}{2} = 1996\ °C$$

Hence, from eq. above

$$t_g^{hot} = t_g^{cold} + 2.9 \times 10^{-3} - 3.22 \times 10^{-6}(1996-25)$$

$$= 0.007 + 0.0029 - 0.00635$$
$$= 0.00355 \text{ cm}$$

First Iteration

$$\frac{k_g}{t_g} = \frac{0.0035}{0.00355} = 0.985 \text{ W/cm}^2 \text{°C}$$

therefore,

$$T_s - T_{ci} = \frac{t_g}{k_g} \bullet \left[\frac{P}{2\pi R}\right]$$

$$= \left(\frac{1}{0.985}\right)\left\{\left(\frac{550}{2\pi}\right) \bullet \frac{1}{0.268\left[1 + 0.000012(996 - 25)\right]}\right\}$$

R_f^{hot} (above)

$$= \left[\frac{550}{2\pi \bullet 0.985 \bullet 0.275}\right]$$

$$= 323 \text{ °C}$$

$$T_s = 637 + 323 = 960 \text{ °C}$$

Now go back to Figure 10.20

$$\int_{0}^{T_s=960} k\,dT = 41\ \text{W/cm}$$

$$\int_{0}^{T_0} k\,dT = 43.8 + 41 = 84.8\ \text{W/cm}$$

$$T_o = 2520\ ^{\circ}\text{C}$$

recalculate,

$$\overline{T_F} = \left(\frac{2520 + 960}{2}\right) = 1740^{\circ}\text{C}$$

and $t_g^{hot} = t_g^{cold} + 2.9 \times 10^{-3} - 3.22 \times 10^{-6}\,(1740 - 25)$

$$= 4.525 \times 10^{-3}\ \text{cm}$$

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Repeat this until convergence occurs

$$T_o = 2530\ ^{\circ}\text{C}$$

$$T_S = 1050\ ^{\circ}$$

Problem 10.6

Slab fuel element between two thin plates of cladding

- ***Initial porosity*** $= P_o$
- ***Power density before restructuring is*** H_o

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a.) Find the value of the heat generation rate such that the center temperature just equals the melting temperature. Assume,

$$k_0 = k_s\left(1 - P^{\frac{2}{3}}\right)$$

using

$$k_0\left(\frac{d^2T}{dx^2}\right) + H_0 = 0$$

or,

$$\frac{dT}{dx} = -\left(\frac{H_o}{k_o}\right)x + A \quad \text{but } \frac{dT}{dx} = 0$$

@ x=0, so A=0

then,

$$T = -\left(\frac{H_o}{2k_o}\right)x^2 + B \qquad @\ x = L,\ T = T_S$$

$$\text{so } \quad B = T_s + \frac{H_o}{2k_o}L^2$$

this gives;

$$T - T_s = \left(\frac{H_0}{2k_0}\right)L^2\left[1 - \left(\frac{x}{L}\right)^2\right]$$

but at x=0, T(0) = T_m

then,

$$H_o = 2k_o\frac{(T_m - T_s)}{L^2}$$

or,

$$H_0 = \frac{2k_s\left(1 - P_0^{\frac{2}{3}}\right)(T_m - T_s)}{L^2}$$

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b.) After restructuring, fuel is converted to a columnar grain structure with a new porosity, P_1

- ***What is the size of the central void, x_o ?***
- ***What is the new power density, H_1, if the temperature at x_o = T_m ?***

$$k_1 = k_s\left(1 - P_1^{\frac{2}{3}}\right)$$

Hole size; $\rho_o L = \rho_1 (L - x_o)$

$$\frac{x_o}{L} = 1 - \frac{\rho_o}{\rho_1}$$

$$\frac{x_o}{L} = 1 - \frac{(1 - P_o)}{(1 - P_1)}$$

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$$\frac{dT}{dx} = -\frac{H_1}{k_1} x + A$$

$$T = -\frac{H_1}{2k_1} x^2 + Ax + B$$

now $\frac{dT}{dx} = 0$ **@** $x = x_o$

so, $A = \frac{H_1 x_o}{k_1}$

and @ $x = L$, $T = T_s$

$$B = T_s + \frac{H_1 L^2}{2k_1} - \frac{H_1 x_o L}{k_1}$$

and $T = T_m$ **@** $x = x_o$

substituting;

$$T_m - T_s = \left(\frac{H_1 x_0^2}{2k_1}\right) + \left(\frac{H_1 L^2}{2k_1}\right) - \left(\frac{H_1 L x_o}{k_1}\right)$$

Solving for H_1

$$H_1 = \frac{T_m - T_s}{\frac{L^2}{2k_1}\left[1 - \left(\frac{2x_0}{L}\right) + \left(\frac{x_0}{L}\right)^2\right]}$$

$$H_1 = \frac{2k_1\left(T_m - T_s\right)}{L^2\left[1 - \left(\frac{x_0}{L}\right)\right]^2}$$

but $\frac{\rho_o}{\rho_1} = \left(1 - \frac{x_o}{L}\right)$ **or,**

$$H_1 = \left(\frac{2k_0\left[T_m - T_s\right]}{L^2}\right)\left(\frac{k_1}{k_0}\right)\left(\frac{L^2}{L^2\left[1 - \left(\frac{x_0}{L}\right)\right]^2}\right)$$

finally,

$$H_1 = H_0\left(\frac{1 - P_1^{2/3}}{1 - P_0^{2/3}}\right)\left[\frac{1 - P_1}{1 - P_0}\right]^2$$

If $P_1 < P_o$ **then** $H_1 > H_o$